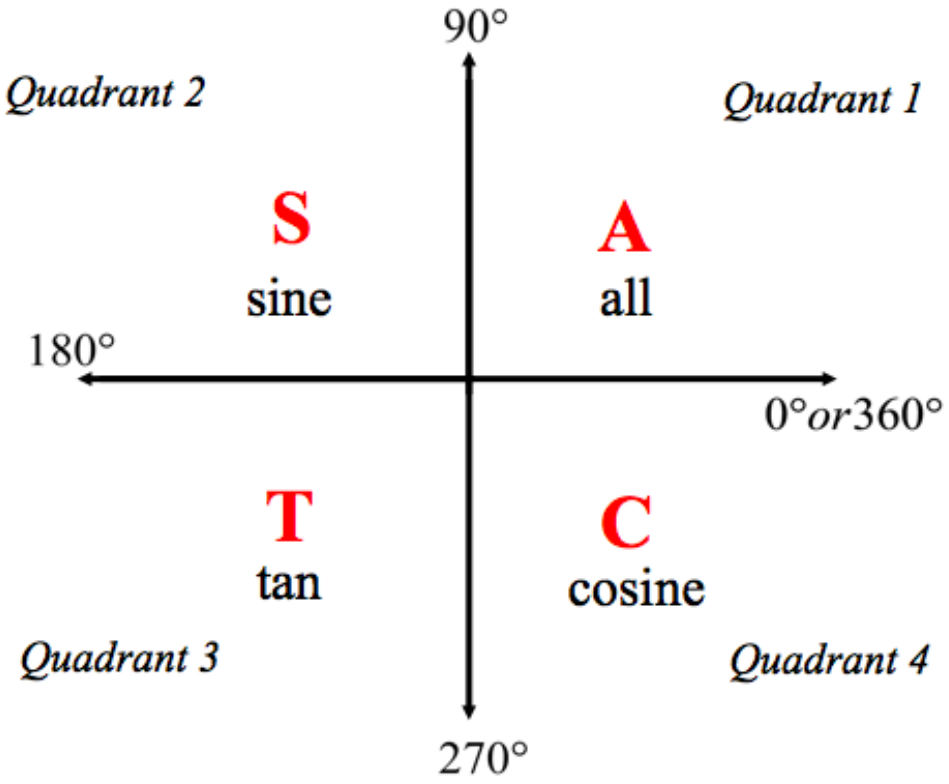
Chapter 4- Trigonometry

Workbook

MCR3U



Chapter 4 Workbook Checklist

Worksheet	Check ✓
4.1 – Special Angles 1 - Worksheet	
4.1 – Special Angles 2 - Worksheet	
4.2 – Related and Co-Terminal Angles – Worksheet	
4.3 – Reciprocal Trig Ratios Worksheet	
4.4 – Problems in 2-Dimensions Worksheet	
4.5 – Problems in 3-Dimensions Worksheet	
The Ambiguous Case of Sine Worksheet	
4.6 – Trig Identities Worksheet 1	
4.6 – Trig Identities Worksheet 2	

W1 – Special Angles
MCR3U
Jensen

1) a) Draw a right triangle that has one angle measuring $30°$. Label the sides using lengths $\sqrt{3}, 2$, and 1.
b) Identify the adjacent and opposite sides relative to the $30°$ angle.

c) Redraw the triangle and identify the adjacent and opposite sides relative to the $60°$ angle.

2) a) Draw a right triangle that has one angle measuring $45°$. Label the sides using the lengths 1, 1, and $\sqrt{2}$.
b) Identify the adjacent and opposite sides relative to one of the $45°$ angles.

