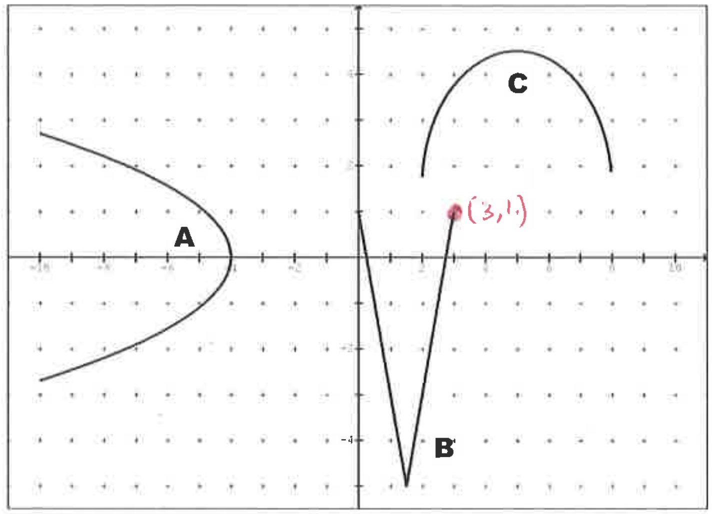
SOLUTIONS

MCR3U EXAM REVIEW

Chapter 1 – Functions

1) Use the graph to answer the following questions



a) List which graphs above are the graphs of functions, and which are not.

Functions: B and C

Not a function: A

b) Describe how you can tell whether a given graph is the graph of a function.

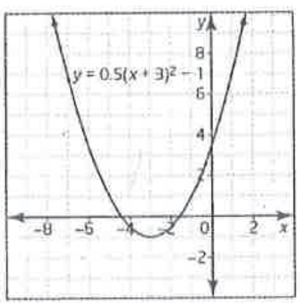
Functions pass the vertical line test; each value of x corresponds to only 1 value of y.

c) For graph **B**, if $y = f(x)$, what is the value of $f(3)$?

$f(3) = 1$

2) State the domain and range for each relation. Determine if each relation is a function.

a)



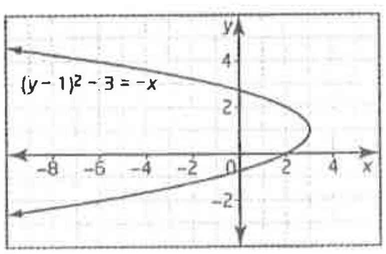
Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid y \geq -1\}$

Is the relation a function?

YES

b)



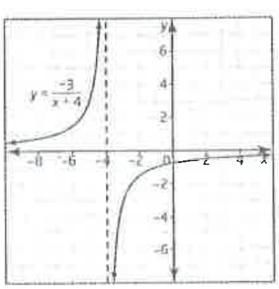
Domain: $\{x \in \mathbb{R} \mid x \leq 3\}$

Range: $\{y \in \mathbb{R}\}$

Is the relation a function?

NO

c)



Domain: $\{x \in \mathbb{R} \mid x \neq -4\}$

Range: $\{y \in \mathbb{R} \mid y \neq 0\}$

Is the relation a function?

YES

d) {(-6, 2), (-5, 2), (-4, 2), (-3, 2)}

Domain: $\{x = -6, -5, -4, -3\}$

Range: $\{y = 2\}$

Is the relation a function?

YES

3) Suppose $f(x) = -2x^2 + 6$, find each of the following...

a) $f(5)$ $= -2(5)^2 + 6$
$= -2(25) + 6$
$= -50 + 6$
$= -44$

b) $f(0)$ $= -2(0)^2 + 6$
$= 0 + 6$
$= 6$

c) $f\left(\frac{3}{4}\right)$ $= -2\left(\frac{3}{4}\right)^2 + 6$
$= -2\left(\frac{9}{16}\right) + 6$
$= -\frac{9}{8} + \frac{48}{8}$
$= \frac{39}{8}$

4) Determine the vertex of the quadratic function $f(x) = x^2 + 4x + 1$ by completing the square. Verify your answer using partial factoring. Then state if the vertex is a max or min point.

Completing the Square

$f(x) = (x^2 + 4x) + 1$

$f(x) = (x^2 + 4x + 4 - 4) + 1$

$f(x) = (x^2 + 4x + 4) - 4 + 1$

$f(x) = (x + 2)^2 - 3$

Partial Factoring

$1 = x^2 + 4x + 1$

$0 = x^2 + 4x$

$0 = x(x + 4)$

$x_1 = 0$ $\quad x + 4 = 0$

$x_2 = -4$

$x\text{-vertex} = \frac{0 + (-4)}{2}$
$= -2$

$y\text{-vertex} = (-2)^2 + 4(-2) + 1$
$= 4 - 8 + 1$
$= -3$

Vertex: $(-2, -3)$

Max or Min? Min

5) Determine the vertex of the quadratic function $f(x) = -2x^2 + 12x + 7$ by completing the square. Verify your answer using partial factoring. Then state if the vertex is a max or min point.

Completing the Square

$f(x) = (-2x^2 + 12x) + 7$

$f(x) = -2(x^2 - 6x) + 7$

$f(x) = -2(x^2 - 6x + 9 - 9) + 7$

$f(x) = -2(x^2 - 6x + 9) + 18 + 7$

$f(x) = -2(x-3)^2 + 25$

Partial Factoring

$7 = -2x^2 + 12x + 7$

$0 = -2x^2 + 12x$

$0 = -2x(x-6)$

$-2x = 0$ $\quad$ $x - 6 = 0$

$x_1 = 0$ $\quad$ $x_2 = 6$

$x\text{-vertex} = \frac{0+6}{2}$

$= 3$

$y\text{-vertex} = -2(3)^2 + 12(3) + 7$

$= -18 + 36 + 7$

$= 25$

Vertex: $(3, 25)$

Max or Min? Max

6) Determine the vertex of the quadratic function $f(x) = -\frac{1}{2}x^2 - 4x - 3$ by completing the square. Verify your answer using partial factoring. Then state if the vertex is a max or min point.

Completing the Square

$f(x) = (-\frac{1}{2}x^2 - 4x) - 3$

$f(x) = -\frac{1}{2}(x^2 + 8x) - 3$

$f(x) = -\frac{1}{2}(x^2 + 8x + 16 - 16) - 3$

$f(x) = -\frac{1}{2}(x^2 + 8x + 16) + 8 - 3$

$f(x) = -\frac{1}{2}(x+4)^2 + 5$

Partial Factoring

$-3 = -\frac{1}{2}x^2 - 4x - 3$

$0 = -\frac{1}{2}x^2 - 4x$

$0 = -\frac{1}{2}x(x+8)$

$-\frac{1}{2}x = 0$ $\quad$ $x + 8 = 0$

$x_1 = 0$ $\quad$ $x_2 = -8$

$x\text{-vert} = \frac{0 + (-8)}{2}$

$= -4$

$y\text{-vert} = -\frac{1}{2}(-4)^2 - 4(-4) - 3$

$= -8 + 16 - 3$

$= 5$

Vertex: $(-4, 5)$

Max or Min? MAX

7) The student council is organizing a trip to a rock concert. All proceeds from ticket sales will be donated to charity. Tickets to the concert cost \$31.25 per person if a minimum of 104 people attend. For every 8 extra people that attend, the price will decrease by \$1.25 per person.

a) How many tickets need to be sold to maximize the donation to charity?

Revenue = (cost)(#sold)

$R = (31.25 - 1.25x)(104 + 8x)$

* find x-vertex by averaging x-intercepts *

$0 = (31.25 - 1.25x)(104 + 8x)$

$31.25 - 1.25x = 0$ $104 + 8x = 0$

$x_1 = 25$ $x_2 = -13$

$x\text{-vertex} = \frac{25 + (-13)}{2} = 6$

Number sold to maximize revenue:

$\# \text{ sold} = 104 + 8x = 104 + 8(6) = 152$

$\boxed{152 \text{ tickets}}$

b) What is the price of each ticket that maximizes the donation?

$\text{cost} = 31.25 - 1.25(6) = 23.75$

$\boxed{\$23.75 \text{ per ticket}}$

c) What is the maximum donation?

$R = (\text{cost})(\# \text{ sold})$

$R = (23.75)(152)$

$R = \$3610$

$\boxed{\$3610}$

8) Simplify each of the following expressions involving radicals as much as possible

a) $\sqrt{54}$

$= (\sqrt{9})(\sqrt{6})$

$= 3\sqrt{6}$

b) $\sqrt{84}$

$= (\sqrt{4})(\sqrt{21})$

$= 2\sqrt{21}$

c) $2(7\sqrt{3})$

$= 14\sqrt{3}$

d) $-3\sqrt{3}(5\sqrt{2})$

$= -15\sqrt{6}$

e) $5\sqrt{12} - 2\sqrt{48} - 7\sqrt{75}$

$= 5(\sqrt{4})(\sqrt{3}) - 2(\sqrt{16})(\sqrt{3}) - 7(\sqrt{25})(\sqrt{3})$

$= 5(2)(\sqrt{3}) - 2(4)(\sqrt{3}) - 7(5)(\sqrt{3})$

$= 10\sqrt{3} - 8\sqrt{3} - 35\sqrt{3}$

$= -33\sqrt{3}$

f) $2\sqrt{12} + 4\sqrt{20} - 3\sqrt{27} - 5\sqrt{45}$

$= 2(\sqrt{4})(\sqrt{3}) + 4(\sqrt{4})(\sqrt{5}) - 3(\sqrt{9})(\sqrt{3}) - 5(\sqrt{9})(\sqrt{5})$

