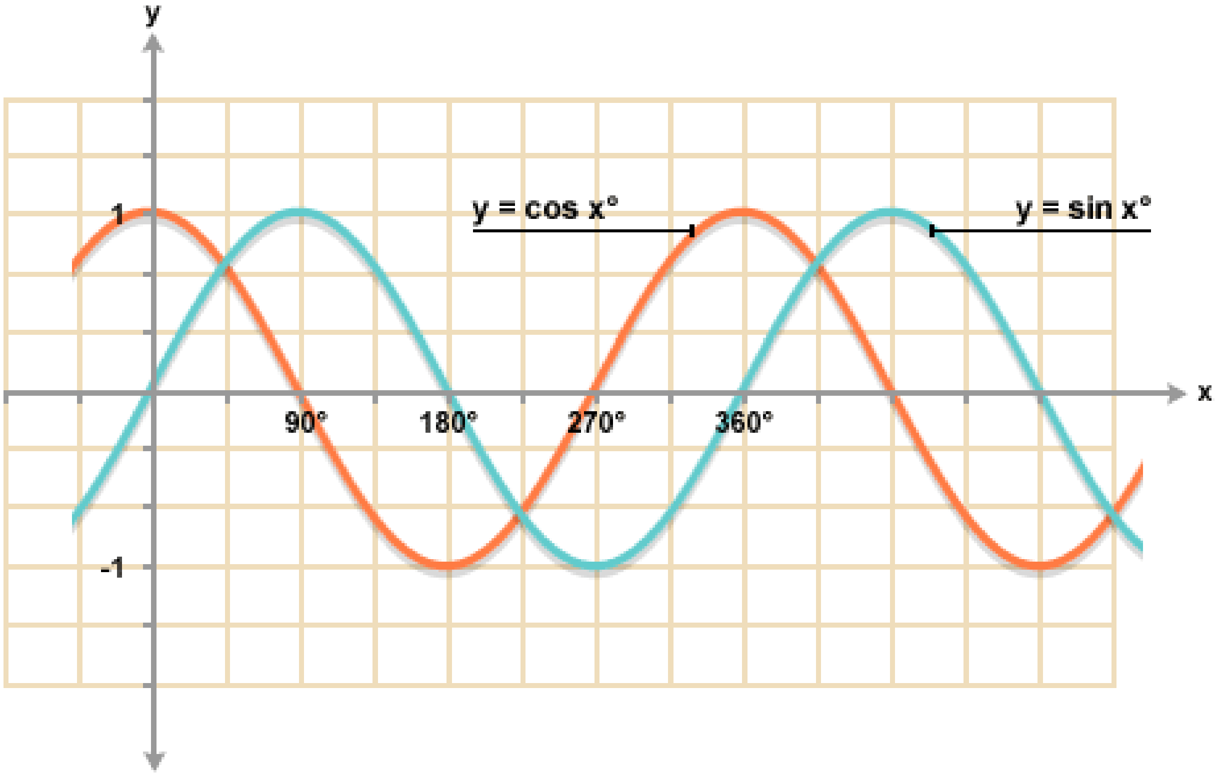
Chapter 5- Trig Functions

Workbook

MCR3U



Chapter 5 Workbook Checklist

Worksheet	Check
5.1 – Modeling Periodic Behaviour	
Graphing Sine and Cosine Functions Worksheet	
5.3 – Transformations of Sine and Cosine Worksheet #1	
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5.5/5.6 Applications of Sine and Cosine Worksheet #1	

5.1 Periodic Behaviour
WORKSHEET SOLUTIONS

1 and 2

a) Yes it is periodic.