3) State the exact values

a) $\sin 60°$

b) $\cos 30°$

c) $\tan 45°$

d) $\cos 45°$

4) Determine the exact value of each trig expression.

a) $sin30° \times tan60° - cos30°$

b) $1 - \frac{sin45°}{cos45°}$

5) Using exact values, show that $sin^2\theta + cos^2\theta = 1$ for each angle

a) $\theta = 30°$

b) $\theta = 45°$

c) $\theta = 60°$

6) Using the appropriate special triangle, determine θ if $0° \leq \theta \leq 90°$

a) $\sin\theta = \frac{\sqrt{3}}{2}$

b) $\sqrt{3}\tan\theta = 1$

c) $2\sqrt{2}\cos\theta = 2$

d) $2\cos\theta = \sqrt{3}$

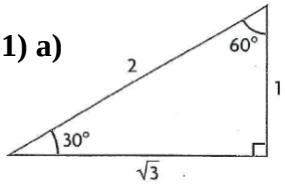
7) A 5 meter stepladder propped against a classroom wall forms an angle of $30°$ with the wall. Exactly how far is the top of the ladder from the floor? Express your answer in radical form.

8) Simplify the following expressions by rationalizing the denominator

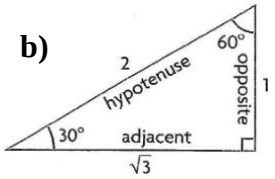
a) $\frac{3\sqrt{2}}{\sqrt{5}}$ **b)** $\frac{6\sqrt{5}}{\sqrt{3}}$ **c)** $\frac{4-5\sqrt{3}}{1+\sqrt{2}}$

Answers

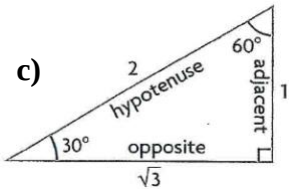
1) a)



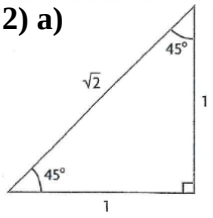
b)



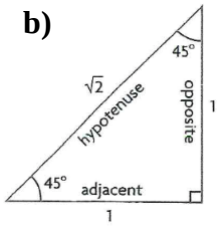
c)



2) a)



b)



3) a) $\frac{\sqrt{3}}{2}$ **b)** $\frac{\sqrt{3}}{2}$ **c)** 1 **d)** $\frac{\sqrt{2}}{2}$

4) a) 0 **d)** 0

5) a) $\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2=1$ **b)** $\left(\frac{\sqrt{2}}{2}\right)^2+\left(\frac{\sqrt{2}}{2}\right)^2=1$ **c)** $\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2=1$

6) a) $60°$ **b)** $30°$ **c)** $45°$ **d)** $30°$

7) $\frac{5\sqrt{3}}{2}$ m

8) a) $\frac{3\sqrt{10}}{5}$ **b)** $2\sqrt{15}$ **c)** $-4+4\sqrt{2}+5\sqrt{3}-5\sqrt{6}$

W2 – Ratios for Angles Greater than 90°

MCR3U

Jensen

1) Sketch each angle in standard position and determine the related acute angle.

a) 135^0 b) 210^0 c) 315^0 d) -30^0

e) -225^0 f) -330^0 g) 150^0 h) -120^0

i) 105^0 j) -163^0 k) -141^0 l) -280^0

2) State the value of each ratio **exactly**.

a) sin 225°

c) sin 270°

b) cos 240°

d) tan 300°

e) cos 180°

3) Point P (-9, 4) is on the terminal arm of an angle in standard position.

a) Sketch the principal angle, θ.

b) What is the measure of β, the related acute angle to the nearest degree?

c) What is the measure of θ to the nearest degree?

4) Point P (7, -24) is on the terminal arm of an angle in standard position.

a) Sketch the principal angle, θ.

b) What is the measure of β, the related acute angle to the nearest degree?

c) What is the measure of θ to the nearest degree?

