

## Unit 1 Pretest Review

MPM2D

Jensen

Unit 1

1) Solve each linear system by graphing.

a)  $\ell_1: y = 4x - 5$

$\ell_2: y = \frac{2}{3}x + 5$

Solution:  $x=3, y=7$

Line 1

$$y = 4x - 5$$

$$\text{slope } m = \frac{4}{1}$$

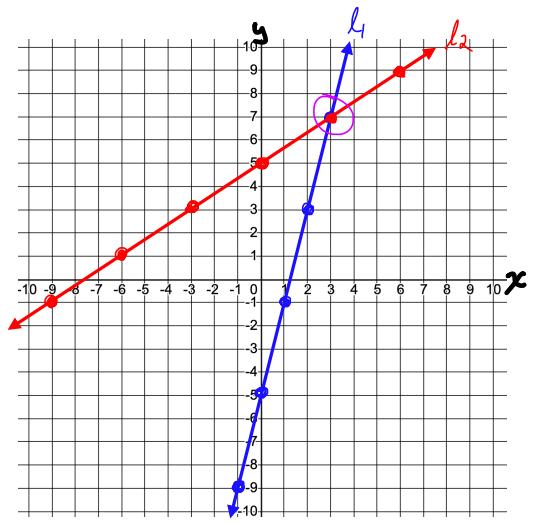
$$y\text{-int } b = -5$$

Line 2

$$y = \frac{2}{3}x + 5$$

$$\text{slope } m = \frac{2}{3}$$

$$y\text{-int } b = 5$$



b)  $\ell_1: 3x + y = 1$

$\ell_2: x + 4y = 4$

solution:  $x=0, y=1$

Line 1

$$3x + y = 1$$

$$y = -3x + 1$$

$$\text{slope } m = -\frac{3}{1}$$

$$y\text{-int } b = 1$$

Line 2

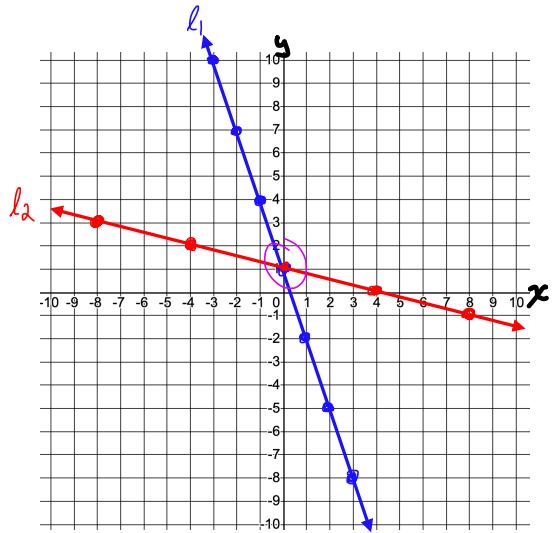
$$x + 4y = 4$$

$$4y = -x + 4$$

$$y = -\frac{1}{4}x + 1$$

$$\text{slope } m = -\frac{1}{4}$$

$$y\text{-int } b = 1$$



c)  $\ell_1: y = -2x + 5$

$\ell_2: y = \frac{1}{2}x - 5$

solution:  $x=4, y=-3$

Line 1

$$y = -2x + 5$$

$$\text{slope } m = -\frac{2}{1}$$

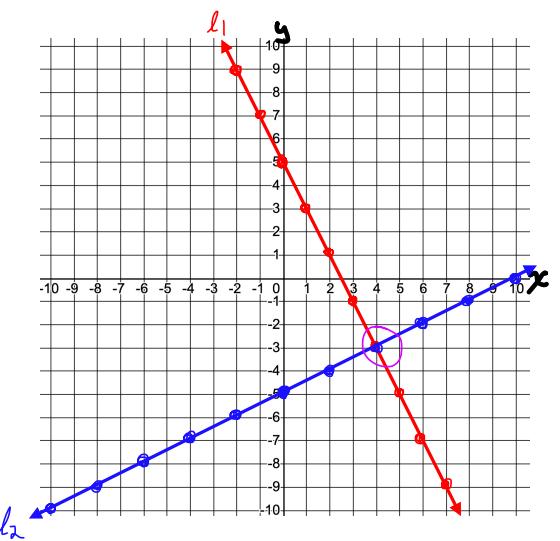
$$y\text{-int } b = 5$$

Line 2

$$y = \frac{1}{2}x - 5$$

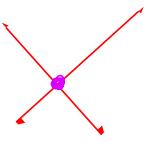
$$\text{slope } m = \frac{1}{2}$$

$$y\text{-int } b = -5$$

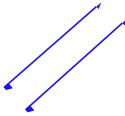


2) What are the different possibilities for the number of solutions to a linear system. Explain each scenario.

