L7 – Trig Identities
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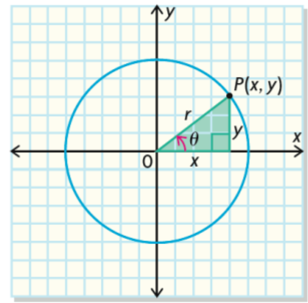
Identity: A mathematical equation that is true for ALL values of the given variables.

Part 1: Proving the Pythagorean and Quotient Identities

For this part you will need to remember that trig ratios can be written in terms of x and y



Example 1: Prove the quotient identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$

LS

$= \tan\theta$

$= \frac{y}{x}$

RS

$= \frac{\sin\theta}{\cos\theta}$

$= \frac{\left(\frac{y}{r}\right)}{\left(\frac{x}{r}\right)}$

$= \left(\frac{y}{r}\right)\left(\frac{r}{x}\right)$

$= \frac{y}{x}$

LS = RS

Example 2: Prove the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$

LS

$= \sin^2\theta + \cos^2\theta$

$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$

$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$

$= \frac{y^2+x^2}{r^2}$

$= \frac{r^2}{r^2}$

$= 1$

RS

$= 1$

LS = RS

Fundamental Trigonometric Identities		
Reciprocal Identities	**Quotient Identities**	**Pythagorean Identities**
$\boldsymbol{csc\ \theta = \frac{1}{sin\ \theta}}$ $\boldsymbol{sec\ \theta = \frac{1}{cos\ \theta}}$ $\boldsymbol{cot\ \theta = \frac{1}{tan\ \theta}}$	$\boldsymbol{\frac{sin\ \theta}{cos\ \theta} = tan\ \theta}$ $\boldsymbol{\frac{\cos\theta}{\sin\theta} = \cot\theta}$	$\boldsymbol{sin^2\theta + cos^2\theta = 1}$

Tips and Tricks		
Reciprocal Identities	**Quotient Identities**	**Pythagorean Identities**
Square both sides $csc^2\ \theta = \frac{1}{sin^2\ \theta}$ $sec^2\ \theta = \frac{1}{cos^2\ \theta}$ $cot^2\ \theta = \frac{1}{tan^2\ \theta}$	Square both sides $\frac{sin^2\ \theta}{cos^2\ \theta} = tan^2\ \theta$ $\frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$	Rearrange the identity $sin^2\theta = 1 - cos^2\theta$ $cos^2\theta = 1 - sin^2\theta$
General tips for proving identities: **i)** Try to change everything to $sin\ \theta$ or $cos\ \theta$ **ii)** If you have to fractions being added or subtracted, find a common denominator and combine the fractions **iii)** Use difference of squares → $1 - sin^2\ \theta = (1 - sin\ \theta)(1 + sin\ \theta)$ **iv)** Use the power rule → $sin\ ^6\ \theta = (sin^2\ \theta)^3$		

We will use the preceding identities to help us prove more complex identities in the following examples.

Example 3: Prove each of the following identities

a) $\frac{\cos\theta\tan\theta}{\sin\theta} = 1$

LS

$= \frac{\cos\theta\tan\theta}{\sin\theta}$

$= \frac{\cancel{\cos\theta}\left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)}{\sin\theta}$

$= \frac{\sin\theta}{\sin\theta}$

$= 1$

RS

$= 1$

LS=RS

b) $\tan^2\theta + 1 = \sec^2\theta$

LS

$= \tan^2\theta + 1$

$= \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}$

$= \frac{1}{\cos^2\theta}$

RS

$= \sec^2\theta$

$= \frac{1}{\cos^2\theta}$

LS=RS

c) $\cos^2 x = (1 - \sin x)(1 + \sin x)$

LS

$= \cos^2\theta$

RS

$= (1 - \sin x)(1 + \sin x)$

$= 1 - \sin^2 x$

$= \cos^2 x$

LS=RS

d) $\frac{\sin^2 x}{1-\cos x} = 1 + \cos x$

LS

$= \frac{\sin^2 x}{1-\cos x}$

$= \frac{1-\cos^2 x}{1-\cos x}$

$= \frac{(\cancel{1-\cos x})(1+\cos x)}{\cancel{1-\cos x}}$

$= 1+\cos x$

RS

$= 1+\cos x$

LS=RS

e) $\sin\theta \sec\theta \cot\theta = 1$

LS

$= \sin\theta \sec\theta \cot\theta$

$= \sin\theta \left(\frac{1}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right)$

$= \frac{\sin\theta \cos\theta}{\sin\theta\cos\theta}$

$= 1$

RS

$= 1$

LS=RS

f) $\frac{1}{1-\sin x} - \frac{1}{1+\sin x} = \frac{2\tan x}{\cos x}$

LS

$= \frac{(1+\sin x)1}{(1+\sin x)1-\sin x} - \frac{1\,(1-\sin x)}{1+\sin x\,(1-\sin x)}$

$= \frac{1+\sin x-(1-\sin x)}{(1+\sin x)(1-\sin x)}$

$= \frac{1+\sin x-1+\sin x}{1-\sin^2 x}$

$= \frac{2\sin x}{\cos^2 x}$

RS

$= \frac{2\tan x}{\cos x}$

$= \frac{2\left(\frac{\sin x}{\cos x}\right)}{\cos x}$

$= \frac{2\sin x}{\cos x} \cdot \frac{1}{\cos x}$

$= \frac{2\sin x}{\cos^2 x}$

LS=RS

g) $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$

LS

$= (\sin x + \cos x)^2 + (\sin x \cdot \cos x)^2$

$= \sin^2 x + 2\sin x \cos x + \cos^2 x + \sin^2 x - 2\sin x \cos x + \cos^2 x$

$= 2\sin^2 x + 2\cos^2 x$

$= 2(\sin^2 x + \cos^2 x)$

$= 2(1)$

$= 2$

RS

$= 2$

LS = RS

h) $\tan x + \frac{\cos x}{1+\sin x} = \sec x$

LS

$= \frac{(1+\sin x)\sin x}{(1+\sin x)\cos x} + \frac{\cos x\,(\cos x)}{(1+\sin x)(\cos x)}$

$= \frac{\sin x(1+\sin x) + \cos x(\cos x)}{(1+\sin x)(\cos x)}$

$= \frac{\sin x + \sin^2 x + \cos^2 x}{(1+\sin x)(\cos x)}$

$= \frac{\cancel{\sin x + 1}}{\cancel{(1+\sin x)}(\cos x)}$

$= \frac{1}{\cos x}$

RS

$= \sec x$

$= \frac{1}{\cos x}$

LS = RS