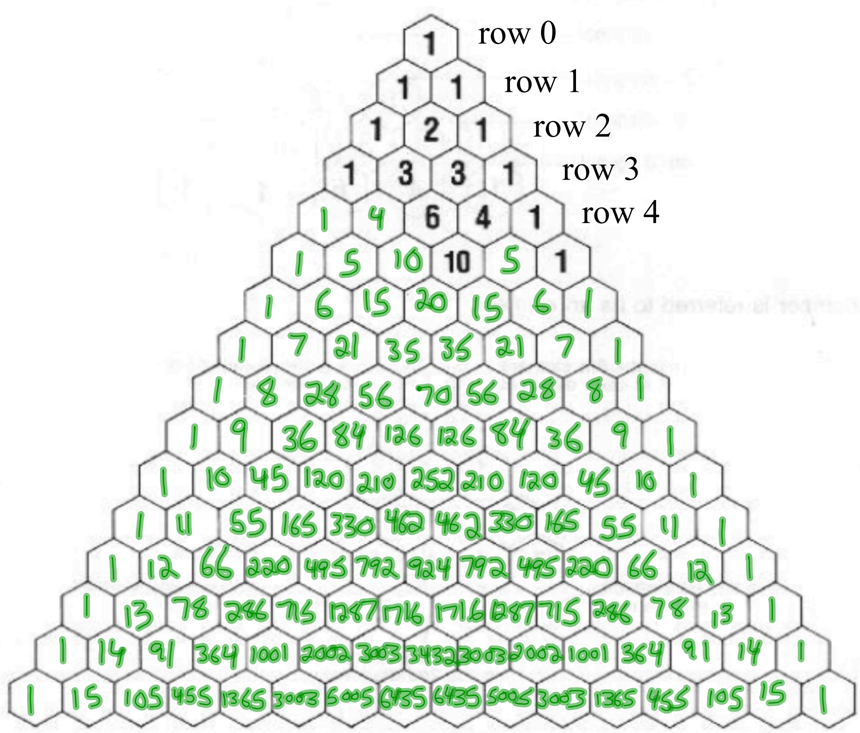
Pascal's Triangle

a) Complete Pascal's Triangle



b) What patterns do you notice in Pascal's Triangle?

Main Pattern:

Each term in Pascal's Triangle is the sum of the two terms directly above it. The first and last terms in each row are 1 since the only term immediately above them is always a 1.

Other Patterns:

- sum of each row is a power of 2 (sum of nth row is 2n, begin count at 0)
- symmetrical down the middle

c) Expand each of the following binomials

$(a+b)^0 = 1$

$(a+b)^1 = 1a + 1b$

$(a+b)^2 = (a+b)(a+b)$
$= 1a^2 + ab + ab + 1b^2$
$= 1a^2 + 2ab + 1b^2$

$(a+b)^3 = (a+b)(a+b)(a+b)$
$= (a+b)(a^2 + 2ab + b^2)$
$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$
$= 1a^3 + 3a^2b + 3ab^2 + 1b^3$

$(a+b)^4 =$ $1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$

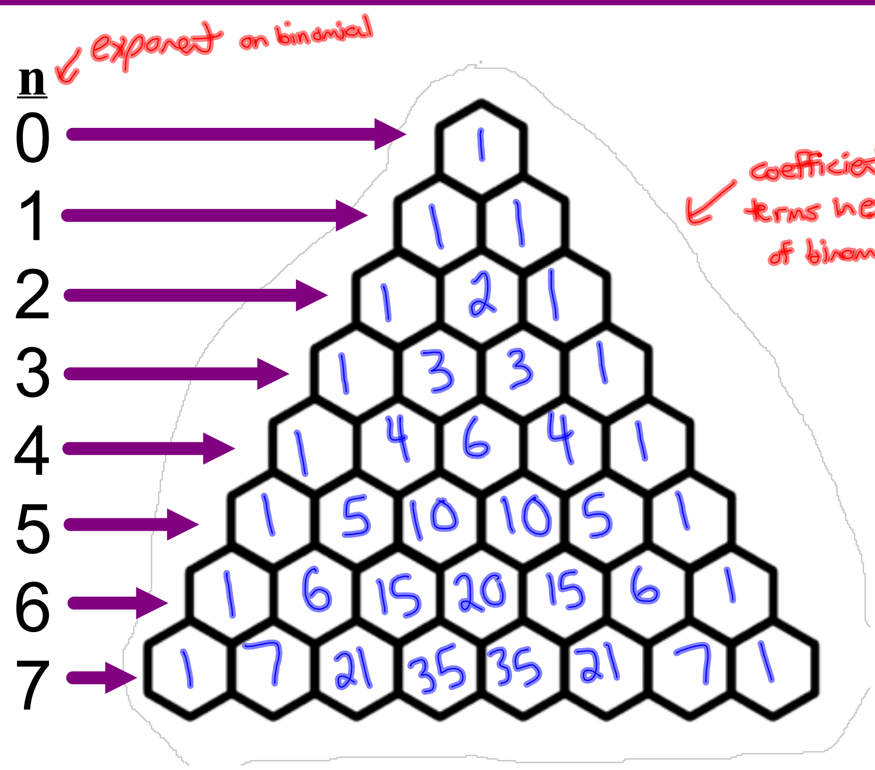
Blaise Pascal (French Mathematician) discovered a pattern in the expansion of $(a+b)^n$.... which patterns do you notice?

The coefficients in the expansion of $(a + b)^n$ can be found in row n of Pascal's triangle.

In each expansion, the exponents of a start at n and decrease by 1 down to zero, while the exponents of b start at zero and increase by 1 up to n.

In each term, the sum of the exponents of a and b is always n.

Pascal's Triangle



Example 1

Expand using the Binomial Theorem:

use row 6 of Pascal's triangle

a) $(a + b)^6$ ← n

$= 1a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + 1b^6$

b) $(2x - 3)^5$

$= 1(2x)^5 + 5(2x)^4(-3)^1 + 10(2x)^3(-3)^2 + 10(2x)^2(-3)^3$

$+ 5(2x)^1(-3)^4 + 1(-3)^5$

$= 32x^5 - 240x^4 + 720x^3 - 1080x^2 + 810x - 243$

c) $(2x + 3y^2)^5$

$$= 1(2x)^5(3y^2)^0 + 5(2x)^4(3y^2)^1 + 10(2x)^3(3y^2)^2$$
$$+ 10(2x)^2(3y^2)^3 + 5(2x)^1(3y^2)^4 + 1(2x)^0(3y^2)^5$$

$$= 1(32)(x^5)(1) + 5(16)(x^4)(3)(y^2) + 10(8)(x^3)(9)(y^4)$$
$$+ 10(4)(x^2)(27)(y^6) + 5(2)(x)(81)(y^8) + 1(1)(243)(y^{10})$$

$$= 32x^5 + 240x^4y^2 + 720x^3y^4 + 1080x^2y^6 + 810xy^8 + 243y^{10}$$

d) $\left(\frac{y}{2} - y^2\right)^4 \leftarrow n$

$$= 1\left(\frac{y}{2}\right)^4(-y^2)^0 + 4\left(\frac{y}{2}\right)^3(-y^2)^1 + 6\left(\frac{y}{2}\right)^2(-y^2)^2$$
$$+ 4\left(\frac{y}{2}\right)^1(-y^2)^3 + 1\left(\frac{y}{2}\right)^0(-y^2)^4$$

$$= \frac{y}{16} + 4\left(\frac{y^3}{8}\right)(-1)(y^2) + 6\left(\frac{y^2}{4}\right)(1)(y^4)$$
$$+ 4\left(\frac{y}{2}\right)(-1)(y^6) + 1(1)(1)(y^8)$$

$$= \frac{y}{16} - \frac{y^5}{2} + \frac{3y^6}{2} - 2y^7 + y^8$$

Example 2

How many terms will there be if you expand $(x + 2y)^{20}$?

number of terms in a binomial expansion is always equal to $n + 1$

$20 + 1 = 21$

$\boxed{21 \text{ terms}}$

Example 3

a) What is the 2nd term in the expansion of $(x+6)^7$

$= 7(x)^6(6)^1$

$= 42x^6$

b) What is the 5th term in the expansion of $(3y - 4)^8$

$= 70(3y)^4(-4)^4$

$= 70(81)(y^4)(256)$

$= 1451520y^4$

Example 4

4th term

a) What is the coefficient of x^3 in the expansion of $(x + 6)^6$

$= 20(x)^3(6)^3$

$= 20x^3(216)$

$= 4320x^3$

b) What is the coefficient of y^4x^2 in the expansion of $(y + 3x)^6$

3rd term

$= 15(y)^4(3x)^2$

$= 15(y^4)(9)(x^2)$

$= 135y^4x^2$

Homework: Worksheet Questions