Period $= 4-(-1)$
$= 5$

Amplitude $= \frac{2-(-1)}{2}$
$= \frac{3}{2}$ OR 1.5

b) Not periodic

c) Yes it is periodic

Period $= 6-0$
$= 6$

Amplitude $= \frac{1-(-1)}{2}$
$= \frac{2}{2}$
$= 1$

d) Yes it is periodic

Period $= 4-0.5$
$= 3.5$

Amplitude $= \frac{1.5-0}{2}$
$= 0.75$

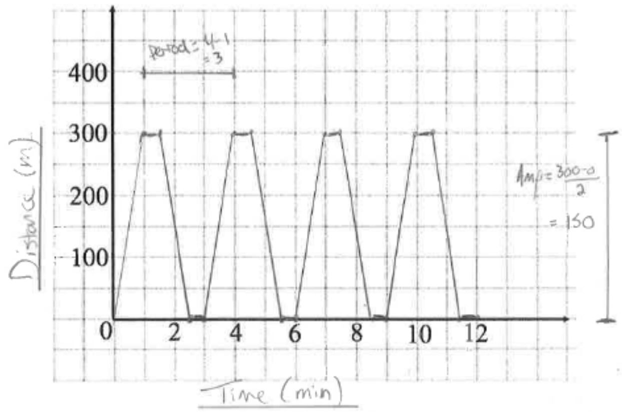
3 a) $f(9) = f(9-8)$
$= f(1)$
$= -3$

b) $f(29) = f[29-3(8)]$
$= f(29-24)$
$= f(5)$
$= 2$

c) $f(63) = f[63-7(8)]$
$= f(63-56)$
$= f(7)$
$= 8$

d) not possible.

4 a)



b) Period = 4-1

= 3 minutes

c) Amplitude = $\frac{max - min}{2}$

$= \frac{300-0}{2}$

= 150 meters

5 a) Period = difference between 6:30 am and 6:50 pm

= 12 hours and 20 minutes

b) Amplitude = $\frac{max - min}{2}$

$= \frac{3.3 - 0.7}{2}$

= 1.3 meters

c) 6 hours and 10 minutes between high tide and low tide; therefore the next low tide is 6 hours and 10 minutes after 6:50 pm

= 1:00 am

Graphing Sine and Cosine Functions Worksheet

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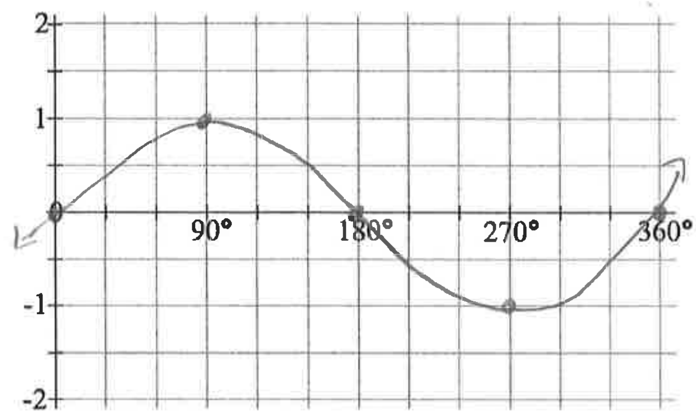
SOLUTIONS

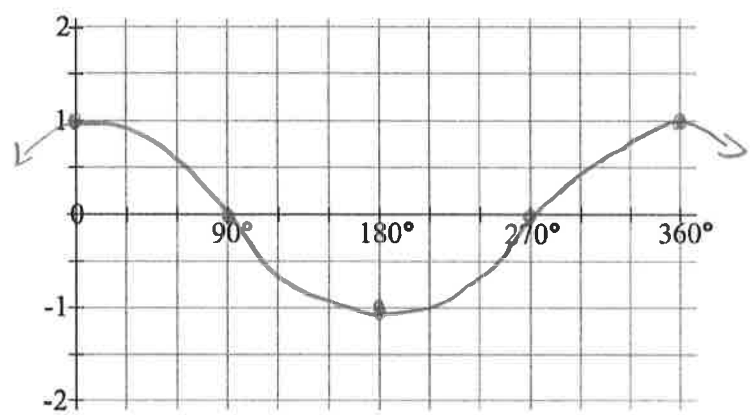
1) Graph the function $y = sinx$ using key points between 0° and 360°.

x	y
0	0
90	1
180	0
270	-1
360	0



2) Graph the function $y = cosx$ using key points between 0° and 360°.

x	y
0	1
90	0
180	-1
270	0
360	1



3) Determine the phase shift and the vertical shift of y=sinx.

a) $y = \sin(x - 50°) + 3$

right 50°

up 3

b) $y = 2\sin(x + 45°) - 1$

left 45°

down 1

4) Determine the phase shift and the vertical shift of y=cosx.

a) $y = -9\cos(x + 120°) - 5$

left 120°

down 5

b) $y = 12\cos[5(x - 150°)] + 7$

right 150°

up 7

5) Determine the amplitude, the period, phase shift, vertical shift, maximum and minimum for each of the following.

a) $y = 5\sin[4(x + 60°)] - 2$

amplitude = 5

period = $\frac{360}{4} = 90°$

shift left 60°

shift down 2

max = 5 - 2 = 3

min = -2 - 5 = -7

b) $y = 2\cos[2(x + 150°)] - 5$

amplitude = 2

period = $\frac{360}{2} = 180°$

shift left 150°

shift down 5

max = 2 - 5 = -3

min = -5 - 2 = -7

c) $y = \frac{1}{2}\sin[\frac{1}{2}(x - 60°)] + 1$

amplitude = $\frac{1}{2}$

period = $\frac{360}{0.5} = 720°$

shift right 60°

shift up 1

max = $\frac{1}{2} + 1 = \frac{3}{2}$ OR 1.5

min = $1 - \frac{1}{2} = \frac{1}{2}$ OR 0.5

d) $y = 0.8\cos[3.6(x - 40°)] - 0.4$

amplitude = 0.8

period = $\frac{360}{3.6} = 100°$

shift right 40°

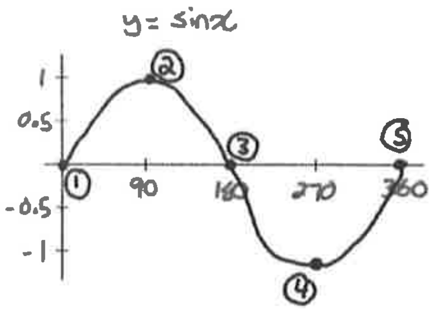
shift down 0.4

max = 0.8 - 0.4 = 0.4

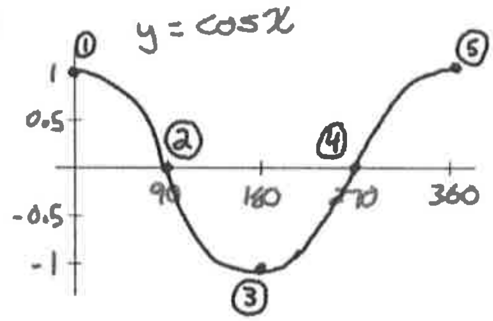
min = -0.4 - 0.8 = -1.2

Answers

1)



2)



3) a) phase shift: right 50°
vertical shift: up 3 units

b) phase shift: left 45°
vertical shift: down one unit

4) a) phase shift: left 120°
vertical shift: down 5 units

b) phase shift: right 150°
vertical shift: up 7 units

5) a) amplitude: 5 period: 90° phase shift: left 60°
vertical shift: down 2 units max: 3 min: -7

b) amplitude: 2 period: 180° phase shift: left 150°
vertical shift: down 5 units max: -3 min: -7

c) amplitude: $\frac{1}{2}$ period: 720° phase shift: right 60°
vertical shift: up 1 unit max: 1.5 min: 0.5

d) amplitude: 0.8 period: 100° phase shift: right 40°
vertical shift: down 0.4 units max: 0.4 min: -1.2

5.3 Transformations of Sine and Cosine Worksheet #1

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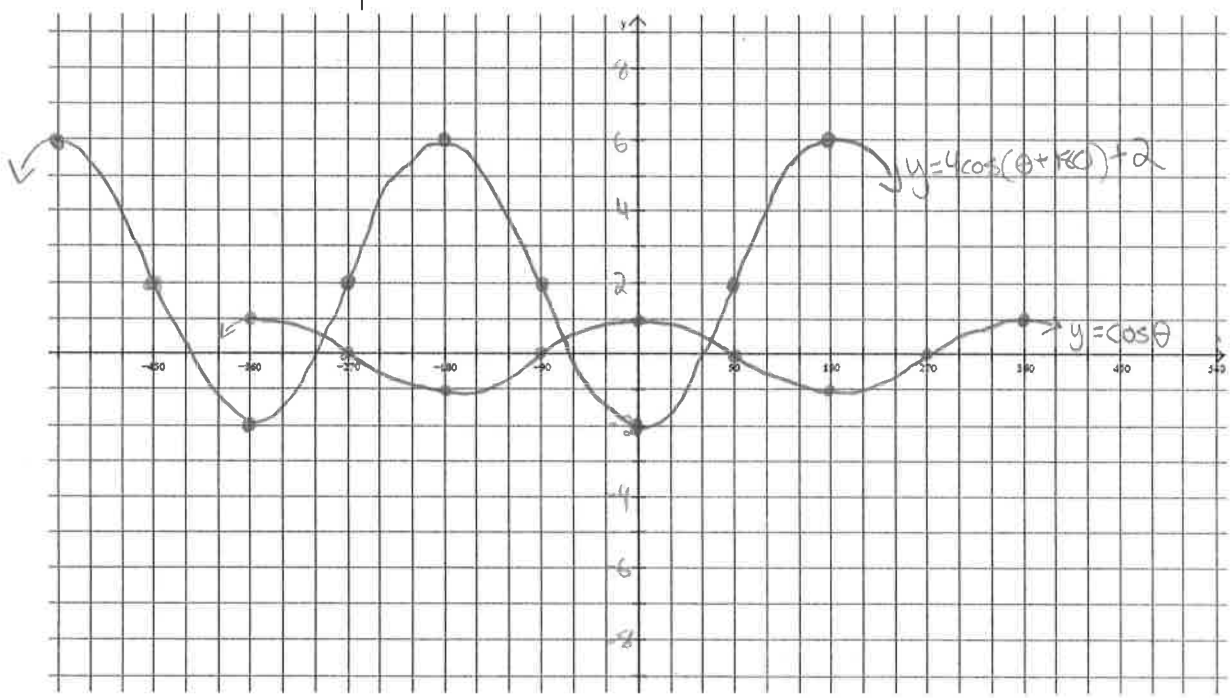
SOLUTIONS

Instructions: For each of the following start with two cycles of the parent graph (-360 to 360). Transform key points to produce the new graph.

1) Using transformations, graph two cycles of the following trigonometric functions. State the period, phase shift, amplitude, and vertical displacement.

