1.5 Solving Quadratic Equations

Part 2: Solve Using QF

Lesson Outline:

Part 1: Do It Now - QF Refresher

Part 2: Discriminant review

Part 3: Find exact solutions of a quadratic with 2 roots

Part 4: Solve a quadratic with 1 solution

Part 5: Solve a quadratic with 0 solutions

Part 6: Use the discriminant to determine the number of solutions (x-intercepts) a quadratic has

Part 7: Application

DO IT NOW!

a) Do you remember the quadratic formula?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

b) Use the quadratic formula to find the x-intercepts of:

$$0 = \overset{a}{2}x^2 + \overset{b}{7}x - \overset{c}{4}$$

Don't forget that to solve a quadratic, it must be set equal to zero because at an x-intercept, the y-coordinate will be zero.

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(2)(-4)}}{2(2)}$$

$$x = \frac{-7 \pm \sqrt{49 + 32}}{4}$$

$$x = \frac{-7 \pm \sqrt{81}}{4}$$

$$x = \frac{-7+9}{4} \quad \text{or} \quad x = \frac{-7-9}{4}$$

$$= \frac{1}{2} \qquad\qquad = -4$$

Part 2: Discriminant Review

Do all parabolas have two x-intercepts?

NO

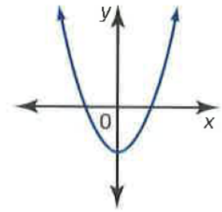
What are the three different scenarios?

0, 1, or 2 solutions

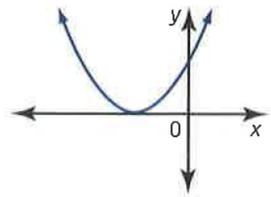
The way to determine how many x-intercepts a parabola might have is by evaluating the $b^2 - 4ac$ part of the quadratic formula (called the "**discriminant**")

Discriminant: the value under the square root

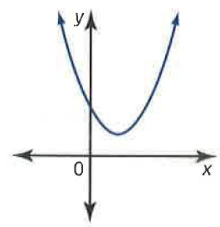
If $b^2 - 4ac > 0$, there are two solutions



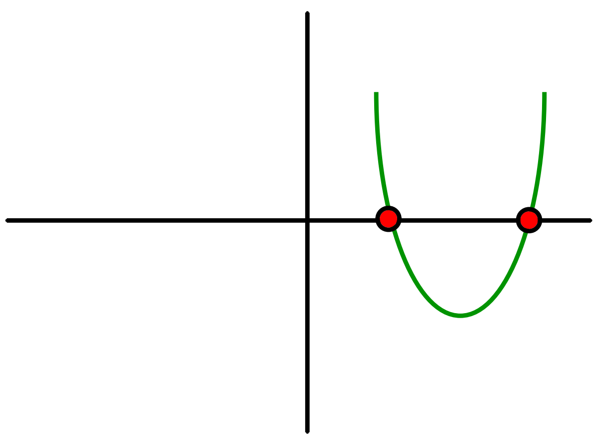
If $b^2 - 4ac = 0$, there is one solution



If $b^2 - 4ac < 0$, there are no solutions



Part 3: Solve a Quadratic With 2 Roots



Objective: Determine the roots of a quadratic using the quadratic formula and leave as EXACT answers

<u>Exact answer:</u> as a radical or fraction. Exact answers do not have decimals.

Example 1: Find the exact solutions of

$$3x^2 - 10x + 5 = 0$$

$$x = \frac{10 \pm \sqrt{(-10)^2 - 4(3)(5)}}{2(3)}$$

$$= \frac{10 \pm \sqrt{40}}{6}$$ (simplify the radical)

$$= \frac{10 \pm \sqrt{4}(\sqrt{10})}{6}$$

$$= \frac{10 \pm 2\sqrt{10}}{6}$$ (common factor)

$$= \frac{{}^{1}\cancel{2}(5 \pm \sqrt{10})}{3\cancel{6}}$$ (reduce)

$$= \frac{5 \pm \sqrt{10}}{3}$$

$$x = \frac{5 + \sqrt{10}}{3} \quad \text{or} \quad x = \frac{5 - \sqrt{10}}{3}$$

