6.2 Recursive Functions

In earlier sections we used function notation to write an explicit formula to determine the value of any term in a sequence. Sometimes it is easier to calculate one term in a sequence using the previous terms.

Recursion formula:

a formula by which each term of a sequence is generated from the preceding term or terms.

Recursive Functions

Functions that get new terms in the sequence by using earlier terms.

t_n = the value of term '*n*'
t_{n-1} = the value before t_n

Example 1: Write the first 4 terms of the sequence.

a) $t_n = t_{n-1} - 2$ where $t_1 = 7$

$t_2 = t_{2-1} - 2$
$= t_1 - 2$
$= 7 - 2$
$= 5$

$t_3 = t_2 - 2$
$= 5 - 2$
$= 3$

$t_4 = t_3 - 2$
$= 3 - 2$
$= 1$

The first four terms of the sequence are 7, 5, 3, 1.

b) $t_n = 2t_{n-1} + 4$ where $t_1 = 5$

$t_2 = 2t_1 + 4$
$= 2(5) + 4$
$= 14$

$t_3 = 2t_2 + 4$
$= 2(14) + 4$
$= 32$

$t_4 = 2t_3 + 4$
$= 2(32) + 4$
$= 68$

The first four terms of the sequence are 5, 14, 32, 68.

You may also see questions asked in function notation.

Example 2: Find the first 4 terms.

$f(n)=2f(n-1) - 7$ where $f(1) = 2$

$f(2) = 2f(1) - 7$
$= 2(2) - 7$
$= -3$

$f(3) = 2 \cdot f(2) - 7$
$= 2(-3) - 7$
$= -13$

$f(4) = 2 \cdot f(3) - 7$
$= 2(-13) - 7$
$= -33$

The first four terms of the sequence are 2, -3, -13, -33.

Example 3: Find the first 7 terms of the sequence.

$t_n = t_{n-2} + t_{n-1}$ where $t_1 = 1$ where $t_2 = 1$

$t_3 = t_{3-2} + t_{3-1}$
$= t_1 + t_2$
$= 1+1$
$= 2$

$t_4 = t_2 + t_3$
$= 1+2$
$= 3$

$t_5 = t_3 + t_4$
$= 2+3$
$= 5$

Note: *each term in the sequence is the sum of the previous two terms. This is the Fibonacci sequence!*

The first seven terms in this sequence are 1, 1, 2, 3, 5, 8, 13.

Example 4:

Write a recursion formula for each sequence

$\times(-2)$

a) -3, 6, -12, 24,....

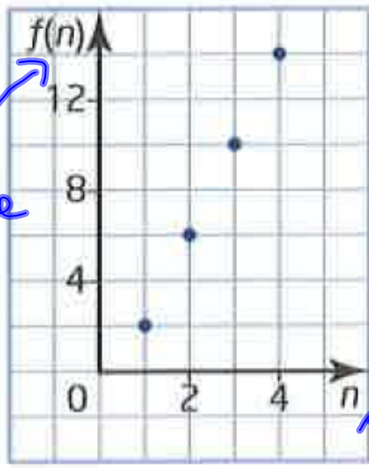
$t_n = -2 \cdot t_{n-1}$

Look for a pattern in the terms:

$t_1 = -3$
$t_2 = t_1 \times (-2)$
$t_3 = t_2 \times (-2)$
$t_4 = t_3 \times (-2)$

b)

term value



term #

+4

2, 6, 10, 14

$$\boxed{t_n = t_{n-1} + 4}$$

+2 +3 +4

c) 3, 5, 8, 12,...

$$\boxed{t_n = t_{n-1} + n}$$