a) $y = 4\cos(\theta + 180^{0}) + 2$

period = 360 amplitude = 4
left 180°
up 2



$y = \cos\theta$

θ	y
0	1
90	0
180	-1
270	0
360	1

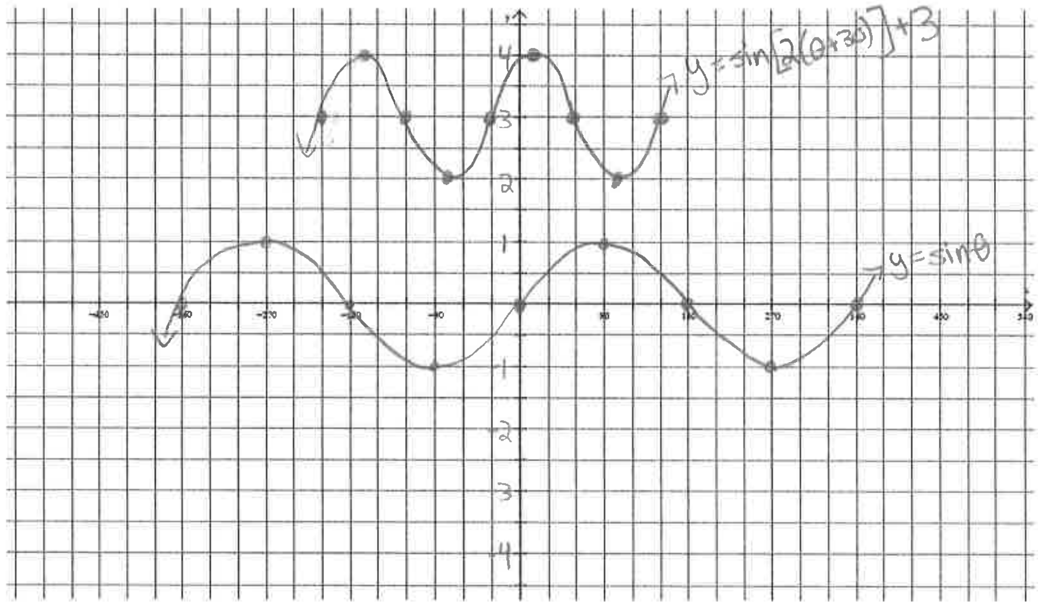
$y = 4\cos(\theta + 180) + 2$

$\theta - 180$	$4y + 2$
-180	6
-90	2
0	-2
90	2
180	6

b) $y = \sin\left[2\left(\theta + 30^{0}\right)\right] + 3$

amplitude = 1
period = $\frac{360}{2}$
= 180

left 30°
up 3



y=sinθ

θ	y
0	0
90	1
180	0
270	-1
360	0

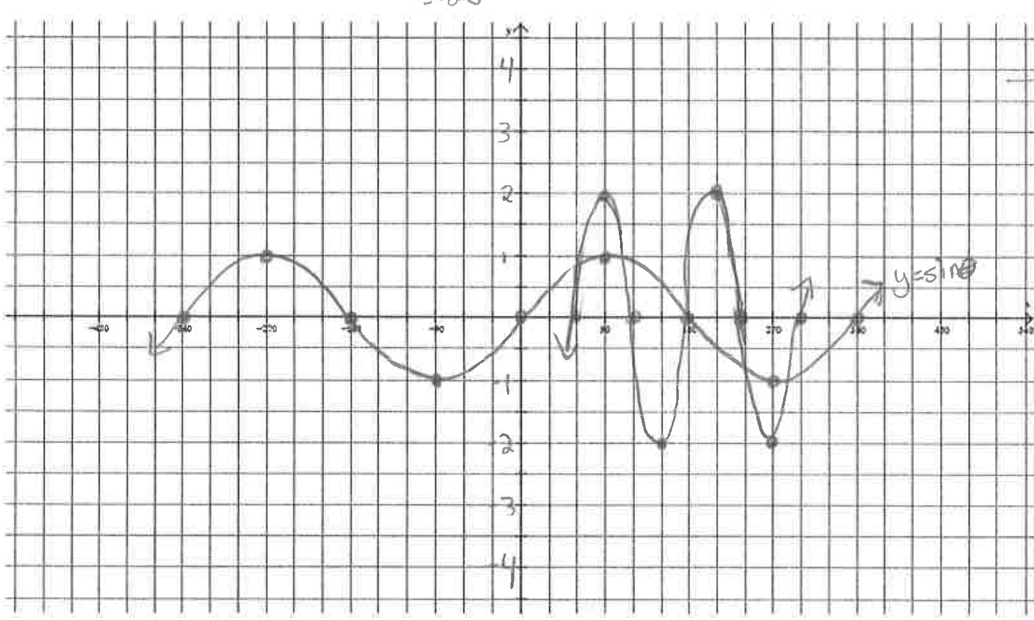
y = sin[2(θ+30)]+3

$\frac{\theta}{2}$ -30	y+3
-30	3
15	4
60	3
105	2
150	3

c) $y = 2\sin\left[3\left(\theta - 180^{0}\right)\right]$

amplitude = 2
period = $\frac{360}{3}$
= 120°

right 180°



y=sinθ

θ	y
0	0
90	1
180	0
270	-1
360	0

y=2sin[3(θ-180)]