- 1 solution if the lines have different slopes.



- 0 solutions if the lines are parallel (same slope) but have different y-intercepts.



Parallel and distinct.

- Infinitely many solutions if the lines have the same slope and same y-intercepts.



Parallel and coincident

3) Solve each linear system using the method of substitution.

a)  $\ell_1: y = 2x + 4$

$\ell_2: x - 4y = -9$

$$\begin{aligned} \textcircled{1} \quad y &= 2x + 4 \\ y &= 2(-1) + 4 \\ y &= -2 + 4 \\ y &= 2 \end{aligned} \quad \begin{aligned} \textcircled{2} \quad x - 4y &= -9 \\ x - 4(2x + 4) &= -9 \\ x - 8x - 16 &= -9 \\ -7x &= 7 \\ x &= -1 \end{aligned}$$

solution:  $x = -1, y = 2$

b)  $\ell_1: 2x = 7 - y$

$\ell_2: 3x - 2y = 21$

$$\begin{aligned} \textcircled{1} \quad 2x &= 7 - y \\ y &= 7 - 2x \\ y &= 7 - 2(5) \\ y &= -3 \end{aligned} \quad \begin{aligned} \textcircled{2} \quad 3x - 2y &= 21 \\ 3x - 2(7 - 2x) &= 21 \\ 3x - 14 + 4x &= 21 \\ 7x &= 35 \\ x &= 5 \end{aligned}$$

solution:  $x = 5, y = -3$

c)  $\ell_1: 3m + 9n = 1$

$\ell_2: m + 3n = 2$

$$\begin{aligned} \textcircled{1} \quad 3m + 9n &= 1 \\ 3(2-3n) + 9n &= 1 \\ 6 - 9n + 9n &= 1 \\ 0n &= -5 \end{aligned} \quad \begin{aligned} \textcircled{2} \quad m + 3n &= 2 \\ m &= 2 - 3n \end{aligned}$$

So NO solutions.

The lines are parallel and distinct.

d)  $\ell_1: 2x - 3y = 6$

$\ell_2: 2x - y = 7$

$$\begin{aligned} \textcircled{1} \quad 2x - 3y &= 6 \\ 2x - 3(2x - 7) &= 6 \\ 2x - 6x + 21 &= 6 \\ -4x &= -15 \\ x &= \frac{15}{4} \end{aligned} \quad \begin{aligned} \textcircled{2} \quad 2x - y &= 7 \\ 2x - 7 &= y \\ 2\left(\frac{15}{4}\right) - 7 &= y \\ \frac{15}{2} - \frac{14}{2} &= y \\ \frac{1}{2} &= y \end{aligned}$$

solution:  $x = \frac{15}{4}, y = \frac{1}{2}$

e)  $\ell_1: 3x - 8 = -2y$   
 $\ell_2: 2x + 3y = 7$

$$\begin{aligned} \textcircled{1} \quad 3x - 8 &= -2y \\ -\frac{3}{2}x + 4 &= y \\ -\frac{3}{2}(2) + 4 &= y \\ -3 + 4 &= y \\ y &= 1 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad 2x + 3y &= 7 \\ 2x + 3(-\frac{3}{2}x + 4) &= 7 \\ 2x - \frac{9}{2}x + 12 &= 7 \\ \frac{4}{2}x - \frac{9}{2}x &= -5 \\ -\frac{5}{2}x &= -5 \\ -5x &= -10 \\ x &= 2 \end{aligned}$$

Solution:  $x=2, y=1$

f)  $\ell_1: 2x = 6 - y$   
 $\ell_2: 3x - 2y = 2$

$$\begin{aligned} \textcircled{1} \quad 2x &= 6 - y \\ y &= 6 - 2x \\ y &= 6 - 2(2) \\ y &= 2 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad 3x - 2y &= 2 \\ 3x - 2(6 - 2x) &= 2 \\ 3x - 12 + 4x &= 2 \\ 7x &= 14 \\ x &= 2 \end{aligned}$$