Example 2: Find the exact solutions of

$$-2x^2 + 8x - 5 = 0$$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(-2)(-5)}}{2(-2)}$$

$$= \frac{-8 \pm \sqrt{24}}{-4}$$ (simplify the radical)

$$= \frac{-8 \pm \sqrt{4}(\sqrt{6})}{-4}$$

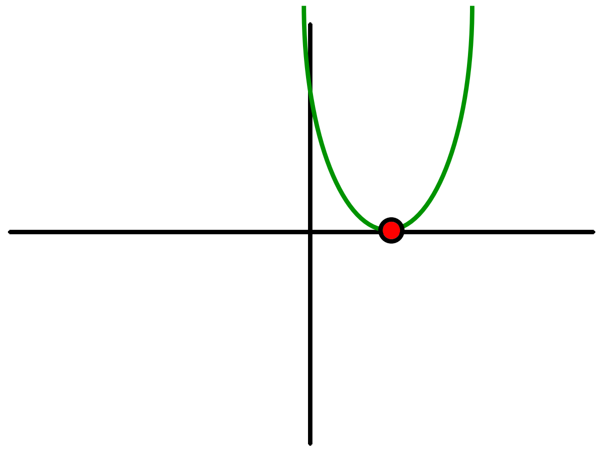
$$= \frac{-8 \pm 2\sqrt{6}}{-4}$$ (common factor)

$$= \frac{{}^{1}\cancel{-2}(4 \mp \sqrt{6})}{2\,\cancel{-4}}$$ (reduce)

$$= \frac{4 \mp \sqrt{6}}{2}$$

$$x = \frac{4 - \sqrt{6}}{2} \quad \text{or} \quad x = \frac{4 + \sqrt{6}}{2}$$

Part 4: Solving a Quadratic With 1 Root



Note: *when a quadratic only has 1 solution, the x-intercept is also the vertex*

Example 3: Find the exact roots of

$$4x^2 + 24x + 36 = 0$$

$4(x^2+6x+9)=0$ (common factor)

$x^2+6x+9=0$

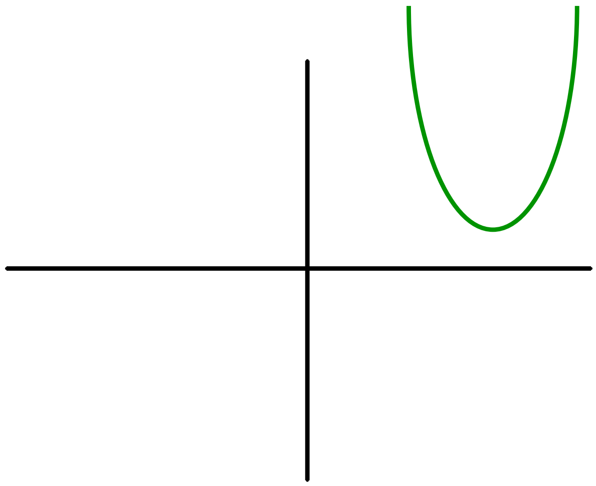
$$x = \frac{-6 \pm \sqrt{(6)^2-4(1)(9)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{0}}{2}$$

$$x = \frac{-6}{2}$$

$$x = -3$$

Part 5: Solving Quadratics With 0 Roots



2 Scenarios causing 0 roots:
i) vertex is above the x-axis and opens up
ii) vertex is below the x-axis and opens down

Example 4: Find the x-intercepts of

$$8x^2 - 11x + 5 = 0$$

$$x = \frac{11 \pm \sqrt{(-11)^2 - 4(8)(5)}}{2(8)}$$

$$x = \frac{11 \pm \sqrt{-39}}{16}$$

$\therefore$ no solutions

Part 6: Use the Discriminant to Determine the Number of Roots

Example 5: For each of the following quadratics, use the discriminant to state the number of roots it will have.

a) $\overset{a}{2}x^2 + \overset{b}{5}x - \overset{c}{5} = 0$

$b^2-4ac = 5^2-4(2)(-5)$
$= 25+40$
$= 60$
$60>0$; ∴ 2 solutions

b) $\overset{a}{3}x^2 - \overset{b}{7}x + \overset{c}{5} = 0$

$b^2-4ac = (-7)^2-4(3)(5)$
$= 49-60$
$= -11$
$-11<0$; ∴ no solutions

c) $-4x^2 + 12x - 9 = 0$

$b^2-4ac = (12)^2-4(-4)(-9)$
$= 144-144$
$= 0$
$0=0$; ∴ 1 solution

Part 7: Application

Example 6: A ball is thrown and the equation below model it's path:

$$h = -0.25d^2 + 2d + 1.5$$

'*h*' is the height in meters above the ground and '*d*' is the horizontal distance in meters from the person who threw the ball.

a) At what height was the ball thrown from? solve for 'h' when d=0

$h = -0.25(0)^2 + 2(0) + 1.5$
$= 1.5$ meters

b) How far has the ball travelled horizontally when it lands on the ground?

$$0 = -0.25d^2 + 2d + 1.5$$

$$0 = -0.25(d^2 - 8d - 6)$$

$$0 = d^2 - 8d - 6$$

$$d = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{88}}{2}$$

$$d = \frac{8 + \sqrt{88}}{2} \quad \text{or} \quad d = \frac{8 - \sqrt{88}}{2}$$

$$= 8.7 \text{ meters} \qquad = -0.7 \text{ meters}$$

reject