Answers

1) a) 45^0 b) 30^0 c) 45^0 d) 30^0 e) 45^0 f) 30^0 g) 30^0 h) 60^0 i) 75^0 j) 17^0 k) 39^0 l) 80^0

2) a) $-\frac{1}{\sqrt{2}}$ b) $-\frac{1}{2}$ c) -1 d) $-\sqrt{3}$ e) -1

3) b) 24^0 c) 156^0

4) b) 74^0 c) 286^0

W3 – Solving Trig Equations

MCR3U

Jensen

1. For each point given on the terminal arm of the angle, determine the exact primary values for the trigonometric ratios of the angle.

a) (1, 3)

b) (-3, 4)

c) $(-6, -5)$

d) $(6, -10)$

2. One of the primary trigonometric ratios for an angle is given, as well as the quadrant in which each angle is located. Find the other two trigonometric ratios of the angle.

a) $\sin A = \frac{3}{4}$, first quadrant

b) $\cos B = -\frac{2}{3}$, second quadrant

3. Determine any three positive angles that are co-terminal with 150°.

4. Solve the following equations for $0° \leq \theta \leq 360°$. Round answers to the nearest degree.

a) $\sin\theta = \frac{1}{2}$

b) $2\cos\theta = -\sqrt{3}$

c) $\tan\theta = \frac{1}{8.1516}$

d) $5\cos\theta + 3 = 0$

5. Consider $\angle F$ such that $\cos F = \frac{12}{37}$.

a) Which quadrants can $\angle F$ be in?

b) Find the coordinates of a point on the terminal arm of the angle in each quadrant.

c) If you are also told that the sine of the angle is negative, in which quadrant is $\angle F$?

d) Write the other primary trigonometric ratios for $\angle F$ in the quadrant identified in part c.

Answers

1. a) $\sin\theta = \frac{3}{\sqrt{10}}$, $\cos\theta = \frac{1}{\sqrt{10}}$, $\tan\theta = 3$ **b)** $\sin\theta = \frac{4}{5}$, $\cos\theta = -\frac{3}{5}$, $\tan\theta = -\frac{4}{3}$

c) $\sin\theta = -\frac{5}{\sqrt{61}}$, $\cos\theta = -\frac{6}{\sqrt{61}}$, $\tan\theta = \frac{5}{6}$ **d)** $\sin\theta = -\frac{5}{\sqrt{34}}$, $\cos\theta = \frac{3}{\sqrt{34}}$, $\tan\theta = -\frac{5}{3}$

2. a) $\cos A = \frac{\sqrt{7}}{4}$, $\tan A = \frac{3}{\sqrt{7}}$ **b)** $\sin B = \frac{\sqrt{5}}{3}$, $\tan B = -\frac{\sqrt{5}}{2}$

3. Answers may vary. Sample answer: 510°, 870°

4.a) $30°, 150°$ **b)** $150°, 210°$ **c)** $97°, 277°$ **d)** $127°, 233°$

5. a) the first and fourth quadrants
b) first quadrant: (12, 35); fourth quadrant: (12, –35)
c) fourth quadrant
d) $\sin F = -\frac{35}{37}$, $\tan F = -\frac{35}{12}$

W4 – Reciprocal Trig Ratios

MCR3U

Jensen

1) Determine the measure of each angle, to the nearest degree, if the angles are in the first quadrant.

a) cot A = 7

b) sec B = $\frac{7}{3}$

c) csc C = $\frac{11}{8}$

2) Determine the exact expressions for the six trigonometric ratios for 315°. Hint: Draw a diagram of the angle in standard position. Then use special triangles to determine the exact values.