Solution:  $x=2, y=2$

4) Solve each linear system using the method of elimination.

a)  $\ell_1: x - y = 14$   
 $\ell_2: 2x + 5y = -7$

$$\begin{aligned} 2x \textcircled{1} \quad 2x - 2y &= 28 \\ \textcircled{2} \quad 2x + 5y &= -7 \\ \hline -7y &= 35 \\ y &= -5 \end{aligned}$$

$$\begin{aligned} \text{sub } y = -5 \text{ into } \textcircled{1} \\ x - (-5) &= 14 \\ x + 5 &= 14 \\ x &= 9 \end{aligned}$$

Solution:  $x=9, y=-5$

b)  $\ell_1: 9x - 3y = 15$   
 $\ell_2: 3x - y = 5$

$$\begin{aligned} \textcircled{1} \quad 9x - 3y &= 15 \\ 3x \textcircled{2} \quad 9x - 3y &= 15 \\ \hline 0x + 0y &= 0 \end{aligned}$$

Infinitely many solutions.  
The lines are parallel and coincident.

c)  $\ell_1: 3x + 4y = 17$   
 $\ell_2: 7x - 2y = 17$

$$\begin{aligned} \textcircled{1} \quad 3x + 4y &= 17 \\ 2x \textcircled{2} \quad 14x - 4y &= 34 \\ \hline 17x &= 51 \\ x &= 3 \end{aligned}$$

$$\begin{aligned} \text{sub } x = 3 \text{ into } \textcircled{1} \\ 3(3) + 4y &= 17 \\ 9 + 4y &= 17 \\ 4y &= 8 \\ y &= 2 \end{aligned}$$

Solution:  $x=3, y=2$

d)  $\ell_1: 2x + 5y = 18$   
 $\ell_2: 3x + 5y - 17 = 0$

$$\begin{aligned} \textcircled{1} \quad 2x + 5y &= 18 \\ \textcircled{2} \quad 3x + 5y &= 17 \\ \hline -x &= 1 \\ x &= -1 \end{aligned}$$

$$\begin{aligned} \text{sub } x = -1 \text{ into } \textcircled{1} \\ 2(-1) + 5y &= 18 \\ -2 + 5y &= 18 \\ 5y &= 20 \\ y &= 4 \end{aligned}$$

Solution:  $x=-1, y=4$

e)  $\ell_1: 3x = 34 - 2y$   
 $\ell_2: 5x - 3y = -13$

$$\begin{array}{l} \textcircled{1} \quad 3x + 2y = 34 \\ \textcircled{2} \quad 5x - 3y = -13 \\ \hline 3x \textcircled{1} \quad 9x + 6y = 102 \\ 2x \textcircled{2} \quad 10x - 6y = -26 \\ \hline 19x = 76 \\ x = 4 \end{array}$$

$$\begin{array}{l} \text{sub } x=4 \text{ into } \textcircled{2} \\ 5x - 3y = -13 \\ 5(4) - 3y = -13 \\ 20 - 3y = -13 \\ 3y = 33 \\ y = 11 \end{array}$$

solution:  $x = 4, y = 11$

f)  $\ell_1: 5x + 2y = 5$   
 $\ell_2: 2x + 3y = 13$

$$\begin{array}{l} \textcircled{1} \quad 5x + 2y = 5 \\ \textcircled{2} \quad 2x + 3y = 13 \\ \hline 3x \textcircled{1} \quad 15x + 6y = 15 \\ 2x \textcircled{2} \quad 4x + 6y = 26 \\ \hline 11x = -11 \\ x = -1 \\ \text{sub } x = -1 \text{ into } \textcircled{2} \\ 2(-1) + 3y = 13 \\ -2 + 3y = 13 \\ 3y = 15 \\ y = 5 \end{array}$$

solution:  $x = -1, y = 5$

- 5) Petr has \$5000 invested in two plans. One plan pays 5% simple interest per year and the other pays 8%. At the end of the year, Petr receives a total of \$340 in interest. How much did he invest in each plan?

$x$  = amount at 5%

$y$  = amount at 8%

$$\textcircled{1} \quad x + y = 5000 \quad \textcircled{2} \quad 0.05x + 0.08y = 340$$

$$\begin{array}{r} 100 \times \textcircled{2} \quad 5x + 8y = 34000 \\ 5x \textcircled{1} \quad 5x + 5y = 25000 \\ \hline 3y = 9000 \\ y = 3000 \end{array}$$

sub  $y = 3000$  into  $\textcircled{1}$

$$\begin{array}{l} x + y = 5000 \\ x + 3000 = 5000 \\ x = 2000 \end{array}$$

solution: Petr invested \$2000 at 5% and  
\$3000 at 8%.