$= 4\sqrt{3} + 8\sqrt{5} - 9\sqrt{3} - 15\sqrt{5}$

$= -5\sqrt{3} - 7\sqrt{5}$

g) $6\sqrt{6}(3\sqrt{2}-4\sqrt{3})$

$= 18\sqrt{12} - 24\sqrt{18}$

$= 18(\sqrt{4})(\sqrt{3}) - 24(\sqrt{9})(\sqrt{2})$

$= 18(2)(\sqrt{3}) - 24(3)(\sqrt{2})$

$= 36\sqrt{3} - 72\sqrt{2}$

h) $(\sqrt{7}-6)(\sqrt{7}+1)$

$= \sqrt{49} + \sqrt{7} - 6\sqrt{7} - 6$

$= 7 - 5\sqrt{7} - 6$

$= 1 - 5\sqrt{7}$

i) $(3\sqrt{5}-2\sqrt{3})(3\sqrt{5}+2\sqrt{3})$ DOS

$= (3\sqrt{5})^2 - (2\sqrt{3})^2$

$= 9(5) - 4(3)$

$= 45 - 12$

$= 33$

9) Use the discriminant to determine the number of roots for each quadratic equation

a) $f(x) = x^2 - 3x + 1$

$b^2 - 4ac = (-3)^2 - 4(1)(1)$

$= 9 - 4$

$= 5$

$b^2 - 4ac > 0 \therefore 2$ roots

b) $f(x) = 2x^2 - 5x + 7$

$b^2 - 4ac = (-5)^2 - 4(2)(7)$

$= 25 - 56$

$= -31$

$b^2 - 4ac < 0 \therefore 0$ roots

c) $f(x) = 4x^2 + 24x + 36$

$b^2 - 4ac = (24)^2 - 4(4)(36)$

$= 0$

$b^2 - 4ac = 0 \therefore 1$ root

10) Solve each of the following quadratics using the most appropriate method. Give EXACT answers.

a) $0 = x^2 + 7x + 12$

$0 = (x+4)(x+3)$

$x+4=0$ $\quad$ $x+3=0$

$\boxed{x_1 = -4}$ $\quad$ $\boxed{x_2 = -3}$

b) $0 = 3x^2 - 4x - 15$

$0 = 3x^2 - 9x + 5x - 15$

$0 = 3x(x-3) + 5(x-3)$

$0 = (x-3)(3x+5)$

$x-3=0$ $\quad$ $3x+5=0$

$\boxed{x_1 = 3}$ $\quad$ $\boxed{x_2 = -\frac{5}{3}}$

c) $3x^2 + 6x = -1$

$3x^2 + 6x + 1 = 0$

$x = \frac{-6 \pm \sqrt{(6)^2 - 4(3)(1)}}{2(3)}$

$x = \frac{-6 \pm \sqrt{24}}{6}$

$x = \frac{-6 \pm 2\sqrt{6}}{6}$

$x = \frac{2(-3 \pm \sqrt{6})}{6}$

$\boxed{x = \frac{-3 \pm \sqrt{6}}{3}}$

d) $0 = x^2 + 6x + 4$

$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(4)}}{2(1)}$

$x = \frac{-6 \pm \sqrt{20}}{2}$

$x = \frac{-6 \pm 2\sqrt{5}}{2}$

$x = \frac{2(-3 \pm \sqrt{5})}{2}$

$\boxed{x = -3 \pm \sqrt{5}}$

11) Determine algebraically the coordinates of the points of intersection of each pair of functions.

a) $y = x^2 + 4x + 3$ and $y = 5x + 9$

$5x+9 = x^2+4x+3$

$0 = x^2 - x - 6$

$0 = (x-3)(x+2)$

$x-3=0$ $\quad$ $x+2=0$

$x_1 = 3$ $\quad$ $x_2 = -2$

POI #1: $y = 5(3)+9$

$y = 24$

$(3, 24)$

POI #2: $y = 5(-2)+9$

$y = -1$

$(-2, -1)$

b) $y = -x^2 - 4x + 6$ and $y = x - 8$

$x - 8 = -x^2 - 4x + 6$

$x^2 + 5x - 14 = 0$

$(x+7)(x-2) = 0$

$x+7=0$ $\quad$ $x-2=0$

$x_1 = -7$ $\quad$ $x_2 = 2$

POI # 1: $y = -7-8$

$y = -15$

$(-7, -15)$

POI #2: $y = 2-8$

$y = -6$

$(2, -6)$

12) Given the equation of a parabola and the slope of a line that is tangent to the parabola, determine the y-intercept of the tangent line.

$f(x) = -3x^2 + x - 4$, tangent line has slope 13 $\quad y = 13x + k$

$13x + k = -3x^2 + x - 4$

$3x^2 + 12x + 4 + k = 0$

Tangent lines have 1 POI ; $\therefore$ $b^2 - 4ac = 0$

$b^2 - 4ac = 0$

$(12)^2 - 4(3)(4+k) = 0$

$144 - 12(4+k) = 0$

$144 - 48 - 12k = 0$

$96 = 12k$

$k = 8$

The equation of the tangent line is $y = 13x + 8$

The y-intercept is at 8

Chapter 2 part 1 – Rations Expressions

13) Simplify each expression. State all restrictions on x.

a) $\frac{x-7}{x^2-4x-21}$

$= \frac{x-7}{(x-7)(x+3)}$

$= \frac{1}{x+3}$; $x \neq -3, 7$

b) $\frac{2x^2+7x-15}{2x^2+3x-9}$

$= \frac{(x+5)(2x-3)}{(x+3)(2x-3)}$

-30; $\frac{5}{1} = \frac{10}{2}$; $\frac{-3}{2}$; -7

-18; $\frac{3}{1} = \frac{6}{2}$; $\frac{-3}{2}$; 3

$= \frac{x+5}{x+3}$; $x \neq -3, \frac{3}{2}$

c) $\frac{36x^4}{5x^2} \times \frac{80x^3}{12x}$

$= \frac{2880x^7}{60x^3}$

$= 48x^4$; $x \neq 0$

d) $\frac{3x}{32y} \div \frac{27x^2}{96y}$

$= \frac{3x}{32y} \cdot \frac{96y}{27x^2}$

$= \frac{288xy}{864x^2y}$

$= \frac{1}{3x}$; $x \neq 0,\ y \neq 0$

e) $\frac{x-8}{\cancel{x+2}} \times \frac{\cancel{x+2}}{x-6}$

$= \frac{x-8}{x-6}$; $x \neq -2, 6$

f) $\frac{4x-20}{x^2+6x} \times \frac{3x^2}{3x-15}$

$= \frac{4\cancel{(x-5)}}{\cancel{x}(x+6)} \cdot \frac{\cancel{3}x^{\cancel{2}\,1}}{\cancel{3}\cancel{(x-5)}}$

$= \frac{4x}{x+6}$; $x \neq -6$

g) $\frac{x+1}{x} \div \frac{x+1}{2x}$

$= \frac{\cancel{x+1}}{\cancel{x}} \cdot \frac{2\cancel{x}}{\cancel{x+1}}$

$= 2$; $x \neq -1, 0$

h) $\frac{x^2-7x+10}{x^2-4} \div \frac{x^2-4x-5}{3x+6}$

$= \frac{\cancel{(x-2)}\cancel{(x-5)}}{\cancel{(x-2)}\cancel{(x+2)}} \cdot \frac{3\cancel{(x+2)}}{\cancel{(x-5)}(x+1)}$

$= \frac{3}{x+1}$; $x \neq -2, -1, 2, 5$

i) $\frac{2x}{x-2} - \frac{3}{x^2-4}$

$= \frac{(x+2)\,2x}{(x+2)\,x-2} - \frac{3}{(x-2)(x+2)}$

$= \frac{2x(x+2)-3}{(x-2)(x+2)}$

$= \frac{2x^2+4x-3}{(x-2)(x+2)}$; $x \neq -2, 2$

j) $\frac{x-2}{x+2} + \frac{x+10}{x^2+6x+8}$

$= \frac{(x+4)}{(x+4)}\frac{x-2}{x+2} + \frac{x+10}{(x+2)(x+4)}$

$= \frac{(x+4)(x-2)+(x+10)}{(x+2)(x+4)}$

$= \frac{x^2+2x-8+x+10}{(x+2)(x+4)}$

$= \frac{x^2+3x+2}{(x+2)(x+4)}$

$= \frac{\cancel{(x+2)}(x+1)}{\cancel{(x+2)}(x+4)}$

$= \frac{x+1}{x+4}$; $x \neq -4, -2$

k) $\frac{4x^2-20x}{x^2+2x-35} + \frac{3x-6}{x^2-12x+20}$

$= \frac{4x\cancel{(x-5)}}{(x+7)\cancel{(x-5)}} + \frac{3\cancel{(x-2)}}{(x-10)\cancel{(x-2)}}$

$= \frac{(x-10)\,4x}{(x-10)\,x+7} + \frac{3\,(x+7)}{x-10\,(x+7)}$

$= \frac{4x(x-10)+3(x+7)}{(x-10)(x+7)}$

$= \frac{4x^2-37x+21}{(x-10)(x+7)}$; $x \neq -7, 2, 5, 10$

l) $\frac{3x+2}{3-4x} + \frac{2x+1}{4x-3}$

$= \frac{3x+2}{3-4x} - \frac{2x+1}{3-4x}$

$= \frac{3x+2-(2x+1)}{3-4x}$

$= \frac{3x+2-2x-1}{}$

$= \frac{x+1}{3-4x}$; $x \neq \frac{3}{4}$

Chapter 2 – Part 2: Transformations

14) For the function $f(x) = \sqrt{x}$, write the new function equation for each transformation.

a) translation up 4 and right 9.

$g(x) = \sqrt{x-9} + 4$

b) vertical stretch by 6 and translation left 5.

$h(x) = 6\sqrt{x+5}$

c) horizontal reflection in the y-axis and horizontal compression by $\frac{1}{4}$.