3) Find the measure, to the nearest degree, of an angle in the first quadrant that satisfies each ratio. If there is no such angle, explain why.

a) $\sin A = \frac{2}{3}$

b) $\cos B = \frac{3}{5}$

c) $csc\ D = \frac{9}{8}$

d) $sec\ E = \frac{4}{3}$

e) $csc\ G = -\frac{4}{3}$

f) $sec\ H = \frac{2}{5}$

4) Solve each of the following equations for $0° \leq \theta \leq 360°$. Round answers to the nearest degree.

a) $\sec\theta = -\sqrt{2}$

b) $\cot\theta + 1 = 0$

c) $\csc\theta = 3$

5) Each point lies on the terminal arm of an angle in standard position. Determine exact expressions for the six trigonometric ratios for the angle.

a) P(-5, 12)

b) T(9, 40)

c) V(5, -3)

Answers

1) a) $A = 8°$ **b)** $B = 65°$ **c)** $C = 47°$

2) $\sin 315° = -\frac{1}{\sqrt{2}}$ $\cos 315° = \frac{1}{\sqrt{2}}$ $\tan 315° = -1$

$\csc 315° = -\sqrt{2}$ $\sec 315° = \sqrt{2}$ $\cot 315° = -1$

3) a) $42°$ **b)** $53°$ **c)** $63°$ **d)** $41°$ **e)** no solution **f)** no solution

4)a) $135°$ and $225°$ **b)** $135°$ and $315°$ **c)** $19°$ and $161°$

5) a) $\sin\theta = \frac{12}{13}$ $\cos\theta = \frac{-5}{13} = -\frac{5}{13}$ $\tan\theta = \frac{12}{-5} = -\frac{12}{5}$

$\csc\theta = \frac{13}{12}$ $\sec\theta = \frac{13}{-5} = -\frac{13}{5}$ $\cot\theta = \frac{-5}{12} = -\frac{5}{12}$

b) $\sin\theta = \frac{40}{41}$ $\cos\theta = \frac{9}{41}$ $\tan\theta = \frac{40}{9}$

$\csc\theta = \frac{41}{40}$ $\sec\theta = \frac{41}{9}$ $\cot\theta = \frac{9}{40}$

c) $\sin\theta = \frac{-3}{\sqrt{34}} = -\frac{3}{\sqrt{34}}$ $\cos\theta = \frac{5}{\sqrt{34}}$ $\tan\theta = -\frac{3}{5}$

$\csc\theta = \frac{\sqrt{34}}{-3} = -\frac{\sqrt{34}}{3}$ $\sec\theta = \frac{\sqrt{34}}{5}$ $\cot\theta = \frac{5}{-3} = -\frac{5}{3}$

W5 – Problems in 2 and 3-Dimensions
MCR3U
Jensen

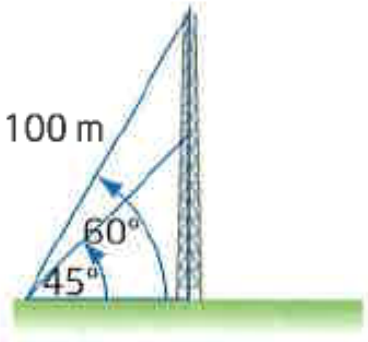
1) The shadow of a tree that is 12 meters tall measures 9 meters in length. Determine the angle of elevation of the sun.

2) Yolanda flies her ultra-light airplane due east for 100 km. She turns right through an angle of 130°, and flies a second leg. Then, she turns right 110° and returns to her starting point.

a) Represent the flight path using an appropriate diagram, labeling all information.

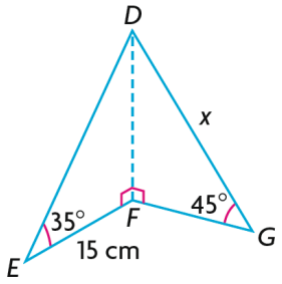
b) Determine the total length of the flight, to the nearest km.

3) A radio antenna is stabilized by two guy wires. One guy wire is 100 m in length and is attached to the top of the antenna. The wire makes an angle of 60° with the ground. One end of the second guy wire is attached to the ground at the same point as the first guy wire. The other end is attached to the antenna such that the wire makes an angle of 45° with the ground. Determine an exact expression for the distance between the points where the two guy wires are attached to the antenna.

