

W4 - Reciprocal Trig Ratios

MCR3U

Jensen

1) Determine the measure of each angle, to the nearest degree, if the angles are in the first quadrant.

a) $\cot A = 7$

$$\tan A = \frac{1}{7}$$

$$A = \tan^{-1}\left(\frac{1}{7}\right)$$

$$A \approx$$

b) $\sec B = \frac{7}{3}$

$$\cos B = \frac{3}{7}$$

$$B = \cos^{-1}\left(\frac{3}{7}\right)$$

$$B \approx 65^\circ$$

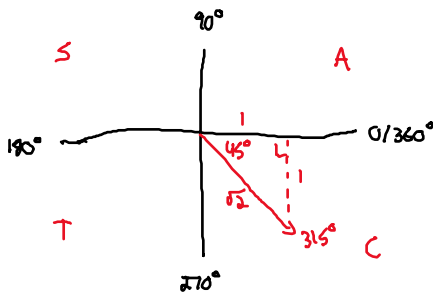
c) $\csc C = \frac{11}{8}$

$$\sin C = \frac{8}{11}$$

$$C = \sin^{-1}\left(\frac{8}{11}\right)$$

$$C \approx 47^\circ$$

2) Determine the exact expressions for the six trigonometric ratios for 315° . Hint: Draw a diagram of the angle in standard position. Then use special triangles to determine the exact values.



$$\sin 315^\circ = -\frac{1}{\sqrt{2}}$$

$$\csc 315^\circ = -\sqrt{2}$$

$$\cos 315^\circ = \frac{1}{\sqrt{2}}$$

$$\sec 315^\circ = \sqrt{2}$$

$$\tan 315^\circ = -1$$

$$\cot 315^\circ = -1$$

3) Find the measure, to the nearest degree, of an angle in the first quadrant that satisfies each ratio. If there is no such angle, explain why.

a) $\sin A = \frac{2}{3}$

$$A = \sin^{-1}\left(\frac{2}{3}\right)$$

$$A \approx 42^\circ$$

b) $\cos B = \frac{3}{5}$

$$B = \cos^{-1}\left(\frac{3}{5}\right)$$

$$B \approx 53^\circ$$

c) $\csc D = \frac{9}{8}$

$$\sin D = \frac{8}{9}$$

$$D = \sin^{-1}\left(\frac{8}{9}\right)$$

$$D \approx 63^\circ$$

d) $\sec E = \frac{4}{3}$

$$\cos E = \frac{3}{4}$$

$$E = \cos^{-1}\left(\frac{3}{4}\right)$$

$$E \approx 41^\circ$$

e) $\csc G = -\frac{4}{3}$

$$\sin G = -\frac{3}{4}$$

No angles in Q1 have a negative sine ratio so no solution.

f) $\sec H = \frac{2}{5}$

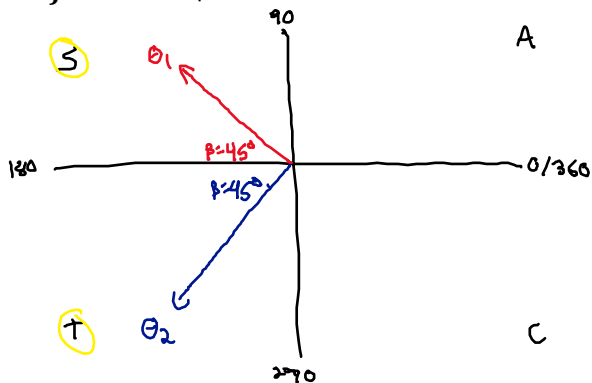
$$\cos H = \frac{5}{2}$$

cosine ratios are always between -1 and 1 because the hypotenuse has to be larger than the adjacent side.

so no solution.