$j(x) = \sqrt{-4x}$

15) List all the transformations, in words, of f(x) for each of the following functions.

a) $g(x) = -f(x-3) - 4$

a = -1; vertical reflection (-1y)
d = 3; shift RIGHT 3 units (x+3)
c = -4; shift DOWN 4 units (y-4)

b) $h(x) = -\frac{1}{3}f(2x) + 10$

$a = \frac{-1}{3}$; vertical reflection and a vertical compression BAFO $\frac{1}{3}$ $\left(\frac{y}{-3}\right)$
K = 2; horizontal compression BAFO $\frac{1}{2}$ $\left(\frac{x}{2}\right)$
c = 10; shift UP 10 units (y+10)

c) $j(x) = 5f(x+4) - 5$

a = 5; vertical stretch BAFO 5 (5y)
d = -4; shift LEFT 4 units (x-4)
c = -5; shift DOWN 5 units (y-5)

d) $k(x) = -2f\left(-\frac{1}{6}x\right) + 6$

a = -2; vertical reflection and a vertical stretch BAFO 2 (-2y)
$K = \frac{-1}{6}$; horizontal reflection and a horizontal stretch BAFO 6 (-6x)
c = 6; shift up 6 units (y+6)

16) For the function $\boxed{g(x) = \frac{1}{2}(x-2)^2 + 5}$:

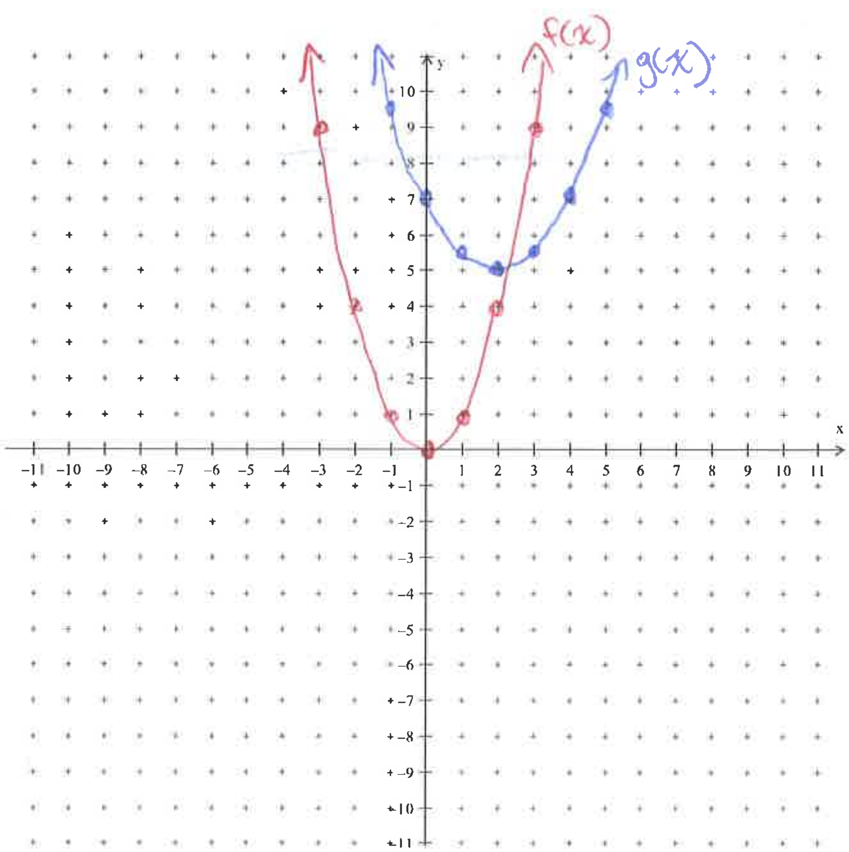
i) state what the parent function is [1 mark]
ii) create a table of values of image points for the transformed function [2 marks]
iii) graph the parent function and the transformed function [2 marks]

Parent Function:

$f(x) = x^2$

$f(x)$	
x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$g(x)$	
x+2	$\frac{y}{2}+5$
-1	9.5
0	7
1	5.5
2	5
3	5.5
4	7
5	9.5



17) For the function $\boxed{h(x) = \frac{1}{2}\sqrt{2x} - 3}$:

i) state what the parent function is [1 mark]
ii) create a table of values of image points for the transformed function [2 marks]
iii) graph the parent function and the transformed function [2 marks]

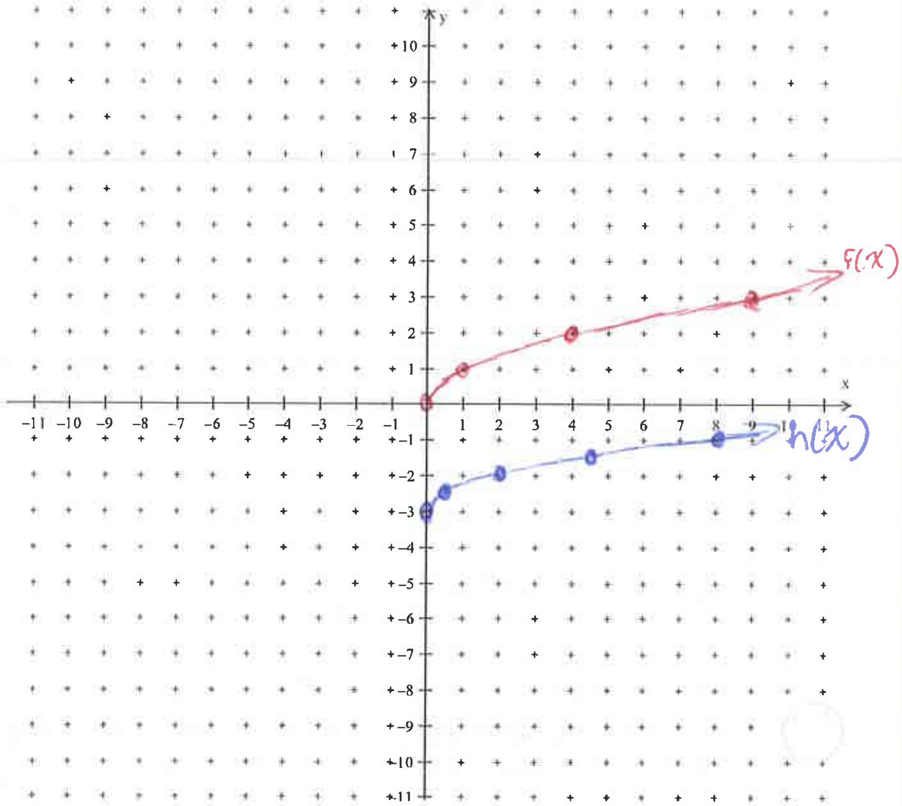
Parent Function:
$f(x) = \sqrt{x}$

$f(x)$

x	y
0	0
1	1
4	2
9	3

$h(x)$

$\frac{x}{2}$	$\frac{y}{2} - 3$
0	-3
0.5	-2.5
2	-2
4.5	-1.5



18) $f(x) = \frac{1}{x}$. For the function $\boxed{g(x) = f(x+2) - 1:}$

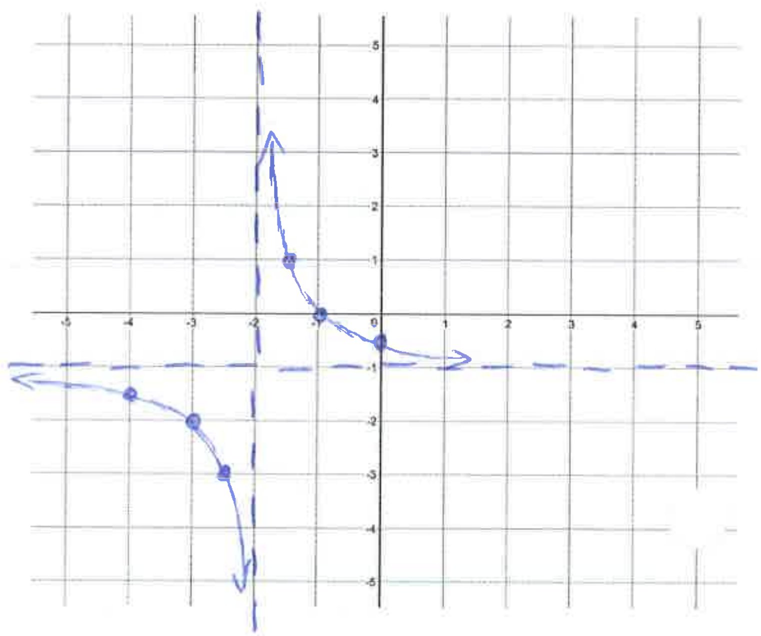
i) create a table of values of image points for the transformed function
ii) graph the ~~parent function and the~~ transformed function
iii) write the equation of the transformed function

$f(x)$

x	y
-2	-0.5
-1	-1
-0.5	-2
0	und.
0.5	2
1	1
2	0.5

$g(x)$

$x-2$	$y-1$
-4	-1.5
-3	-2
-2.5	-3
-2	und.
-1.5	1
-1	0
0	-0.5



Transformed Function: $g(x) = \frac{1}{x+2} - 1$

19) Find the inverse algebraically of the function below

$g(x) = 2(x-1)^2 + 2$

$y = 2(x-1)^2 + 2$

$x = 2(y-1)^2 + 2$

$\frac{x-2}{2} = (y-1)^2$

$\pm\sqrt{\frac{x-2}{2}} = y - 1$

$1 \pm \sqrt{\frac{x-2}{2}} = y$

Equation of Inverse:

$g^{-1}(x) = 1 \pm \sqrt{\frac{x-2}{2}}$

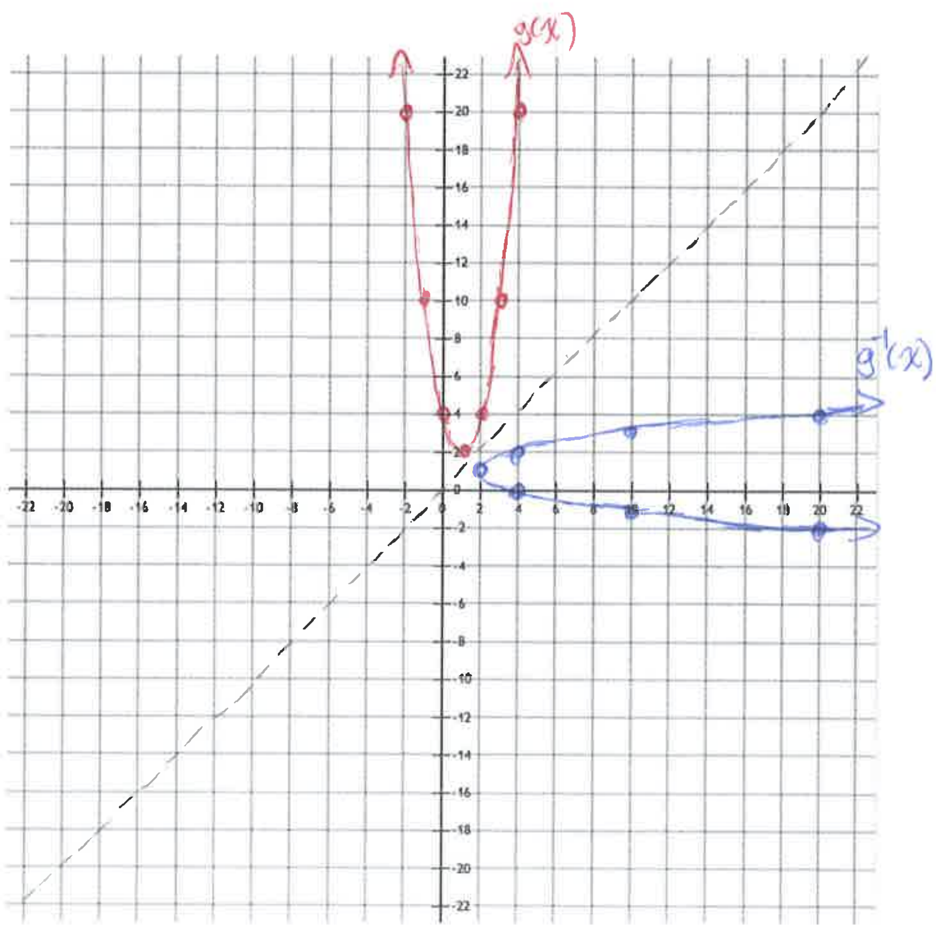
20) For the function $g(x)$ from the previous question

i)	Make a table of values for the parent function $\boxed{f(x) = x^2}$	[1 marks]
ii)	Graph $\boxed{g(x)}$ by creating a table of values of image points	[3 marks]
iii)	Graph $\boxed{g^{-1}(x)}$	[1 mark]

$f(x)$	
x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$g(x)$	
$x+1$	$2y+2$
-2	20
-1	10
0	4
1	2
2	4
3	10
4	20

$g^{-1}(x)$	
x	y
20	-2
10	-1
4	0
2	1
4	2
10	3
20	4



Chapter 3: Exponential Functions

21) An insect colony as an initial population of 15. The number of insects quadruples every day.

a) Determine the equation of a function that models this exponential growth.

$y = 15(4)^x$

b) How many insects will be present in 1 week?

$y = 15(4)^7$

$y = 245\,760$ insects

22) If the population of an ant colony is 213 and it doubles every week,

a) What will the population be in 4 weeks?

$y = 213(2)^x$

$y = 213(2)^4$

$y = 3\,408$ ants

b) How long will it take the population to reach 109 056 ants?

$109\,056 = 213(2)^x$

$512 = 2^x$

$\log(512) = \log(2^x)$

$\log(512) = x\log(2)$

$x = \dfrac{\log(512)}{\log(2)}$

$x = 9$ weeks

23) The population of a town in the Northwest Territories starts off at 20,000 and grows by 13% each year. Find the populations after 10 years.

$y = 20000(1 + 0.13)^x$

$y = 20\,000(1.13)^{10}$

$y = 67\,891.35$

About 67 891 people

24) A bacteria culture starts with a population of 12 000 and doubles every four hours.

a) How many bacteria are present after 12 hours?

$y = 12000(2)^{t/4}$

$y = 12000(2)^{12/4}$

$y = 12000(2)^{3}$

$y = 96000$ bacteria.

b) How many bacteria are present after 1 day?

$y = 12000(2)^{24/4}$

$y = 12000(2)^{6}$

$y = 768000$ bacteria.

c) How long will it take for the population of the bacteria to reach 49 152 000?

$49152000 = 12000(2)^{t/4}$

$4096 = 2^{t/4}$

$\log(4096) = \frac{t}{4}\log(2)$

$\frac{t}{4} = \frac{\log(4096)}{\log(2)}$

$\frac{t}{4} = 12$

$t = 48$ hours or 2 days.

25) Polonium-210 is a radioactive isotope that has a half-life of 20 days. Suppose you start with a 40-mg sample.

a) Write an equation that relates the amount of polonium-210 remaining and time.

$y = 40\left(\frac{1}{2}\right)^{t/20}$

b) How much polonium-210 will remain after 10 weeks?

$y = 40\left(\frac{1}{2}\right)^{70/20}$

$y \simeq 3.54$ mg

c) How long will it take for the amount of polonium-210 to decay to 8% of its initial mass?

$0.08(40) = 40\left(\frac{1}{2}\right)^{t/20}$

$0.08 = \left(\frac{1}{2}\right)^{t/20}$

$\log(0.08) = \frac{t}{20}\log\left(\frac{1}{2}\right)$

$\frac{t}{20} = \frac{\log(0.08)}{\log(0.5)}$

$\frac{t}{20} = 3.64385619$

$t \simeq 72.88$

About 73 days

26) Daniel is very excited about his new motorcycle. Although the motorcycle costs $13 500, its resale value will depreciate by 20% of its current value every year.

a) How much will the motorcycle be worth in 6 years?

$y = 13\,500(1-0.2)^6$

$y = 13\,500(0.8)^6$

$y = \$3\,538.94$

b) How long will it take for Daniel's motorcycle to depreciate to 50% of its original cost?

$0.5(13\,500) = 13\,500(0.8)^x$

$0.5 = 0.8^x$

$\log(0.5) = x\log(0.8)$

$x = \dfrac{\log(0.5)}{\log(0.8)}$

$x \simeq 3.1$ years

27) An investment opportunity is found that makes 7% per year compounded annually. How much should you invest now if you need $13, 450 at the end of 9 years?

$A = P(1+i)^n$

$13450 = P(1.07)^9$

$P = \dfrac{13450}{1.07^9}$

$P = \$7\,315.91$

28) Jacqueline deposits an inheritance of $1500 into an account that earns interest of 3.5% per year, compounded annually.

a) How much is in the account after 8 years?

$A = 1500(1.035)^8$

$A = \$1\,975.21$

b) How long will it take for the money to double (round to the nearest year)?

$2(1500) = 1500(1.035)^n$

$2 = 1.035^n$

$\log(2) = n\log(1.035)$

$n = \dfrac{\log(2)}{\log(1.035)}$

$n \simeq 20.15$ years

29) An investor invests \$5000 into a mutual fund for 10 years at a growth rate of 4% per year. How much is the investment worth after the ~~20~~ 10 years if the interest is compounded...

a) annually

$A = 5000(1.04)^{10}$

$A = \$7401.22$

b) quarterly (4 times a year)

$A = 5000\left(1 + \frac{0.04}{4}\right)^{10(4)}$

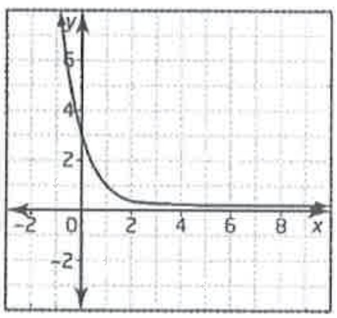
$A = 5000(1.01)^{40}$

$A = \$7444.32$

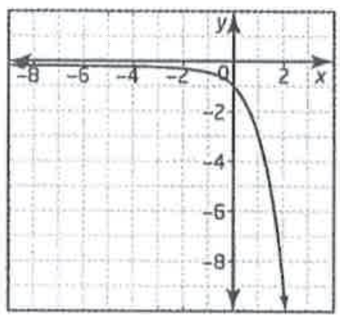
30) Match each graph with its corresponding equation

A $y = 3(3^x)$ **B** $y = 3\left(\frac{1}{3}\right)^x$ **C** $\frac{1}{3}(3^x)$ **D** $y = -3^x$

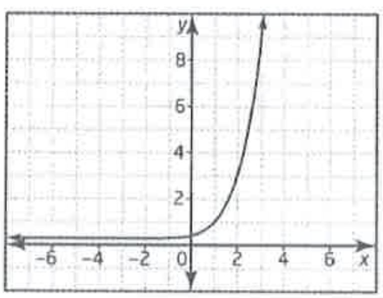
a) B



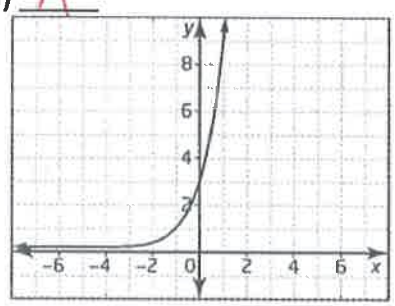
b) D



c) C



d) A



31) For the function $g(x) = -2\left(\frac{1}{2}\right)^{x+1} - 1$ HA: $y = -1$ [5]

a) state what the parent function is
b) create a table of values of image points for the transformed function
c) graph the transformed function making sure to include any asymptotes

a)

