1.7 Solve Linear-Quadratic Systems

Lesson Outline:

Part 1: Do It Now - review of substitution

Part 2: Possible solutions for a lin-quad system

Part 3: Solve linear-quadratic systems

Part 4: Application

DO IT NOW!

Solve the following linear system using the method of substitution:

① $y = 3x + 7$

② $y = 2x - 5$

$3x+7 = 2x-5$

$3x-2x = -5-7$

$x = -12$

sub x-value back in to ① or ② and solve for y.

$y = 3x+7$

$y = 3(-12)+7$

$y = -29$

$\therefore$ the POI is $(-12, -29)$

Recall: *solving a linear system means to find the point of intersection (POI)*

Method of Substitution: *solving a linear system by substituting for one variable from one equation into the other equation.*

Steps to Solving A Linear-Quadratic System

1. Set equations equal to each-other

Line = Parabola

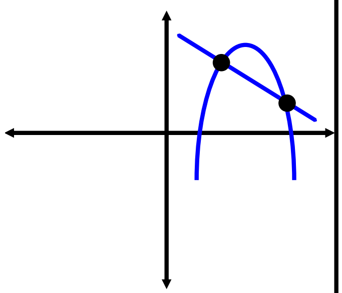
2. Rearrange to set the equation equal to zero

3. Solve for *x* by factoring or using the QF (the solution will tell you for what value of *x* the functions have the same *y* value)

4. Plug this value of *x* back in to either of the original functions to solve for *y*.

Possible solutions for a linear-quadratic system:

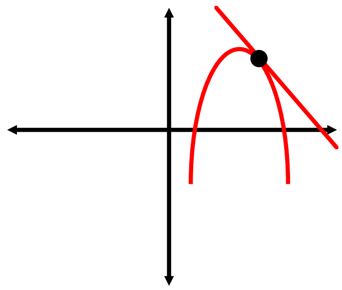
2 intersections



Secant: A line that intersects a curve at two distinct points.

discriminant > 0

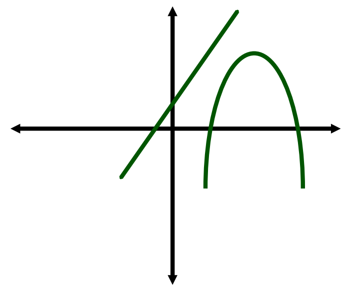
1 intersection



Tangent line: A line that touches a curve at one point and has the slope of the curve at that point.

discriminant = 0

0 intersections



Example 1

a) How many points of intersection are there for the following system of equations?

$$f(x) = \frac{1}{2}x^2 + 2x - 8 \qquad g(x) = 4x - 10$$

set $f(x) = g(x)$

$\frac{1}{2}x^2 + 2x - 8 = 4x - 10$ (set equal to each other)

$\frac{1}{2}x^2 + 2x - 4x - 8 + 10 = 0$ (set equal to zero)

$\frac{1}{2}x^2 - 2x + 2 = 0$

(common factor)

$\frac{1}{2}(x^2 - 4x + 4) = 0$

$x^2 - 4x + 4 = 0$

$b^2 - 4ac = (-4)^2 - 4(1)(4)$ (check discriminant)

$= 0$

$\therefore$ 1 solution

b) Solve the linear-quadratic system (give exact answers)

$x^2 - 4x + 4 = 0$

solve by Factoring.
(hint: it is a perfect square trinomial)

$(x-2)^2 = 0$

$x = 2$

Plug $x = 2$ back in to either original equation

(linear is usually easier)

$g(x) = 4x - 10$

$g(2) = 4(2) - 10$

$g(2) = -2$

$\therefore$ The POI is $(2, -2)$

Example 2

Solve the following linear quadratic system

$y = 3x^2 + 21x - 5$

$y = 10x - 1$

$3x^2 + 21x - 5 = 10x - 1$

$3x^2 + 11x - 4 = 0$

-12, 11; $\frac{4}{1}$ $\cancel{\frac{12}{3}}$, $\frac{-1}{3}$

$(x+4)(3x-1) = 0$

$x + 4 = 0 \qquad 3x - 1 = 0$

$x_1 = -4 \qquad x_2 = \frac{1}{3}$

POI #1

$y = 10(-4) - 1$

$y = -41$

$(-4, -41)$

POI #2

$y = 10\left(\frac{1}{3}\right) - 1$

$y = \frac{10}{3} - \frac{3}{3}$

$y = \frac{7}{3}$

$\left(\frac{1}{3}, \frac{7}{3}\right)$

Part 4: Application

Example 3: If a line with slope 4 has one point of intersection with the quadratic function $y = \frac{1}{2}x^2 + 2x - 8$, what is the y-intercept of the line? Write the equation of the line in slope y-intercept form.

$y = 4x + k$

Recall: equation of a line is $y = mx + k$ where k is the y-intercept and m is the slope.

Recall: *for a lin-quad system to have 1 solution, the discriminant must be zero.*

$4x + k = \frac{1}{2}x^2 + 2x - 8$

$0 = \frac{1}{2}x^2 - 2x - 8 - k$

Then $a = \frac{1}{2}$ $b = -2$ and $c = -8 - k$

$b^2 - 4ac = 0$

$(-2)^2 - 4(\frac{1}{2})(-8-k) = 0$

$4 - 2(-8-k) = 0$

$4 + 16 + 2k = 0$

$2k = -20$

$k = -10$

∴ The equation of the line must be $y = 4x - 10$