4) Solve each of the following equations for $0^\circ \leq \theta \leq 360^\circ$. Round answers to the nearest degree.

a) $\sec \theta = -\sqrt{2}$



$\sec \theta = -\sqrt{2}$

$\cos \theta = -\frac{1}{\sqrt{2}}$

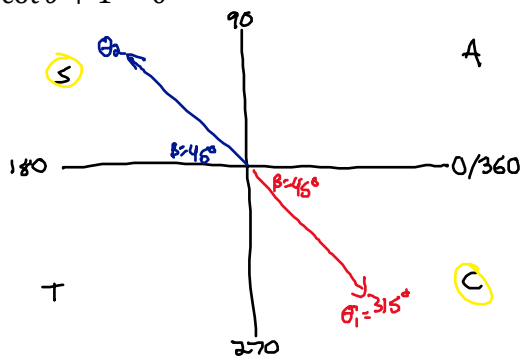
$\theta_1 = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$\theta_1 = 135^\circ$

$\theta_2 = 180^\circ + 45^\circ$

$\theta_2 = 225^\circ$

b) $\cot \theta + 1 = 0$



$\cot \theta = -1$

$\tan \theta = -1$

$\theta_1 = \tan^{-1}(-1)$

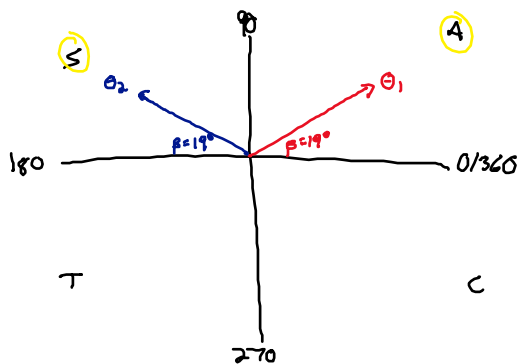
$\theta_1 = -45^\circ + 360^\circ$

$\theta_1 = 315^\circ$

$\theta_2 = 180^\circ - 45^\circ$

$\theta_2 = 135^\circ$

c) $\csc \theta = 3$



$\csc \theta = 3$

$\sin \theta = \frac{1}{3}$

$\theta_1 = \sin^{-1}\left(\frac{1}{3}\right)$

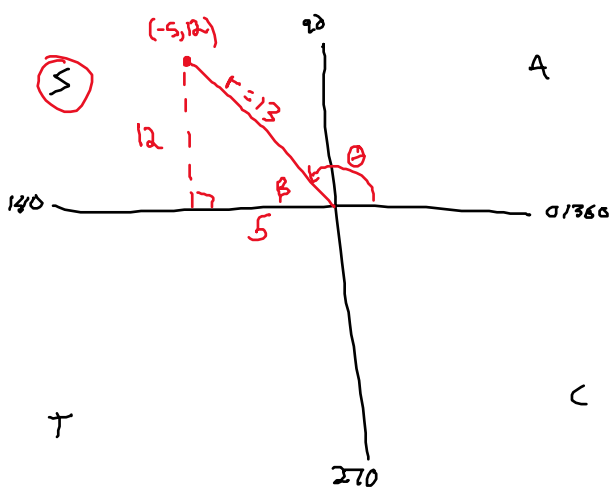
$\theta_1 \approx 19^\circ$

$\theta_2 = 180^\circ - 19^\circ$

$\theta_2 \approx 161^\circ$

5) Each point lies on the terminal arm of an angle in standard position. Determine exact expressions for the six trigonometric ratios for the angle.