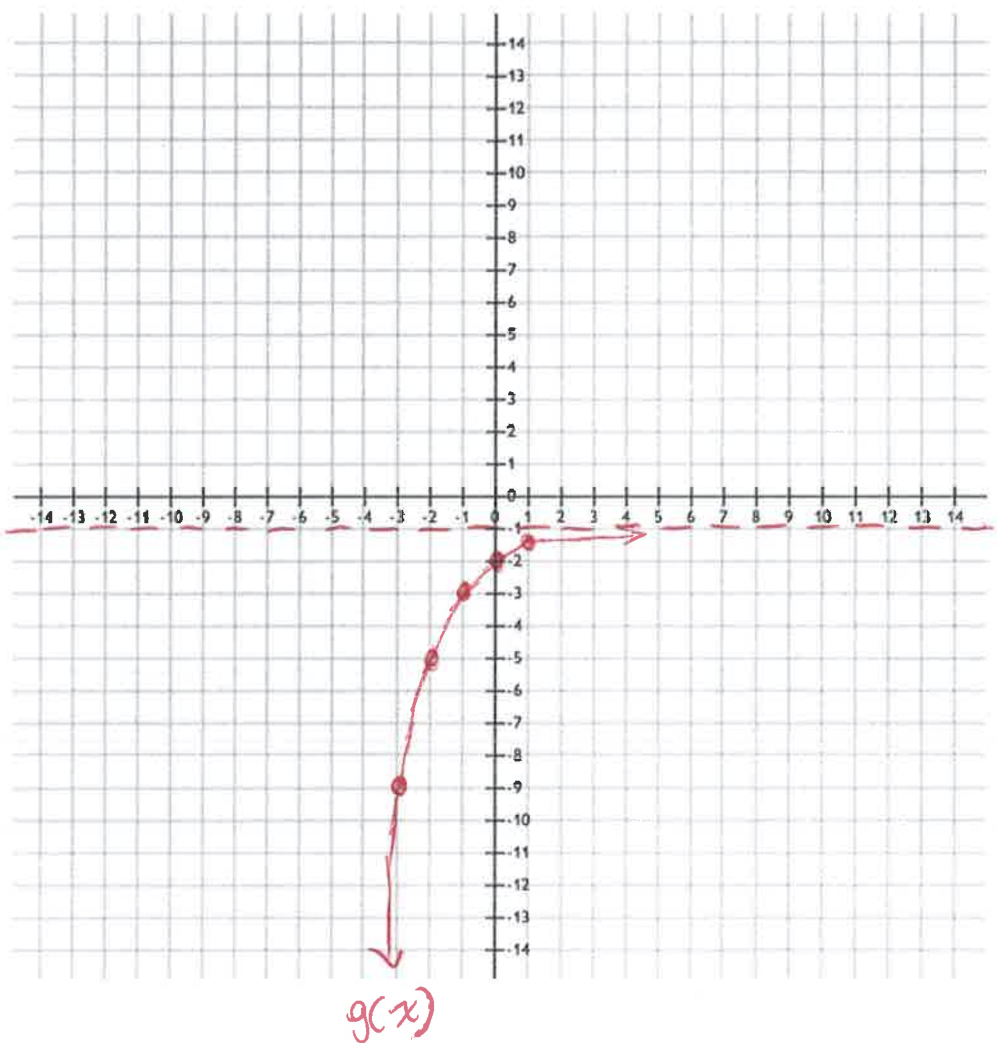
Parent Function: $f(x) = \left(\frac{1}{2}\right)^x$

b)

$f(x)$	
x	y
-2	4
-1	2
0	1
1	0.5
2	0.25

$g(x)$	
$x-1$	$-2y-1$
-3	-9
-2	-5
-1	-3
0	-2
1	-1.5

c)



Chapter 4: Trig Geometry

32) Draw both special triangles learned in this unit. Make sure to label all angles and side lengths.

33) Find a reference angle for the following obtuse angles

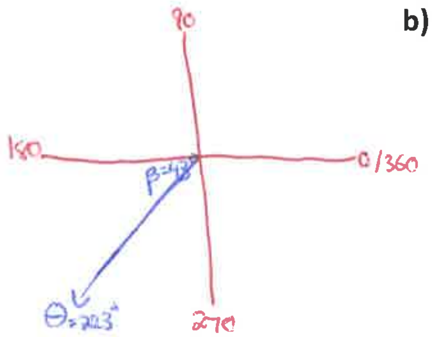
a) $\theta = -137°$

$\theta = -137 + 360$

$\theta = 223°$

$\beta = 223 - 180$

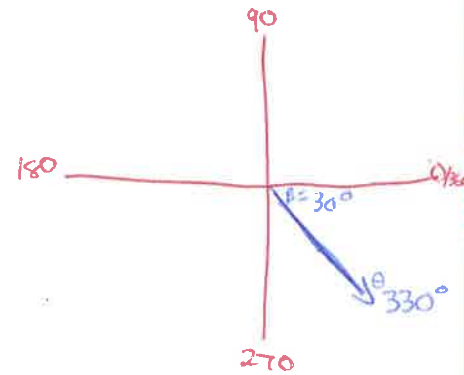
$\boxed{\beta = 43°}$



b) $\theta = 330°$

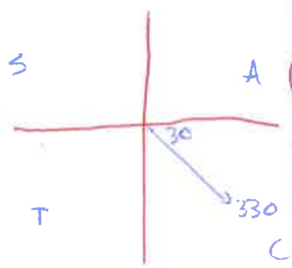
$\beta = 360 - 330$

$\boxed{\beta = 30°}$



34) Determine the exact value of each of the following trig ratios

a) tan 330



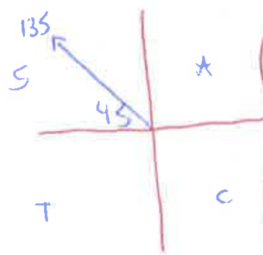
$\tan 330 = -\tan 30$

$= -\frac{1}{\sqrt{3}}$

b) sin 135

$\sin 135 = \sin 45$

$\boxed{= \frac{1}{\sqrt{2}}}$



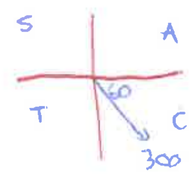
c) cot 300

$\cot 300 = \frac{1}{\tan 300}$

$= \frac{1}{-\tan 60}$

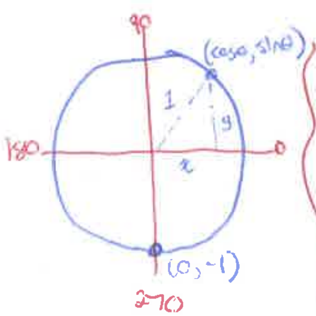
$= \frac{1}{-\sqrt{3}}$

$\boxed{= -\frac{1}{\sqrt{3}}}$



d) sin 270

$\boxed{\sin 270 = -1}$



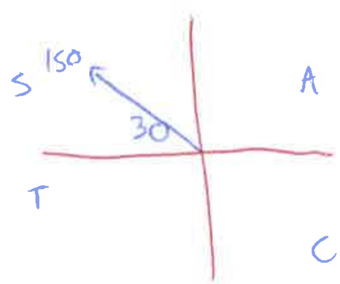
e) csc 150

$\csc 150 = \frac{1}{\sin 150}$

$= \frac{1}{\sin 30}$

$= \frac{1}{(\frac{1}{2})}$

$\boxed{= 2}$



Determine two angles that are co-terminal with angle 30°

$\theta_1 = 30 + 360 = 390°$

$\theta_2 = \theta_1 + 360 = 750°$

36) Determine ***TWO*** angles between 0° and 360° that have the following trigonometric function value. Write each angle to one decimal place.

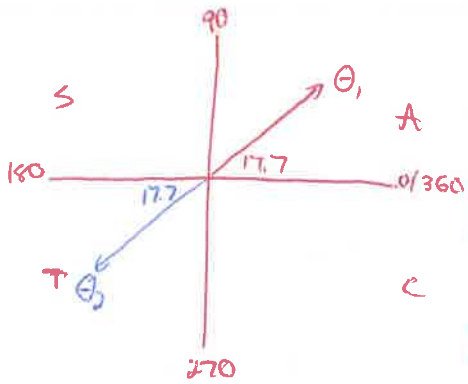
a) $\tan\theta = 0.32$

$\theta_1 = \tan^{-1}(0.32)$

$\boxed{\theta_1 = 17.7^\circ}$

$\theta_2 = 180 + 17.7$

$\boxed{\theta_2 = 197.7^\circ}$



b) $\sin\theta = -0.46$

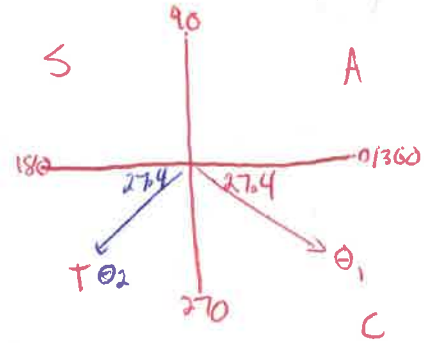
$\theta_1 = \sin^{-1}(-0.46)$

$\theta_1 = -27.4^\circ + 360^\circ$

$\boxed{\theta_1 = 332.6^\circ}$

$\theta_2 = 180 + 27.4$

$\boxed{\theta_2 = 207.4^\circ}$



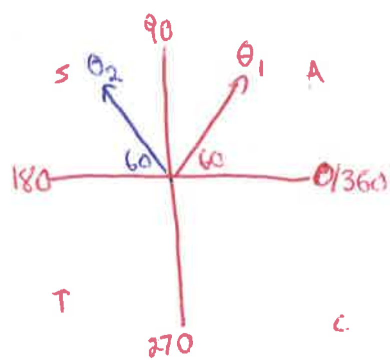
c) $\sin\theta = \frac{\sqrt{3}}{2}$

$\theta_1 = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\boxed{\theta_1 = 60^\circ}$

$\theta_2 = 180 - 60$

$\boxed{\theta_2 = 120^\circ}$



d) $\cot\theta = -\sqrt{3}$

$\tan\theta = \frac{-1}{\sqrt{3}}$

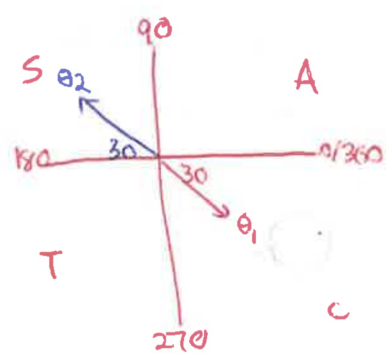
$\theta_1 = \tan\left(\frac{-1}{\sqrt{3}}\right)$

$\theta_1 = -30 + 360$

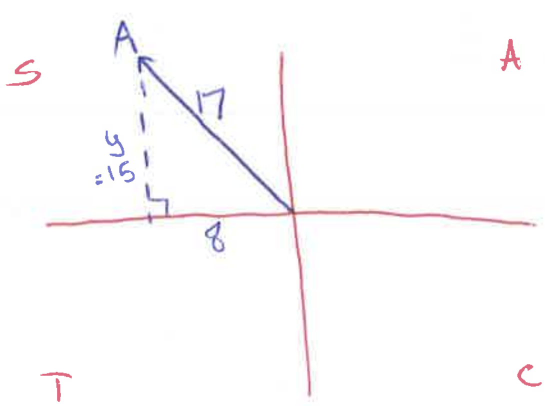
$\boxed{\theta_1 = 330^\circ}$

$\theta_2 = 180 - 30$

$\boxed{\theta_2 = 150^\circ}$



37) If $\cos A = -\frac{8}{17}$ and angle A lies in the second quadrant, find the other two primary trig ratios.



$y^2 + 8^2 = 17^2$

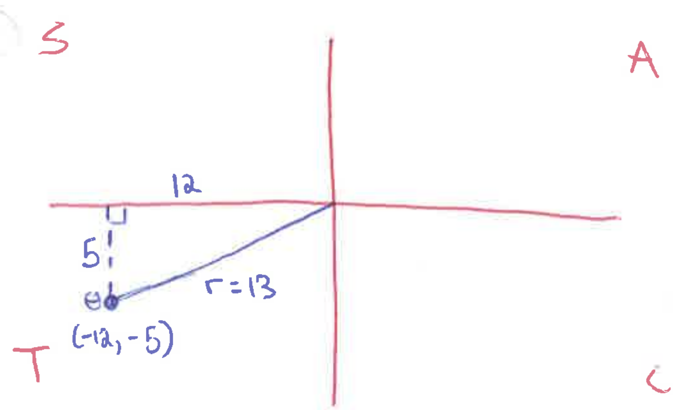
$y^2 = 225$

$y = 15$

$\sin A = \frac{15}{17}$

$\tan A = -\frac{15}{8}$

38) Point (-12, -5) lies on the terminal arm of an angle in standard position. Determine exact expressions for the six trigonometric ratios for the angle.



$5^2+12^2=r^2$

$r^2=169$

$r=13$

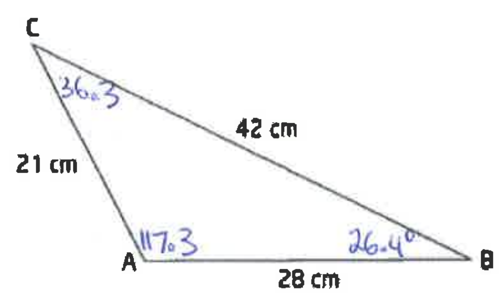
$\sin\theta = -\frac{5}{13}$ $\quad \csc\theta = -\frac{13}{5}$

$\cos\theta = -\frac{12}{13}$ $\quad \sec\theta = -\frac{13}{12}$

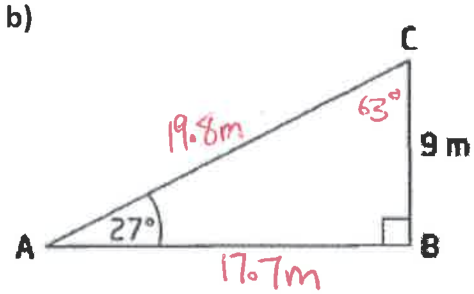
$\tan\theta = \frac{5}{12}$ $\quad \cot\theta = \frac{12}{5}$

39) Solve each of the following triangles.

a)



b)



$\cos A = \frac{a^2-b^2-c^2}{-2bc}$

$\cos A = \frac{42^2-21^2-28^2}{-2(21)(28)}$

$\cos A = \frac{539}{-1176}$

$\angle A = \cos^{-1}\left(\frac{539}{-1176}\right)$

$\boxed{\angle A \simeq 117.3°}$

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{42}{\sin 117.3} = \frac{21}{\sin B}$

$\sin B = \frac{21 \sin 117.3}{42}$

$\angle B = \sin^{-1}\left(\frac{21\sin 117.3}{42}\right)$

$\boxed{\angle B \simeq 26.4°}$

$\angle C = 180-117.3-26.4$

$\boxed{\angle C = 36.3}$

$\angle C = 90-27$

$\boxed{\angle C = 63°}$

$\tan 27 = \frac{9}{c}$

$c = \frac{9}{\tan 27}$

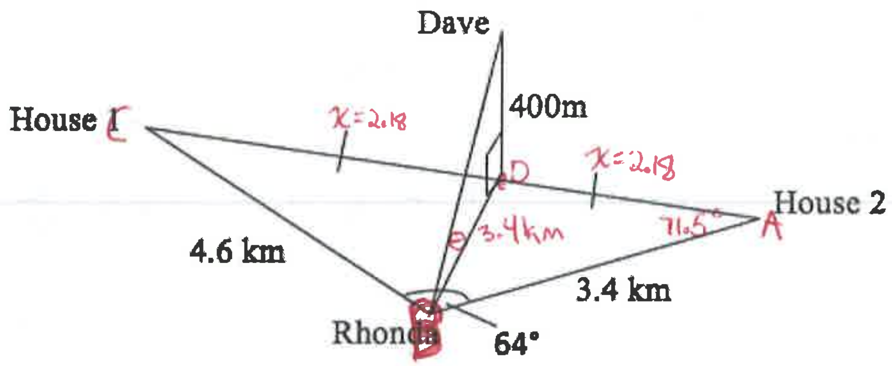
$\boxed{c \simeq 17.7m}$

$\sin 27 = \frac{9}{b}$

$b = \frac{9}{\sin 27}$

$\boxed{b \simeq 19.8m}$

40) Dave is in a hot air balloon 400m in the air exactly halfway between two houses on the ground. His wife, Rhonda, is at her friend's house which is 4.6km from the first house and 3.4km from the second house. The angle of the two houses, from Rhonda's point of view, is 64°. Find the angle of elevation if Rhonda looks up at Dave.



Let H1 to H2 = b

$b^2 = 4.6^2 + 3.4^2 - 2(4.6)(3.4)(\cos 64)$

$b \simeq 4.36$ km

$x = \frac{b}{2}$

$x = \frac{4.36}{2}$

$x = 2.18$ km

Let angle at H2 = $\angle A$

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{4.36}{\sin 64} = \frac{4.6}{\sin A}$

$\sin A = \frac{4.6 \sin 64}{4.36}$

$\angle A \cong 71.5°$

$BD^2 = 2.18^2 + 3.4^2 - 2(2.18)(3.4)(\cos 71.5)$

$BD^2 = 11.60867577$

$BD \cong 3.4$ km

$\tan\theta = \frac{0.4}{3.4}$

$\boxed{\theta \simeq 6.7°}$

41) Prove the following trigonometric identities.

a) $\sec\theta\cos\theta + \sec\theta\sin\theta = 1 + \tan\theta$

LS	RS
$= \sec\theta\cos\theta + \sec\theta\sin\theta$	$= 1 + \tan\theta$
$= \frac{1}{\cancel{\cos\theta}}(\cancel{\cos\theta}) + \frac{1}{\cos\theta}(\sin\theta)$	$= 1 + \frac{\sin\theta}{\cos\theta}$
$= 1 + \frac{\sin\theta}{\cos\theta}$	

LS = RS

b) $tan^2x + cos^2x + sin^2x = \frac{1}{cos^2x}$

LS	RS
$= \tan^2 x + \cos^2 x + \sin^2 x$	$= \frac{1}{\cos^2 x}$
$= \frac{\sin^2 x}{\cos^2 x} + 1$	
$= \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x}$	
$= \frac{\sin^2 x + \cos^2 x}{\cos^2 x}$	
$= \frac{1}{\cos^2 x}$	

LS = RS

c) $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$

LS

$= \frac{(1-\sin\theta)\,1}{(1-\sin\theta)(1+\sin\theta)} + \frac{1}{1-\sin\theta}\cdot\frac{(1+\sin\theta)}{(1+\sin\theta)}$

$= \frac{1-\sin\theta+1+\sin\theta}{(1-\sin\theta)(1+\sin\theta)}$

$= \frac{2}{1-\sin^2\theta}$

$= \frac{2}{\cos^2\theta}$

RS

$= 2\sec^2\theta$

$= \frac{2}{\cos^2\theta}$

LS = RS

d) $\frac{cotx-tanx}{sinxcosx}=csc^2x-sec^2x$

LS

$= \frac{\left(\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}\right)}{\sin x\cos x}$

$= \frac{\left(\frac{\cos^2 x-\sin^2 x}{\sin x\cos x}\right)}{\sin x\cos x}$

$= \frac{\cos^2 x-\sin^2 x}{\sin x\cos x}\cdot\frac{1}{\sin x\cos x}$

$= \frac{\cos^2 x-\sin^2 x}{\sin^2 x\cos^2 x}$

RS

$= \frac{1}{\sin^2 x}-\frac{1}{\cos^2 x}$

$= \frac{\cos^2 x-\sin^2 x}{\sin^2 x\cos^2 x}$

LS = RS

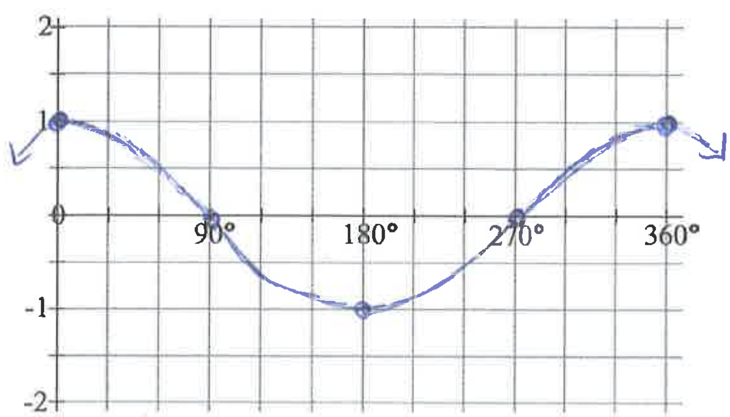
Chapter 5: Trig Functions

42) Graph the function $y = sinx$ using key points between 0° and 360° and then continuing the pattern.

x	y
0	0
90	1
180	0
270	-1
360	0

43) Graph the function $y = cosx$ using key points between 0° and 360°.

