L7 – Trig Identities
MCR3U
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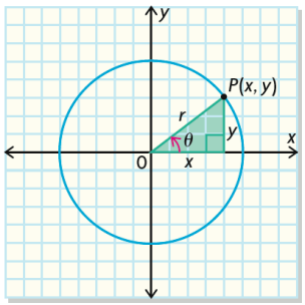
_____**:** A mathematical equation that is true for ALL values of the given variables.

Part 1: Proving the Pythagorean and Quotient Identities

For this part you will need to remember that trig ratios can be written in terms of x and y



Example 1: Prove the quotient identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Example 2: Prove the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$

Fundamental Trigonometric Identities		
Reciprocal Identities	**Quotient Identities**	**Pythagorean Identities**
$\boldsymbol{csc\,\theta = \frac{1}{sin\,\theta}}$ $\boldsymbol{sec\,\theta = \frac{1}{cos\,\theta}}$ $\boldsymbol{cot\,\theta = \frac{1}{tan\,\theta}}$	$\boldsymbol{\frac{sin\,\theta}{cos\,\theta} = tan\,\theta}$ $\boldsymbol{\frac{\cos\theta}{\sin\theta} = \cot\theta}$	$\boldsymbol{sin^2\theta + cos^2\theta = 1}$

Tips and Tricks		
Reciprocal Identities	**Quotient Identities**	**Pythagorean Identities**
Square both sides $csc^2\,\theta = \frac{1}{sin^2\,\theta}$ $sec^2\,\theta = \frac{1}{cos^2\,\theta}$ $cot^2\,\theta = \frac{1}{tan^2\,\theta}$	Square both sides $\frac{sin^2\,\theta}{cos^2\,\theta} = tan^2\,\theta$ $\frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$	Rearrange the identity $sin^2\theta = 1 - cos^2\theta$ $cos^2\theta = 1 - sin^2\theta$

General tips for proving identities:

- **i)** Try to change everything to $sin\,\theta$ or $cos\,\theta$
- **ii)** If you have to fractions being added or subtracted, find a common denominator and combine the fractions
- **iii)** Use difference of squares → $1 - sin^2\,\theta = (1 - sin\,\theta)(1 + sin\,\theta)$
- **iv)** Use the power rule → $sin^{\,6}\,\theta = (sin^2\,\theta)^3$

We will use the preceding identities to help us prove more complex identities in the following examples.

Example 3: Prove each of the following identities

a) $\frac{\cos\theta \tan\theta}{\sin\theta} = 1$

b) $\tan^2\theta + 1 = \sec^2\theta$

c) $\cos^2 x = (1 - \sin x)(1 + \sin x)$

d) $\frac{\sin^2 x}{1-\cos x} = 1 + \cos x$

e) $\sin\theta \sec\theta \cot\theta = 1$

f) $\frac{1}{1-\sin x} - \frac{1}{1+\sin x} = \frac{2\tan x}{\cos x}$

g) $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$

h) $\tan x + \frac{\cos x}{1+\sin x} = \sec x$