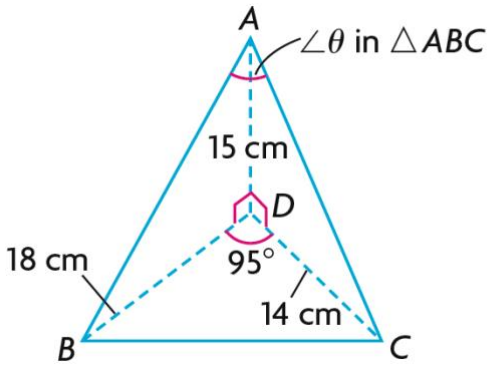


4) A Ferris wheel has a radius of 20 m, with 10 cars spaced around the circumference at equal distances. If the cars are numbered in order, how far is it directly from the first car to the fifth car?

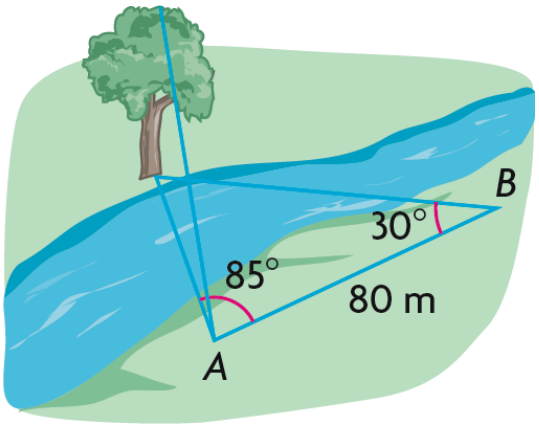
5) Determine the value of x to the nearest cm.



6) Determine the measure of θ to the nearest degree.



7) Bert wants to calculate the height of a tree on the opposite bank of a river. To do this, he lays out a baseline 80 m long and measure the angles as shown. The angle of elevation from A to the top of the tree is 28°. Calculate the height of the tree to the nearest meter.



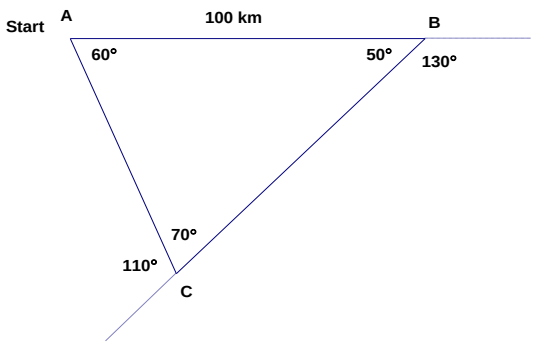
8) The bases on a baseball diamond are 27.4 m apart. The pitcher pitches, and the batter hits a fly ball straight up 15 m. What is the maximum angle of elevation of the ball, to the nearest degree, as seen by the pitcher if he is standing at the center of the diamond?

9) Ranjeet praks his car in a lot on the corner of Park Lane and Main Street. He walks 80 m east to First Aveunue, turns 30° to the left, and follows First Avenue for 100 m to the Metro Building, where he takes the elevator to his office on the 15^{th} floor. Each floor in the building is 4 m in height. From his offie window, Ranjeet can see his car in the lot. How far is Ranjeet from his car, in a direct line to the nearest meter?

Answers

1) 53°

2) a)



b) 274 km

3) $50\left(\sqrt{3}-1\right)$ m.

4) approximately 38 m.

5) 15 cm

6) 93°

7) 23 m

8) 38°

9) 184 m

W6 - The Ambiguous Case of Sine
MCR3U
Jensen

1) In $\Delta ABC, a = 13$ cm, $b = 21$ cm, and $\angle A = 29°$. Draw possible diagrams that match the given measurements. Then calculate the length of side c.

2) In $\Delta ABC, a = 5.9$ m, $b = 7.8$ m, and $\angle A = 36°$. Draw possible diagrams that match the given measurements. Then calculate the length of side c.