x	y
0	1
90	0
180	-1
270	0
360	1



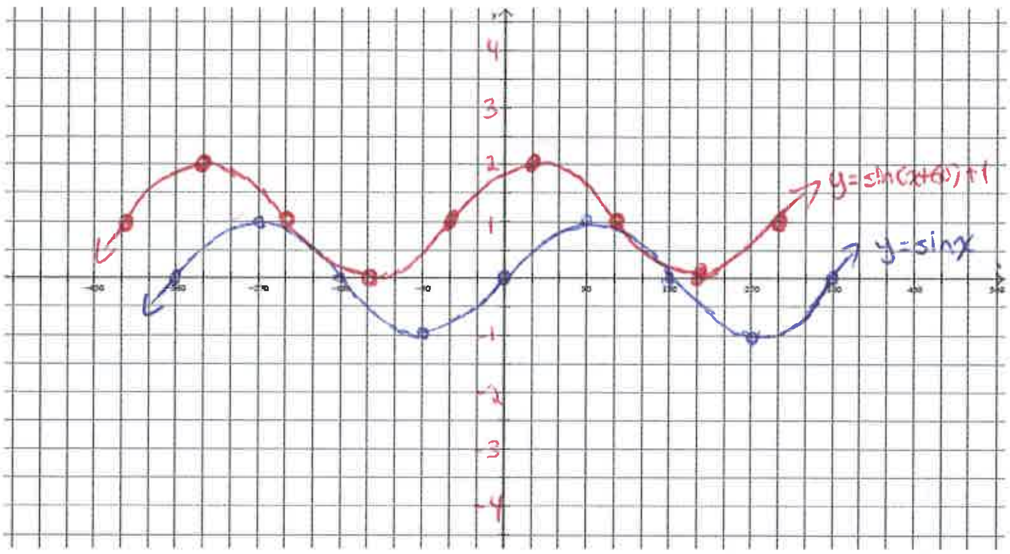
44) For the transformed function $y = \sin(x + 60) + 1$....

a) State the amplitude, the period, the phase shift and the vertical shift of the function with respect to the parent function. Then state the max and min values.

Amplitude = $|a| = 1$ Period = $\frac{360}{|k|} = \frac{360}{1} = 360°$ Phase shift = $60°$ LEFT

Vertical Shift = 1 unit UP Maximum Value = $c + amp = 1+1 = \boxed{2}$ Minimum Value = $c - amp = 1-1 = \boxed{0}$

b) Sketch **two cycles** of the parent function and the transformed function on the graph provided by transforming the key points of the parent function. Make sure to create a scale for the y-axis.



$y = \sin x$

x	y
0	0
90	1
180	0
270	-1
360	0

$y = \sin(x+60)+1$

$x-60$	$y+1$
-60	1
30	2
120	1
210	0
300	1

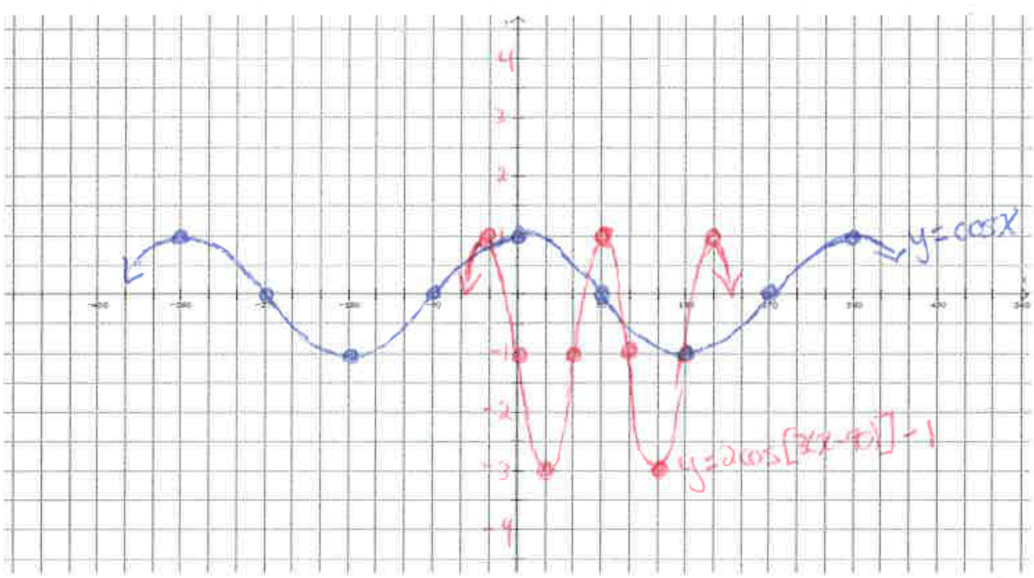
45) For the transformed function $y = 2\cos[3(x - 90)] - 1$

a) State the amplitude, the period, the phase shift and the vertical shift of the function with respect to the parent function. Then state the max and min values.

Amplitude = 2 Period = $\frac{360}{3} = 120$ Phase shift = $90°$ RIGHT

Vertical Shift = 1 DOWN Maximum Value = $-1+2 = 1$ Minimum Value = $-1-2 = -3$

b) Sketch **two cycles** of the parent function and the transformed function on the graph provided by transforming the key points of the parent function. Make sure to create a scale for the y-axis.



$y = \cos x$

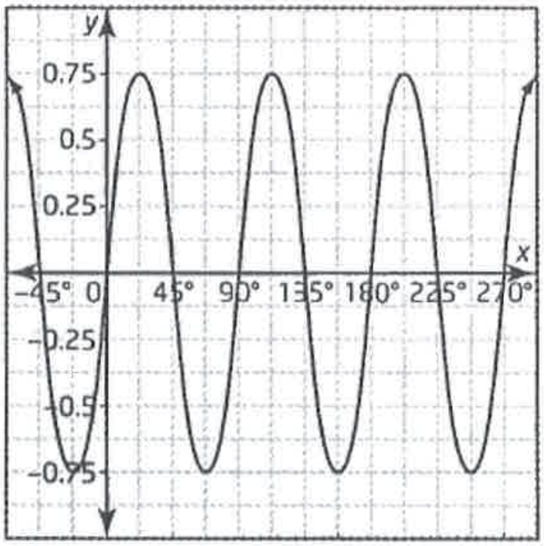
x	y
0	1
90	0
180	-1
270	0
360	1

$y = 2\cos[3(x-90)]-1$

$\frac{x}{3}+90$	$2y-1$
90	1
120	-1
150	-3
180	-1
210	1

46) Write the equations of sine function and a cosine function to match each graph.

a)



$a = \frac{max - min}{2} = \frac{0.75 - (-0.75)}{2} = 0.75$

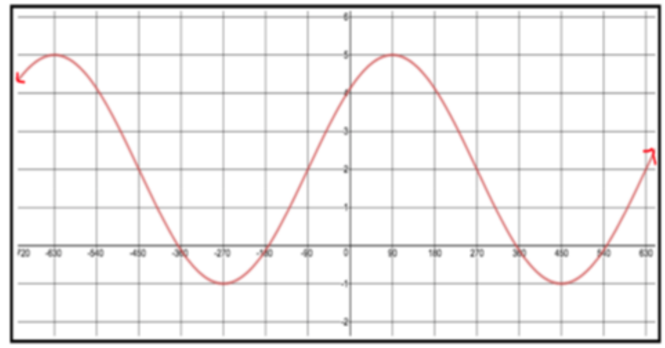
$k = \frac{360}{period} = \frac{360}{90} = 4$

$c = max - amp = 0.75 - 0.75 = 0$

$d_{sin} = 0 \qquad d_{cos} = 22.5$

$$\boxed{\begin{array}{l} y = 0.75 \sin(4x) \\ y = 0.75 \cos[4(x - 22.5)] \end{array}}$$

b)



$a = \frac{5-(-1)}{2} = 3$

$k = \frac{360}{720} = \frac{1}{2}$

$c = 5 - 3 = 2$

$d_{cos} = 90$

$d_{sin} = -90$

$y = 3\cos[\frac{1}{2}(x - 90)] + 2$

$y = 3\sin[\frac{1}{2}(x + 90)] + 2$

47) Determine two equations for a sinusoidal wave that has a maximum at (0, 5), vertical shift of 2 down, and a period of 120.

$c = -2$

$a = max - c = 5 - (-2) = 7$

$k = \frac{360}{period} = \frac{360}{120} = 3$

$d_{cos} = 0$

$d_{sin} = d_{cos} - \frac{90}{k} = 0 - \frac{90}{3} = -30$

$$\boxed{\begin{array}{l} y = 7\cos(3x) - 2 \\ y = 7\sin[3(x + 30)] - 2 \end{array}}$$

48) Pitt Lake is a freshwater lake in southern British Columbia with the highest tidal change of any freshwater lake in the world. In a daily period, the highest tide is traditionally at 8:00 am, reaching 5.2 m, and the lowest tide is traditionally at 8:00 pm, reaching only 0.6 m. Consider the cosine function that gives the tidal height of the lake, *y*, in terms of the hours after midnight, *x*.

a) Draw a sketch of the function. What are the period, amplitude, phase shift and vertical shift of the function?

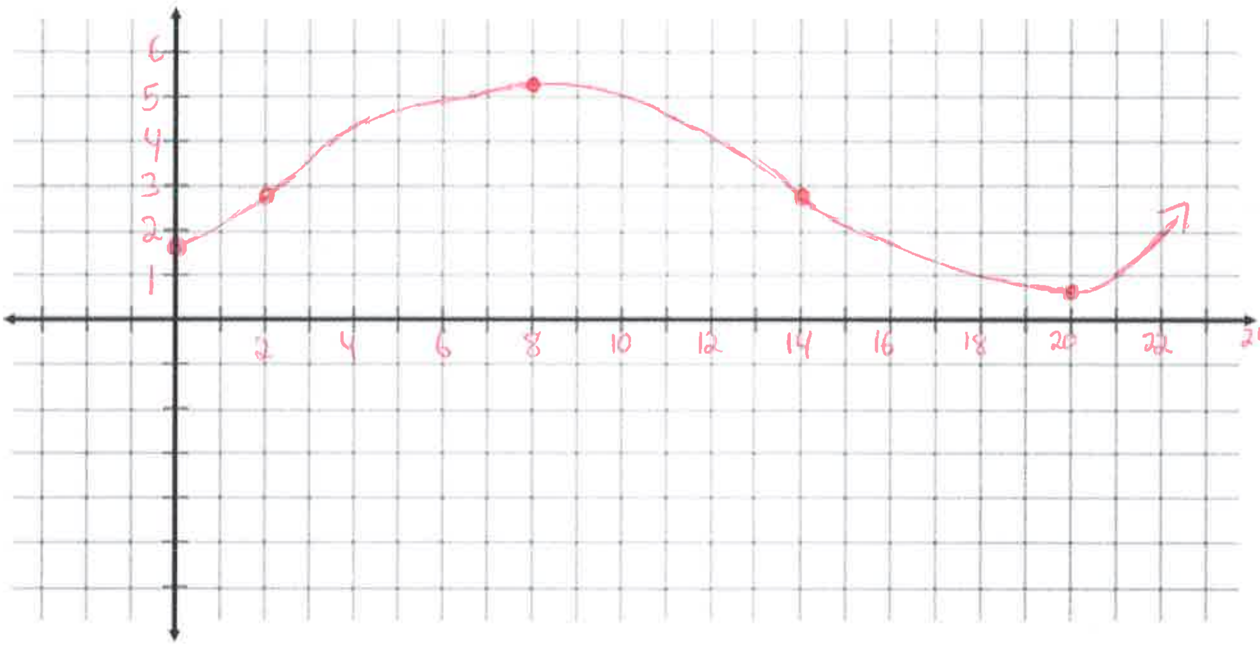
period = 24 hours

amp = $\frac{5.2 - 0.6}{2} = 2.3$

vertical shift = C = 5.2 − 2.3 = 2.9 UP

phase shift = d_{cos} = 8 RIGHT

y-int: (0, 1.75)



b) What is the function equation in the form $y = a\cos k(x-d)+c$?

$a = 2.3$

$k = \frac{360}{24} = 15$

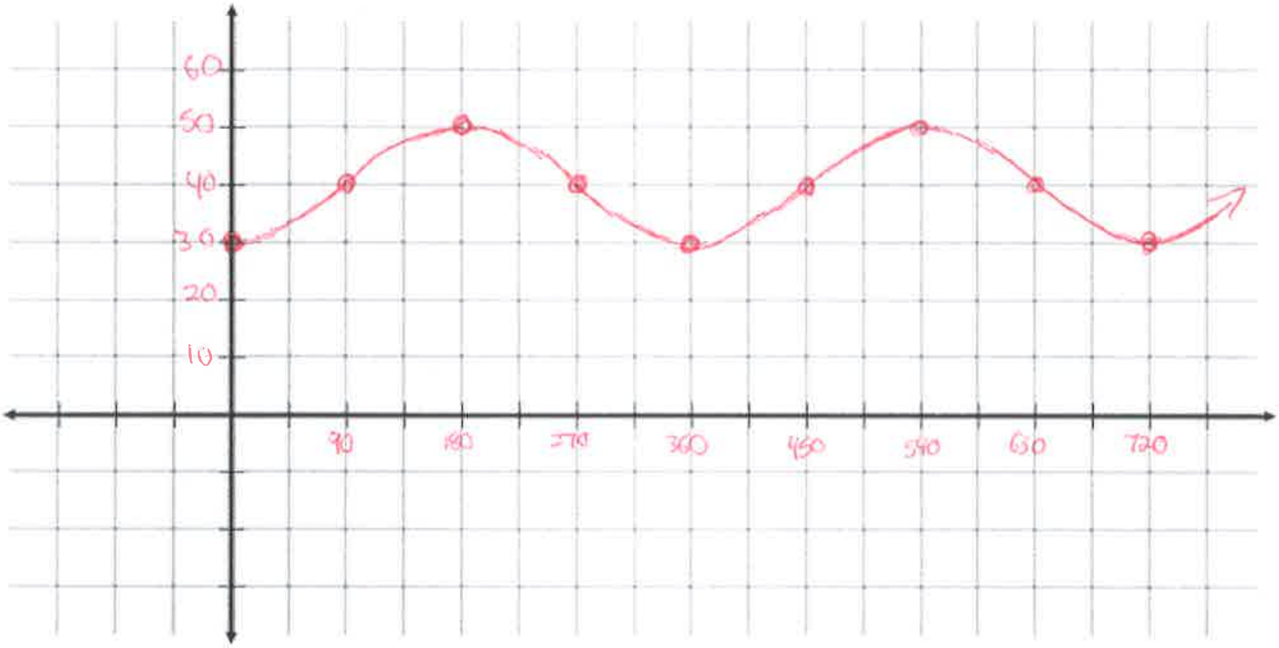
$c = 2.9$

$d = 8$

$$\boxed{y = 2.3\cos[15(x-8)] + 2.9}$$

49) A windmill is 40 meters tall and has three blades each measuring 10m.

a) Graph the height of the tip of a blade that starts at the bottom of the windmill and rotates around counter clockwise. Graph two rotations.



b) Determine a sine and cosine function to represent the motion of the blade.

$a = \frac{50-30}{2} = 10$

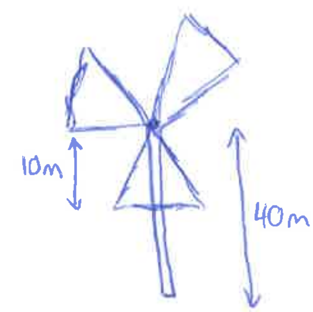
$k = \frac{360}{360} = 1$

$c = 50 - 10 = 40$

$d_{sin} = 90$

$d_{cos} = 180$

$$\boxed{\begin{array}{l} y = 10\sin(x-90)+40 \\ y = 10\cos(x-180)+40 \end{array}}$$



Chapter 6: Discrete Functions

50) For each of the following sequences…

i) state if it is arithmetic or geometric

ii) write an explicit formula for the general term

iii) calculate t_{10} using your formula

a) 9, 15, 21, 27, … $+6$

i) Arithmetic

ii) $t_n = 9 + (n-1)(6)$

iii) $t_{10} = 9 + (10-1)(6)$

$t_{10} = 63$

b) $-1, 2, -4, 8, \ldots$

i) Geometric

ii) $t_n = -1(-2)^{n-1}$

iii) $t_{10} = -1(-2)^{10-1}$

$t_{10} = 512$

51) In an arithmetic series of 50 terms, the 17th term is 53 and the 28th term is 86. Determine, a, d and S_{50}.

$t_{17} = 53$

$53 = a + (17-1)d$

① $53 = a + 16d$

$t_{28} = 86$

$86 = a + (28-1)d$

② $86 = a + 27d$

② $86 = a + 27d$
① $53 = a + 16d$ −

$33 = 11d$

$\boxed{d = 3}$

sub $d=3$ into ①

$53 = a + 16(3)$

$53 = a + 48$

$\boxed{a = 5}$

$S_{50} = \frac{50}{2}\left[2(5) + (50-1)(3)\right]$

$S_{50} = 25(157)$

$\boxed{S_{50} = 3925}$

52) The fifth term of a geometric series is 405 and the sixth term is 1215. Find the sum of the first nine terms.

$t_5 = 405$

$405 = a(r)^{5-1}$

① $405 = a(r)^4$

$t_6 = 1215$

$1215 = a(r)^{6-1}$

② $1215 = a(r)^5$

solve using substitution:

$\frac{405}{r^4} = a$

$1215 = \left(\frac{405}{r^4}\right)(r^5)$

$1215 = 405r$

$r = \frac{1215}{405}$

$\boxed{r = 3}$

$\frac{405}{(3)^4} = a$

$\frac{405}{81} = a$

$\boxed{a = 5}$

$S_9 = \frac{5\left[(3)^9 - 1\right]}{3-1}$

$S_9 = \frac{98410}{2}$

$\boxed{S_9 = 49205}$

53) Find the sum of each of the following series:

a) 251 + 243 + 235 +…–205

$-205 = 251 + (n-1)(-8)$

$-456 = (n-1)(-8)$

$57 = n - 1$

$n = 58$

$S_{58} = \frac{58}{2}(251 - 205)$

$\boxed{S_{58} = 1334}$

b) –4 – 12 – 36– … –8748

$-8748 = -4(3)^{n-1}$

$2187 = 3^{n-1}$

$\log(2187) = (n-1)\log(3)$

$\frac{\log(2187)}{\log(3)} = n-1$

$7 = n - 1$

$n = 8$

$S_8 = \frac{-4(3^8 - 1)}{3-1}$

$S_8 = \frac{-26240}{2}$

$\boxed{S_8 = -13120}$

54) Write the first four terms for the recursive sequence: $t_1 = -6;\ t_n = 2t_{n-1} + 3$

$t_1 = -6$

$t_2 = 2(-6) + 3 = -9$

$t_3 = 2(-9) + 3 = -15$

$t_4 = 2(-15) + 3 = -27$

55) Determine a recursive formula for the sequence 3, 8, 13, 18, 23, 28, 33, 38

$$t_n = t_{n-1} + 5 \; ; \; t_1 = 3$$

56) Expand the following binomials using Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

a) $(x^2 - 2y)^4$

$$= 1(x^2)^4(-2y)^0 + 4(x^2)^3(-2y)^1 + 6(x^2)^2(-2y)^2 + 4(x^2)^1(-2y)^3 + 1(x^2)^0(-2y)^4$$

$$= 1(x^8)(1) + 4(x^6)(-2)(y) + 6(x^4)(4)(y^2) + 4(x^2)(-8)(y^3) + 1(1)(16)(y^4)$$

$$= x^8 - 8x^6y + 24x^4y^2 - 32x^2y^3 + 16y^4$$

b) $(4x + 2x^3)^5$

$$= 1(4x)^5(2x^3)^0 + 5(4x)^4(2x^3)^1 + 10(4x)^3(2x^3)^2 + 10(4x)^2(2x^3)^3 + 5(4x)^1(2x^3)^4 + 1(4x)^0(2x^3)^5$$

$$= 1(1024)(x^5)(1) + 5(256)(x^4)(2)(x^3) + 10(64)(x^3)(4)(x^6) + 10(16)(x^2)(8)(x^9) + 5(4)(x)(16)(x^{12}) + 1(1)(32)(x^{15})$$

$$= 1024x^5 + 2560x^7 + 2560x^9 + 1280x^{11} + 320x^{13} + 32x^{15}$$