

Sequences (part 2)

Sequences Questions

What is the difference between a sequence and a series?

Sequence - a list of numbers that change by a constant value

Series - the sum of values in a sequence

What is the difference between Arithmetic and Geometric?

arithmetic: + or - to get future terms

geometric: \times to get future terms

Formulas for general terms of a sequence

Arithmetic
Sequence

$$t_n = a + (n - 1)d$$

Geometric
Sequence

$$t_n = a \cdot r^{n-1}$$

Example 1: -10, -4, 2, ...


a) Determine whether the sequence is arithmetic or geometric.

Arithmetic

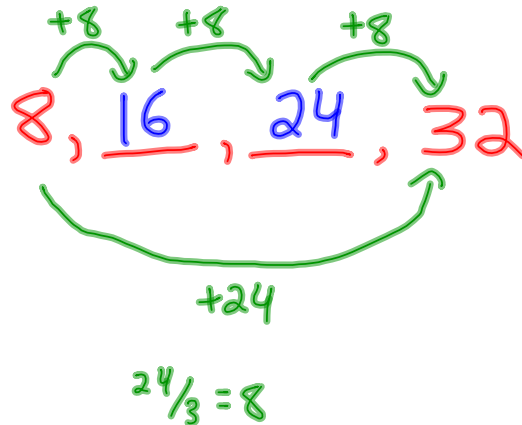
b) Determine an equation for the sequence.

$$\begin{aligned}t_n &= a + (n-1)d \\ &= -10 + (n-1)(6)\end{aligned}$$

c) Find the value of t_{21}

$$\begin{aligned}t_{21} &= -10 + (21-1)(6) \\ &= 110\end{aligned}$$

Example 2: Insert two numbers between 8 and 32 so the four numbers form an arithmetic sequence.



Example 3: An arithmetic sequence is 8, 14, 20, 26,.... Which term has the value 92? Prove mathematically.

$$t_n = a + (n-1)d$$

$$a = 8$$

$$d = 6$$

$$92 = 8 + (n-1)(6)$$

$$84 = (n-1)(6)$$

$$\frac{84}{6} = n-1$$

$$14 = n-1$$

$$15 = n$$

$$\sum_{i=1}^{15} t_i = 92$$

Example 4: 200, -100, 50, ...

a) Is the sequence arithmetic or geometric?

geometric

b) Find an equation to represent the sequence.

$$t_n = a \cdot r^{n-1}$$

$$t_n = 200 \left(\frac{1}{2}\right)^{n-1}$$

c) Find t_{14} .

$$t_{14} = 200 \left(\frac{1}{2}\right)^{14-1}$$

$$= 200 \left(\frac{1}{8192}\right)$$

$$= \frac{-25}{1024}$$

Example 5: Complete the geometric sequence:

320, 160, 80, 40, 20, 10

Example 6: The 50th term of an arithmetic sequence is 238 and the 93rd term is 539. Find a general equation to represent the sequence.

$$t_{50} = a + (50-1)d \quad t_{93} = a + (93-1)d$$

$$\textcircled{1} 238 = a + 49d \quad \textcircled{2} 539 = a + 92d$$

* solve the system of equations *

$$\begin{array}{r} \textcircled{2} 539 = a + 92d \\ \textcircled{1} 238 = a + 49d \quad - \\ \hline 301 = 43d \\ 7 = d \end{array}$$

sub $d=7$ into $\textcircled{1}$

$$\begin{array}{r} 238 = a + 49(7) \\ 238 - 343 = a \\ a = -105 \end{array}$$

∴ the general equation is $t_n = -105 + (n-1)(7)$

Example 7: Determine the number of terms in the geometric sequence: 5, -10, 20,, -10 240

$$t_n = -10240$$

$$n = ?$$

$$a = 5$$

$$r = -2$$

$$t_n = a \cdot r^{n-1}$$

$$-10240 = 5(-2)^{n-1}$$

write as a power w base (-2). $\rightarrow -2048 = (-2)^{n-1}$

$$(-2)^n = (-2)^{n-1}$$

$$\textcircled{8} 11 = n-1$$

$$12 = n$$

∴ there are 12 terms in the sequence.

