More Arithmetic and Geometric Series Questions

DO IT NOW!

In an arithmetic sequence, $t_3 = 25$ and $t_9 = 43$. Determine the formula for the general term of this sequence.

$\boxed{t_3}$

$t_n = a + (n-1)d$

$25 = a + (3-1)d$

① $25 = a + 2d$

$\boxed{t_9}$

$43 = a + (9-1)d$

② $43 = a + 8d$

* solve using elimination

② $43 = a + 8d$

① $25 = a + 2d$ −

$18 = 6d$

$3 = d$

* sub $d = 3$ into ①

$25 = a + 2(3)$

$25 = a + 6$

$19 = a$

$\boxed{\text{The general formula is } t_n = 19 + (n-1)(3)}$

Arithmetic

Sequence:

$$t_n = a + (n-1)d$$

Series:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{n}{2}(a + t_n)$$

Geometric

Sequence:

$$t_n = a \cdot r^{n-1}$$

Series:

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Example 1: In an amphitheatre, seats are arranged in 50 semicircular rows facing a domed stage. The first row contains 23 seats, and each row contains 4 more seats than the previous row. How many seats are in the amphitheatre?

$a = 23 \quad d = 4 \quad n = 50 \quad S_{50} = ?$

$$S_{50} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{50}{2}[2(23) + (50-1)(4)]$$

$$= 25(242)$$

$$= 6050$$

***Note:** use formula for arithmetic series because difference between consecutive rows is a constant.*

The amphitheatre has 6050 seats.

Example 2: Determine the sum of $-31 \underbrace{-35}_{-4} \underbrace{-39}_{-4}-.....-403$

arithmetic series

last term. $n=?$

Start by determining what term # the last term is.

$$t_n = a+(n-1)d$$
$$-403 = -31+(n-1)(-4)$$
$$-372 = (n-1)(-4)$$
$$93 = n-1$$
$$94 = n$$

$\therefore t_{94} = -403$

Next, find S_{94}:

$$S_n = \frac{n}{2}(a+t_n)$$
$$S_{94} = \frac{94}{2}(-31-403)$$
$$= 47(-434)$$
$$= -20\ 398$$

Example 3: Determine the sum of the first 20 terms of the arithmetic series in which the 15th term is 107 and the terms decrease by 3.

Start by finding the value of 'a' using $t_{15} = 107$

$$t_n = a+(n-1)d$$
$$107 = a+(15-1)(-3)$$
$$107 = a-42$$
$$149 = a$$

Now find S_{20}:

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$= \frac{20}{2}[2(149)+(20-1)(-3)]$$
$$= 10(241)$$
$$= 2410$$

Example 4: The 10th term of an arithmetic series is 34, and the sum of the first 20 terms is 710. Determine the 25th term.

$t_{10} = 34 \quad S_{20} = 710 \quad t_{25} = ?$

$\underline{t_{10}}$

$t_n = a + (n-1)d$

$34 = a + (10-1)d$

① $34 = a + 9d$

$\underline{S_{20}}$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$710 = \frac{20}{2}[2a + (20-1)d]$

$710 = 10(2a + 19d)$

② $710 = 20a + 190d$

* solve using substitution or elimination.

$$\begin{array}{lr} ② \longrightarrow & 710 = 20a + 190d \\ ① \times 20 \rightarrow & 680 = 20a + 180d \quad - \\ \hline & 30 = 10d \\ & 3 = d \end{array}$$

* sub $d = 3$ into ①

$34 = a + 9(3)$

$34 = a + 27$

$7 = a$

* now solve for t_{25}: $t_n = a + (n-1)d$

$t_{25} = 7 + (25-1)(3)$

$t_{25} = 79$

Example 5: Determine the sum of the first seven terms of the geometric series in which $t_5 = 5$ and $t_8 = -40$.

$\boxed{\text{method 1}}$

$\frac{5}{16}$	$-\frac{5}{8}$	$\frac{5}{4}$	$-\frac{5}{2}$	5	-10	20	-40
1st	2nd	3rd	4th	5th	6th	7th	8th

$\div(-2)$ $\quad \times(-2)$

$\therefore r = -2$ and $a = \frac{5}{16}$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_7 = \frac{\left(\frac{5}{16}\right)\left[(-2)^7 - 1\right]}{-2-1}$$

$$= \frac{\left(\frac{5}{16}\right)(-129)}{-3}$$

$$= \frac{5}{16}(43)$$

$$= \frac{215}{16}$$

Example 5: Determine the sum of the first seven terms of the geometric series in which $t_5 = 5$ and $t_8 = -40$.