a) $P(-5, 12)$



$\sin \theta = \frac{12}{13}$

$\csc \theta = \frac{13}{12}$

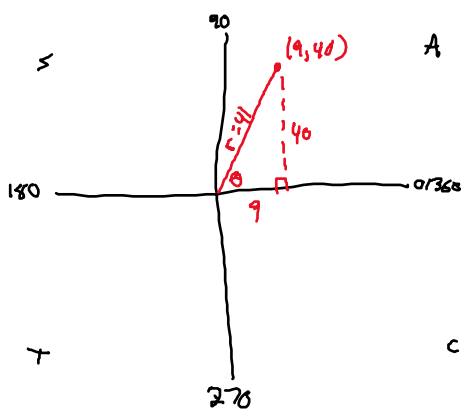
$\cos \theta = -\frac{5}{13}$

$\sec \theta = -\frac{13}{5}$

$\tan \theta = -\frac{12}{5}$

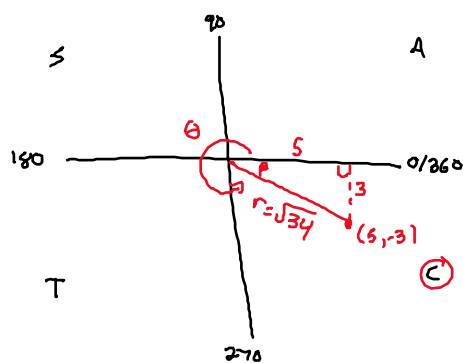
$\cot \theta = -\frac{5}{12}$

b) T(9, 40)



$$\begin{aligned} \sin \theta &= \frac{40}{41} & \csc \theta &= \frac{41}{40} \\ \cos \theta &= \frac{9}{41} & \sec \theta &= \frac{41}{9} \\ \tan \theta &= \frac{40}{9} & \cot \theta &= \frac{9}{40} \end{aligned}$$

c) V(5, -3)



$$\begin{aligned} \sin \theta &= -\frac{3}{\sqrt{34}} & \csc \theta &= -\frac{\sqrt{34}}{3} \\ \cos \theta &= \frac{5}{\sqrt{34}} & \sec \theta &= \frac{\sqrt{34}}{5} \\ \tan \theta &= -\frac{3}{5} & \cot \theta &= -\frac{5}{3} \end{aligned}$$

Answers

1) a) $A = 8^\circ$ b) $B = 65^\circ$ c) $C = 47^\circ$

2) $\sin 315^\circ = -\frac{1}{\sqrt{2}}$ $\cos 315^\circ = \frac{1}{\sqrt{2}}$ $\tan 315^\circ = -1$
 $\csc 315^\circ = -\sqrt{2}$ $\sec 315^\circ = \sqrt{2}$ $\cot 315^\circ = -1$

3) a) 42° b) 53° c) 63° d) 41° e) no solution f) no solution

4) a) 135° and 225° b) 135° and 315° c) 19° and 161°

5) a) $\sin \theta = \frac{12}{13}$ $\cos \theta = \frac{-5}{13}$ $\tan \theta = \frac{12}{-5}$
 $\csc \theta = \frac{13}{12}$ $\sec \theta = \frac{13}{-5}$ $\cot \theta = \frac{-5}{12}$
 $\csc \theta = \frac{13}{12}$ $\sec \theta = \frac{13}{-5}$ $\cot \theta = \frac{-5}{12}$
 $\csc \theta = \frac{13}{12}$ $\sec \theta = \frac{13}{-5}$ $\cot \theta = \frac{-5}{12}$

b) $\sin \theta = \frac{40}{41}$ $\cos \theta = \frac{9}{41}$ $\tan \theta = \frac{40}{9}$
 $\csc \theta = \frac{41}{40}$ $\sec \theta = \frac{41}{9}$ $\cot \theta = \frac{9}{40}$

c) $\sin \theta = \frac{-3}{\sqrt{34}}$ $\cos \theta = \frac{5}{\sqrt{34}}$ $\tan \theta = -\frac{3}{5}$
 $\csc \theta = \frac{\sqrt{34}}{-3}$ $\sec \theta = \frac{\sqrt{34}}{5}$ $\cot \theta = \frac{5}{-3}$
 $\csc \theta = \frac{\sqrt{34}}{-3}$ $\sec \theta = \frac{\sqrt{34}}{5}$ $\cot \theta = \frac{5}{-3}$
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