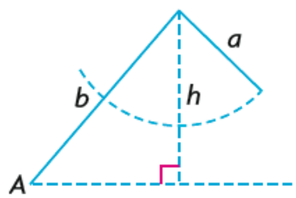
L6 – Ambiguous Case of Sine
MCR3U
Jensen

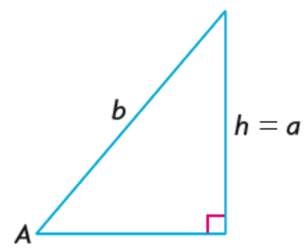
If you are given **2 sides and an opposite angle** in a triangle, there are many scenarios to consider.

If $\angle A$, a, and b are given and $\angle A$ is acute, there are 4 scenarios to consider:

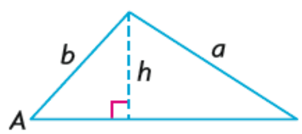
If $\angle A$ is acute and $a < h$, no triangle exists.



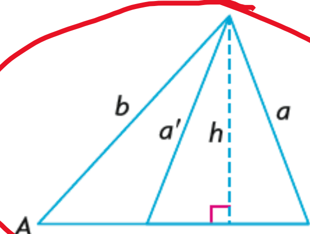
If $\angle A$ is acute and $a = h$, one right triangle exists.



If $\angle A$ is acute and $a > b$, one triangle exists.

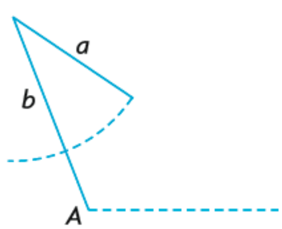


If $\angle A$ is acute and $h < a < b$, two triangles exist.

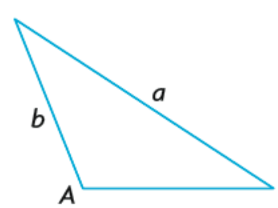


If $\angle A$, a, and b are given and $\angle A$ is obtuse, there are 2 scenarios to consider:

If $\angle A$ is obtuse and $a < b$ or $a = b$, no triangle exists.



If $\angle A$ is obtuse and $a > b$, one triangle exists.



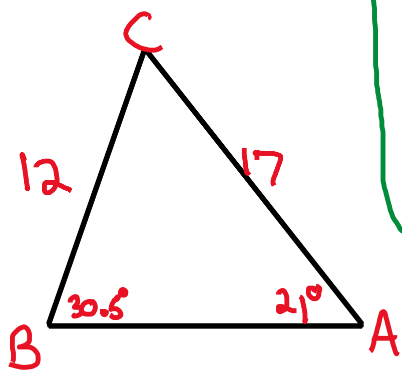
The ambiguous case only occurs when two possible triangles exist for the same given information. This means, the ambiguous case must be considered if $\angle A$ is acute and $h < a < b$.

Test: $\sin A = \frac{h}{b}$

$h = b\sin A$

Example 1: In triangle ABC, side a = 12 cm, side b = 17 cm, and A = 21°. Find the measure of angle B.

CASE 1



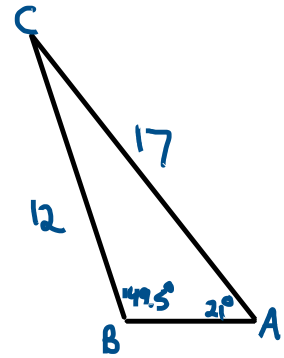
test: $\sin 21 = \frac{h}{17}$

$h = 17\sin 21$

$h \approx 6.1$

$h < a < b$

CASE 2



$\frac{12}{\sin 21} = \frac{17}{\sin B}$

$\sin B = \frac{17\sin 21}{12}$

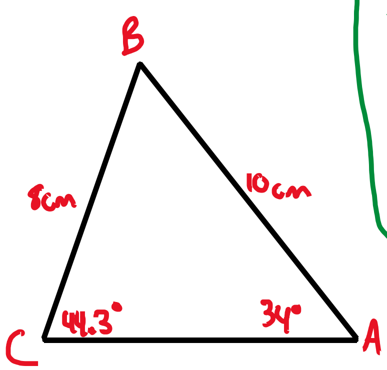
$\angle B = \sin^{-1}\left(\frac{17\sin 21}{12}\right) \approx 30.5°$

$\angle B = 180° -$ case 1

$= 180° - 30.5°$

$\approx 149.5°$

Example 2: In triangle ABC, side a = 8 cm, side c = 10 cm, and A = 34°. Find angle C.

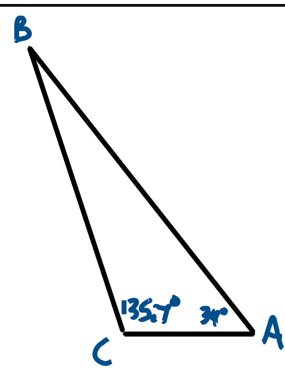


test: $\sin 34° = \frac{h}{10}$

$h = 10\sin 34°$

$h \approx 5.6$cm

$h < a < c$



$\frac{10}{\sin C} = \frac{8}{\sin 34°}$

$\sin C = \frac{10\sin 34°}{8}$

$\angle C = \sin^{-1}\left(\frac{10\sin 34°}{8}\right)$

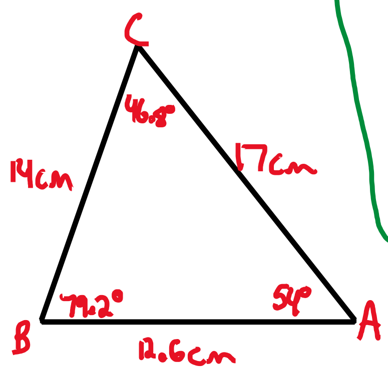
$\angle C \approx 44.3°$

$\angle C = 180° -$ case 1

$\angle C = 180° - 44.3°$

$\angle C = 135.7°$

Example 3: In triangle ABC, side a = 14 cm, side b = 17 cm, and A = 54°. Find the measure of all missing sides and angles.



test: $\sin 54 = \frac{h}{17}$

$h = 17 \sin 54°$

$h \approx 13.8cm$

$h < a < b$

$\frac{14}{\sin 54} = \frac{17}{\sin B}$

$\sin B = \frac{17 \sin 54}{14}$

$\angle B = \sin^{-1}\left(\frac{17 \sin 54}{14}\right)$

$\boxed{\angle B \approx 79.2°}$

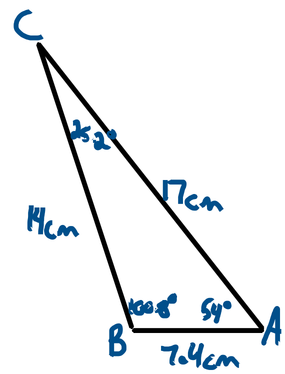
$\angle C = 180 - 79.2 - 54$

$\boxed{\angle C = 46.8°}$

$c^2 = 14^2 + 17^2 - 2(14)(17)(\cos 46.8)$

$c^2 \approx 159.1555776$

$\boxed{c \approx 12.6cm}$



$\angle B = 180° - \text{case 1}$

$\angle B = 180° - 79.2°$

$\boxed{\angle B = 100.8°}$

$\angle C = 180 - 100.8 - 54$

$\boxed{\angle C = 25.2°}$

$c^2 = 14^2 + 17^2 - 2(14)(17)(\cos 25.2)$

$c^2 \approx 54.30232303$

$\boxed{c \approx 7.4cm}$