$\boxed{\text{method 2}}$

$\underline{t_5}$

$5 = a(r)^{5-1}$

$5 = a(r)^4$

$a = \frac{5}{r^4}$

$\underline{t_8}$

$-40 = a(r)^{8-1}$

$-40 = a(r)^7$

* solve using substitution

$-40 = \left(\frac{5}{r^4}\right)(r^7)$

$-40 = 5r^3$

$-8 = r^3$

$\sqrt[3]{(-8)} = r$

$-2 = r$

$a = \frac{5}{r^4}$

$a = \frac{5}{(-2)^4}$

$a = \frac{5}{16}$

* geo. series formula

$S_7 = \frac{a(r^n - 1)}{r-1}$

$S_7 = \frac{\left(\frac{5}{16}\right)\left[(-2)^7 - 1\right]}{-2-1}$

$S_7 = \frac{215}{16}$

Example 6: Calculate the sum of the geometric series, $960 + 480 + 240 + ... + 15$

$\times \frac{1}{2}$

Method 1: write out full series

$= 960 + 480 + 240 + 120 + 60 + 30 + 15$

$= 1905$

Method 2: Solve using logarithms

Figure out how many terms are in the series by solving for n in the formula: $t_n = a \cdot r^{n-1}$

$a = 960 \quad r = \frac{1}{2}$

$t_n = a(r)^{n-1}$

$15 = 960\left(\frac{1}{2}\right)^{n-1}$ last term = 15

$\frac{15}{960} = \left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{64} = \left(\frac{1}{2}\right)^{n-1}$

$\log\left(\frac{1}{64}\right) = \log\left(\frac{1}{2}\right)^{n-1}$

$\log\left(\frac{1}{64}\right) = (n-1)\cdot\log\left(\frac{1}{2}\right)$

$\frac{\log\left(\frac{1}{64}\right)}{\log\left(\frac{1}{2}\right)} = n-1$

$6 = n-1$

$7 = n$

Now solve S_7:

$S_n = \frac{a(r^n-1)}{r-1}$

$S_7 = \frac{960\left[\left(\frac{1}{2}\right)^7-1\right]}{\left(\frac{1}{2}\right)-1}$

$= \frac{-952.5}{-0.5}$

$= 1905$

Method 3: Solve using powers with the same base

$a = 960 \quad r = \frac{1}{2}$

$t_n = a(r)^{n-1}$

$15 = 960\left(\frac{1}{2}\right)^{n-1}$ last term = 15

$\frac{15}{960} = \left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{64} = \left(\frac{1}{2}\right)^{n-1}$

$\left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^{n-1}$

$6 = n-1$

$7 = n$

Now solve S_7:

$S_n = \frac{a(r^n-1)}{r-1}$

$S_7 = \frac{960\left[\left(\frac{1}{2}\right)^7-1\right]}{\left(\frac{1}{2}\right)-1}$

$= \frac{-952.5}{-0.5}$

$= 1905$

Example 7: A tennis tournament has 128 entrants. A player is dropped from the competition after losing one match. Winning players go on to another match. What is the total number of matches that will be played in this tournament?

$a = 64 \quad r = \frac{1}{2}$ last term is 1.

Note: The first term is 128/2 = 64 because 2 players participate in one match. The last term is 1 but we don't know what term number it is.

Start by determining the # of terms in the series

$$t_n = a(r)^{n-1}$$

$$1 = 64\left(\tfrac{1}{2}\right)^{n-1}$$

$$\frac{1}{64} = \left(\tfrac{1}{2}\right)^{n-1}$$

$$\left(\tfrac{1}{2}\right)^6 = \left(\tfrac{1}{2}\right)^{n-1}$$

$$6 = n - 1$$

$$7 = n$$

Next, calculate S_7

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_7 = \frac{64\left[\left(\tfrac{1}{2}\right)^7 - 1\right]}{\tfrac{1}{2} - 1}$$

$$= \frac{-63.5}{-0.5}$$

$$= 127$$

$\therefore$ 127 matches will be played in the tournament.