- 6) A physics contest has 30 multiple choice questions. A correct answer gains 4 points, while a wrong answer loses 1 point. Rolly answered every question and scored 55 points. How many questions did he answer correctly?

$x$  = # correct  
 $y$  = # wrong

$$\begin{array}{l} \textcircled{1} \quad x + y = 30 \\ \textcircled{2} \quad 4x - y = 55 \\ \hline 5x = 85 \\ x = 17 \end{array}$$

solution: Rolly got 17 correct.

7) One lawn fertilizer is 24% nitrogen, and another is 12% nitrogen. How much of each fertilizer should be mixed to obtain 100 kg of fertilizer that is 21% nitrogen?

$x$  = amount of 24% nitrogen

$y$  = amount of 12% nitrogen

$$\textcircled{1} \quad x + y = 100$$

$$\textcircled{2} \quad 0.24x + 0.12y = 0.21(100)$$

$$100 \times \textcircled{2} \quad 24x + 12y = 2100$$

$$12 \times \textcircled{1} \quad 12x + 12y = 1200 -$$

$$\underline{12x = 900}$$

$$x = 75$$

sub  $x=75$  into \textcircled{1}

$$x + y = 100$$

$$75 + y = 100$$

$$y = 25$$

Solution: 75 kg of 24% nitrogen and 25 kg of 12% should be mixed.

8) A small plane took 3 hours to fly 960 km from Ottawa to Halifax with a tail wind. On the return trip, flying into the wind, the plane took 4 hours. Find the wind speed and the speed of the plane in still air.

$x$  = speed of plane in still air

$y$  = wind speed.

$$\textcircled{1} \quad 3(x+y) = 960$$

$$3x + 3y = 960$$

$$\textcircled{2} \quad 4(x-y) = 960$$

$$4x - 4y = 960$$

remember:

distance = speed  $\times$  time

$$\frac{1}{3} \times \textcircled{1} \quad x + y = 320$$

$$\frac{1}{4} \times \textcircled{2} \quad x - y = 240 -$$

$$\underline{2y = 80}$$

$$y = 40$$

sub  $y=40$  into \textcircled{1}

$$x + y = 320$$

$$x + 40 = 320$$

$$x = 280$$

Solution: The speed of the plane in still air is 280 km/h  
The speed of the wind is 40 km/h

9) The Outdoors Club held a car wash to raise money. They washed cars for \$5 each and vans for \$7 each. They washed 45 vehicles and earned \$243. How many of each type of vehicle did they wash?

$x$  = # of cars

$y$  = # of vans

$$\textcircled{1} \quad x + y = 45$$

$$\textcircled{2} \quad 5x + 7y = 243$$

$$5x \times \textcircled{1} \quad \underline{5x + 5y = 225} -$$

$$2y = 18$$

$$y = 9$$

sub  $y=9$  into \textcircled{1}

$$x + y = 45$$

$$x + 9 = 45$$

$$x = 36$$

Solution: They washed 36 cars and 9 vans

## Answers

- 1)a)**  $x = 3, y = 7$  **b)**  $x = 0, y = 1$  **c)**  $x = 4, y = -3$   
**2)** no solutions if the lines are parallel and distinct; 1 solution if the lines are not parallel; infinitely many solutions if the lines are parallel and coincident.  
**3)a)**  $x = -1, y = 2$  **b)**  $x = 5, y = -3$  **c)** no solutions **d)**  $x = \frac{15}{4}, y = \frac{1}{2}$  **e)**  $x = 2, y = 1$  **f)**  $x = 2, y = 2$   
**4)a)**  $(9, -5)$  **b)** infinite solutions **c)**  $(3, 2)$  **d)**  $(-1, 4)$  **e)**  $(4, 11)$  **f)**  $(-1, 5)$   
**5)** \$2000 at 5% and \$3000 at 8%  
**6)** 17  
**7)** 75 kg of 24% nitrogen, 25 kg of 12% nitrogen  
**8)** wind 40 km/h; plane in still air 280 km/h  
**9)** 36 cars and 9 vans