Arithmetic and Geometric Sequences

DO IT NOW!

How much can you figure out about this list of numbers?

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233,...

https://www.youtube.com/watch?v=SjSHVDfXHQ4

Definitions

Formula for general term (explicit formula):

A formula that represents any term in a sequence relative to the term number (n)

Sequence:

an ordered list of numbers identified by a pattern or rule that may stop at some number of continue indefinitely

Ex.

3, 7, 11, 15

2, 6, 18, 54, … the three dots indicate that it continues forever

> **Note:** the terms of a sequence represent the range (*y*-values) of a function.

Example 1: Write the first three terms of each sequence, given the explicit formula for the *n*th term of the sequence.

a) $t_n = 3n^2 - 1$

$t_1 = 3(1)^1 - 1 = 2$ $\quad t_2 = 3(2)^2 - 1 = 11$ $\quad t_3 = 3(3)^2 - 1 = 26$

The first three terms are 2, 11, 26

b) $t_n = \dfrac{n-1}{n}$

$t_1 = \dfrac{1-1}{1} = 0$ $\quad t_2 = \dfrac{2-1}{2} = \dfrac{1}{2}$ $\quad t_3 = \dfrac{3-1}{3} = \dfrac{2}{3}$

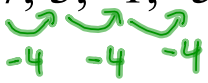
The first three terms are $0, \frac{1}{2}, \frac{2}{3}$

Arithmetic Sequences

Examples of sequences:

a) 14, 18, 22, 26,...

b) 7, 3, -1, -5,...



These are called ***arithmetic*** sequences because they increase by a constant difference (+ or -)

Arithmetic Sequences

Formula for General Term of an Arithmetic Sequence

$$t_n = a + (n-1)d$$

n: *the term number*

a: *the value of the first term*

t_n: *a term in the sequence*

d: *the common difference*

Example 2

a) Determine a formula for the general term of the following arithmetic sequence.

14, 18, 22, 26,...

$a = 14$

$d = 4$

$t_n = a + (n-1)d$

$t_n = 14 + (n-1)(4)$

b) What is the value of t_{30}

$t_{30} = 14 + (30-1)(4)$

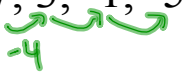
$= 14 + 116$

$= 130$

Example 3:

a) Determine a formula for the general term of the following arithmetic sequence.

7, 3, -1, -5,…



$t_n = a + (n-1)d$

$t_n = 7 + (n-1)(-4)$

$a = 7$

$d = -4$

b) What is the value of t_{41}

$t_{41} = 7 + (41-1)(-4)$

$= 7 - 160$

$= -153$

Geometric Sequences

Examples of sequences:

a) 2, 6, 18, 54,…

b) 80, 40, 20, 10,… * No division is used *

These are called ***geometric*** sequences because the ratio of consecutive terms is constant.

Increase/decrease by a constant multiple

Geometric Sequences

Formula for the General Term of a Geometric Sequence

$$t_n = a \cdot r^{n-1}$$

n: *term number*

a: *the first term*

t_n: *a term in the sequence*

r: *the constant multiple*

Note: no division is used → $\div 2 = \times \frac{1}{2}$

Example 4: Determine a formula for the general term of the following geometric sequence.

$a=2$

$r=3$

$t_n = a \cdot r^{n-1}$

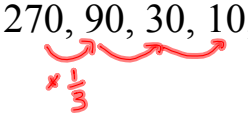
$= 2(3)^{n-1}$

b) What is the value of t_9

$t_9 = 2(3)^{9-1}$

$= 13\,122$

Example 5: Determine a formula for the general term of the following geometric sequence.



$t_n = a \cdot r^{n-1}$

$t_n = 270\left(\frac{1}{3}\right)^{n-1}$

$a=270$

$r=\frac{1}{3}$

b) What is the value of t_9

$t_9 = 270\left(\frac{1}{3}\right)^{9-1}$

$= 270\left(\frac{1}{3^8}\right)$

$= 270\left(\frac{1}{6561}\right)$

$= \frac{10}{243}$