3) In $\Delta ABC, a = 2.4$ cm, $c = 3.2$ cm, and $\angle A = 28°$. Determine two possible measures for $\angle C$ and for the length of side b.

4) In $\Delta DEF, d = 3$ cm, $e = 5$ cm, and $\angle D = 30°$. Determine two possible measures for $\angle E$ and for the length of side f.

5) Two ships, S and T, are 120 km apart when they pick up a distress call from a yacht. Ship T estimates that the yacht is 70 km away and that the angle between the line from T to S and the line from S to the yacht is 28°. What are two possible distances, to the nearest tenth of a km, from ship S to the yacht?

Answers

1) 26.5 cm or 10.3 cm
2) 10 m or 2.6 m
3) $\angle C = 39°$ and $b = 4.7$ cm; $\angle C = 141°$ and $b = 1.0$ cm
4) $\angle E = 56°$ and $f = 6$ cm; $\angle E = 124°$ and $f = 2.7$ cm
5) 147.5 km or 64.4 km

W7a - 4.6 Trig Identities
MCR3U
Jensen

1) Prove each identity

a) $sin\theta = cos\theta\ tan\theta$

c) $cos\theta = sin\theta\ cot\theta$

b) $csc\theta = sec\theta\ cot\theta$

d) $sec\theta = csc\theta\ tan\theta$

2) Prove each identity

a) $1 + csc\theta = csc\theta(1 + sin\theta)$

b) $cot\theta\ sin\theta\ sec\theta = 1$

c) $cos\theta(sec\theta - 1) = 1 - cos\theta$

d) $1 + sin\theta = sin\theta(1 + csc\theta)$

3) Prove that $1 - sin^2\theta = sin\theta\ cos\theta\ cot\theta$

4) Prove that $csc^2\theta = cot^2\theta + 1$

5) Prove that $\frac{cos\theta}{1+sin\theta} = \frac{1-sin\theta}{cos\theta}$

6) Prove that $\frac{cos\theta}{1-sin\theta} + \frac{cos\theta}{1+sin\theta} = \frac{2}{cos\theta}$

7) Prove that $csc^2\theta\ cos^2\theta = csc^2\theta - 1$

8) Prove that $tan^2\theta - sin^2\theta = sin^2\theta\ tan^2\theta$

W7b – Extra Practice of Trig Identities
MCR3U
Jensen

Prove each of the following identities on the next page:

1) $cos\theta \times tan\theta = sin\theta$

2) $\frac{cot\theta}{tan\theta} = \frac{1-sin^2\theta}{1-cos^2\theta}$

3) $cot^2\theta = \frac{cos^2\theta}{1-cos^2\theta}$

4) $\frac{csc\theta}{sec\theta} = cot\theta$

5) $(sin\theta + cos\theta)^2 = 1 + 2\ sin\theta\ cos\theta$

6) $2\ sin^2\theta - 1 = sin^2\theta - cos^2\theta$

7) $\frac{1}{sin^2\theta} + \frac{1}{cos^2\theta} = \frac{1}{sin^2\theta\ cos^2\theta}$

8) $cos^2\theta = sin^2\theta + 2\ cos^2\theta - 1$

9) $tan\theta = tan^2\theta \times cot\theta$

10) $sec^2\theta + csc^2\theta = sec^2\theta \times csc^2\theta$

11) $\frac{1}{1+sin\theta} + \frac{1}{1-sin\theta} = 2\ sec^2\theta$

12) $tan^2\theta - sin^2\theta = sin^2\theta \times tan^2\theta$

13) $\frac{1+2sin\theta\ cos\theta}{sin\theta+cos\theta} = sin\theta + cos\theta$

14) $\frac{sec\theta+1}{sec\theta-1} + \frac{cos\theta+1}{cos\theta-1} = 0$

15) $\frac{csc\theta}{csc\theta-1} + \frac{csc\theta}{csc\theta+1} = 2\ sec^2\theta$

Solutions: