Arithmetic and Geometric Series

Definitions

Formula for general term (explicit formula):

A formula that represents any term in a sequence relative to the term number (n)

Sequence:

an ordered list of numbers identified by a pattern or rule that may stop at some number of continue indefinitely

Arithmetic Sequence:

sequence in which the difference between consecutive terms is a constant

Geometric Sequence:

sequence in which the ratio of consecutive terms is constant

Series:

the indicated **sum** of the terms of a sequence

Example 1: Find S_4 of the sequence represented by:

↖ sum of the first 4 terms of the sequence

$S_4 = t_1 + t_2 + t_3 + t_4$

$t_n = 1+(n-1)3$

$t_1 = 1+(1-1)(3)$
$= 1$

$t_2 = 1+(2-1)(3)$
$= 4$

$t_3 = 1+(3-1)(3)$
$= 7$

$t_4 = 1+(4-1)(3)$
$= 10$

$S_4 = 1+4+7+10$
$= 22$

Arithmetic Series

general form

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

because the general form for an arithmetic sequence is

$$tn = a+(n-1)d$$

OR we can rewrite S_n as:

$$S_n = \frac{n}{2}[a+t_n]$$

use this version when given the value of the last term of the series (t_n)

Example 2: For the series 1+3+5+7+... find S_{23}

$S_n = \frac{n}{2}[2a+(n-1)d]$

$a=1$
$d=2$
$n=23$

$S_{23} = \frac{23}{2}[2(1)+(23-1)(2)]$

$= \frac{23}{2}(46)$

$= 529$

Example 3: An arithmetic series with 52 terms starts with -7 and ends with 102. Find the sum of the series.

$S_{52} = \frac{52}{2}(-7+102)$

$= 26(95)$

$= 2470$

Note: Since we know t_{52}, it would be easier to use this version of the formula...

$$S_n = \frac{n}{2}(a + t_n)$$

Geometric Series

general form

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Example 4: For the geometric sequence -1 + 2 - 4 + 8 - 16,...

×(-2)

a = -1

r = -2

a) Find S_5.

$S_5 = -1 + 2 - 4 + 8 - 16$

$= -11$

b) Find S_{13}

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{13} = \frac{-1[(-2)^{13} - 1]}{-2 - 1}$$

$$= \frac{8193}{-3}$$

$$= -2731$$

Example 5: A student is offered a job with a math teacher that will last 20 hours. It pays $4.75 for the first hour, $5 for the second hour, $5.25 for the next hour, and so on. How much will the student earn in total?

$a = 4.75$

$d = 0.25$

$n = 20$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{20} = \frac{20}{2}[2(4.75) + (20-1)(0.25)]$$

$$S_{20} = 10(14.25)$$

$$= \$142.50$$

The student will be paid $142.50 for 20 hours of work.