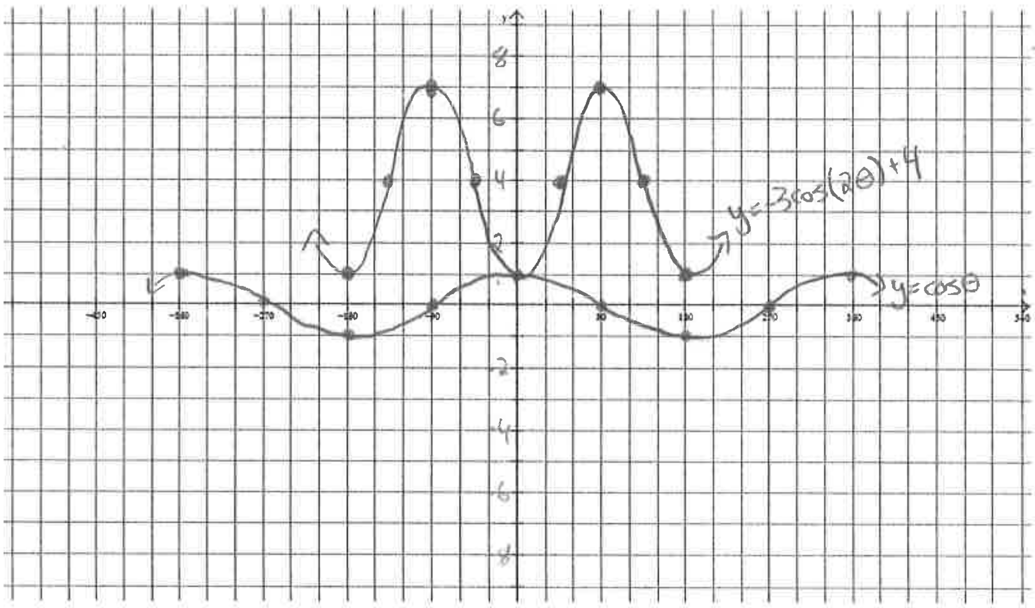
$\frac{\theta}{3}$ +180	2y
180	0
210	2
240	0
270	-2
300	0

d) $y = -3\cos(2\theta) + 4$

amplitude = 3

period = $\frac{360}{2}$ = 180°

up 4



$y = \cos\theta$

θ	y
0	1
90	0
180	-1
270	0
360	1

$y = -3\cos(2\theta) + 4$

$\frac{\theta}{2}$	$-3y+4$
0	1
45	4
90	7
135	4
180	1

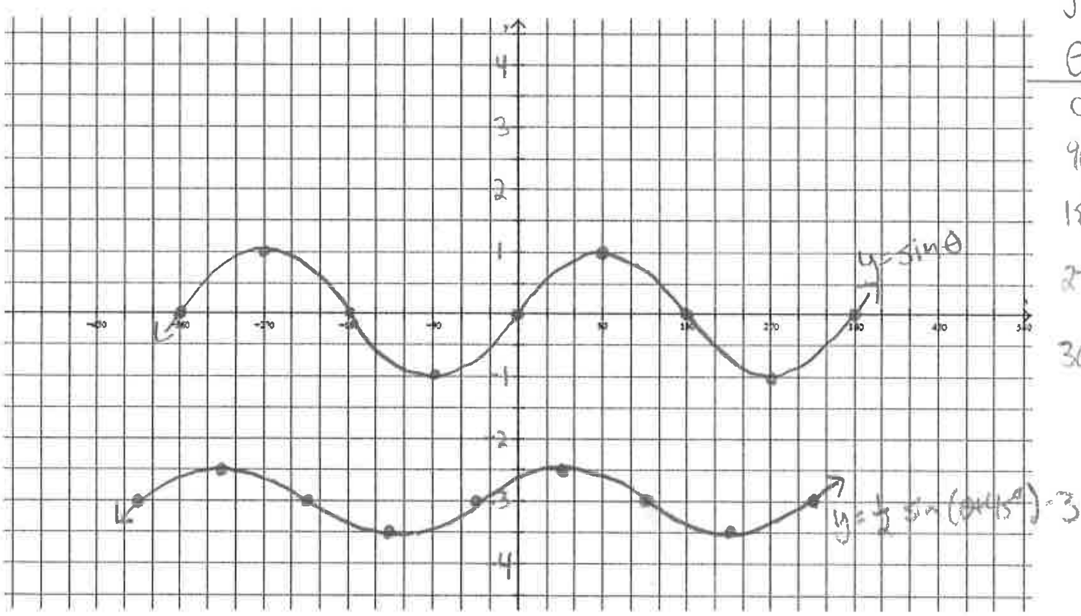
e) $y = \frac{1}{2}\sin(\theta + 45^0) - 3$

amplitude = ½

period = 360

left 45°

down 3



$y = \sin\theta$

θ	y
0	0
90	1
180	0
270	-1
360	0

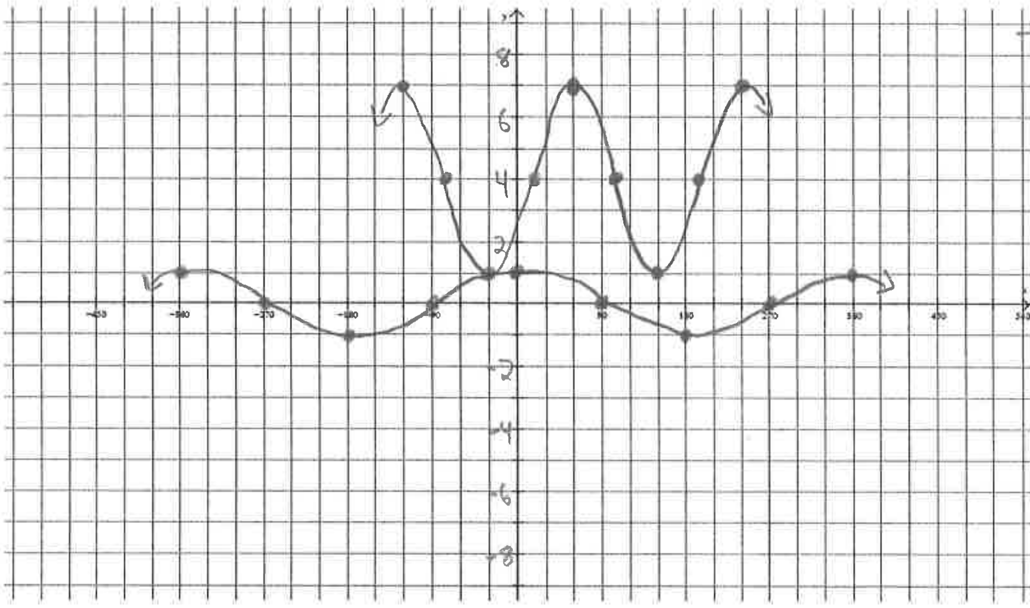
$y = \frac{1}{2}\sin(\theta + 45°) - 3$

$\theta - 45°$	$\frac{y}{2} - 3$
-45	-3
45	-2.5
135	-3
225	-3.5
315	-3

f) $y = 3\cos\left[2\left(\theta - 60^0\right)\right] + 4$

amplitude = 3
period = $\frac{360}{2}$ = 180°

right 60°
up 4



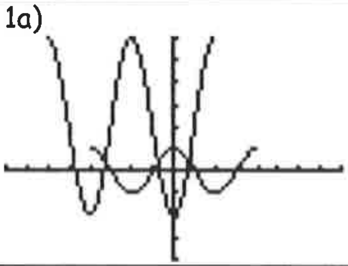
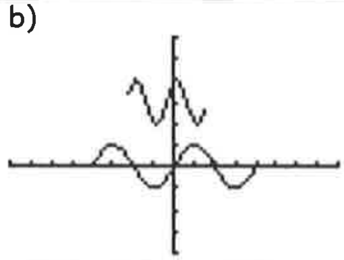
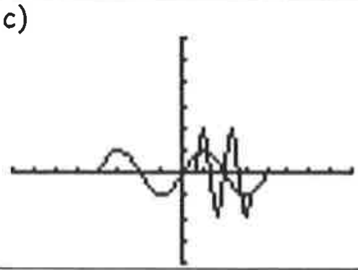
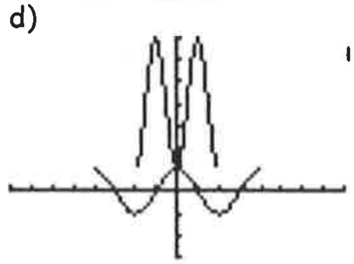
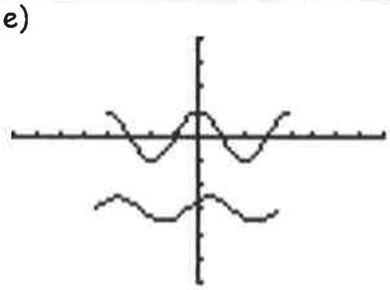
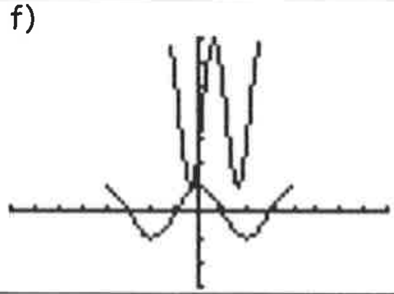
$y = \cos\theta$

θ	y
0	1
90	0
180	-1
270	0
360	1

$y = 3\cos[2(\theta - 60°)] + 4$

$\frac{\theta}{2} + 60$	$3y + 4$
60	7
105	4
150	1
195	4
240	7

Answers

1a) 	b) 
c) 	d) 
e) 	f) 

a) period= 360°
phase shift = left 180°
amplitude = 4
vertical shift = up 2

b) period = 180°
phase shift = left 30°
amplitude = 1
vertical shift = up 3

c) period = 120°
phase shift = right 180°
amplitude = 2
vertical shift = none

d) period = 180°
phase shift = none
amplitude = 3
vertical shift = up 4

e) period = 360°
phase shift = left 45°
amplitude = $\frac{1}{2}$
vertical shift = down 3

f) period = 180°
phase shift = right 60°
amplitude = 3
vertical shift = up 4

5.3 Transformations of Sine and Cosine Worksheet #2

SOLUTIONS

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1) A sinusoidal function has an amplitude of 5 units, a period of 120°, and a maximum at (0, 3).

a) Represent the function with an equation using a sine function

$a = 5$

$k = \frac{360}{period} = \frac{360}{120} = 3$

$c = max - |a| = 3 - 5 = -2$

$d_{sin} = d_{cos} - \frac{90}{k} = 0 - \frac{90}{3} = -30$

$y = 5\sin[3(x+30)] - 2$

b) Represent the function with an equation using a cosine function

$d_{cos} = 0$

$y = 5\cos(3x) - 2$

2) A sinusoidal function has an amplitude of $\frac{1}{2}$ units, a period of 720°, and a maximum at $\left(0, \frac{3}{2}\right)$.

a) Represent the function with an equation using a sine function

$a = 0.5$

$k = \frac{360}{period} = \frac{360}{720} = 0.5$

$c = max - |a| = 1.5 - 0.5 = 1$

$d_{sin} = d_{cos} - \frac{90}{|k|} = 0 - \frac{90}{0.5} = -180$

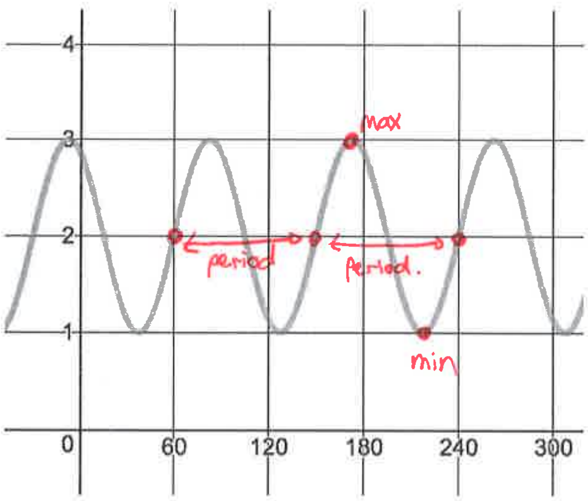
$y = 0.5\sin[0.5(x+180)] + 1$

b) Represent the function with an equation using a cosine function

$d_{cos} = 0$

$y = 0.5\cos[0.5x] + 1$

3) Determine the equation of a cosine function that represents the graph shown.



$a = \frac{max - min}{2} = \frac{3-1}{2} = 1$

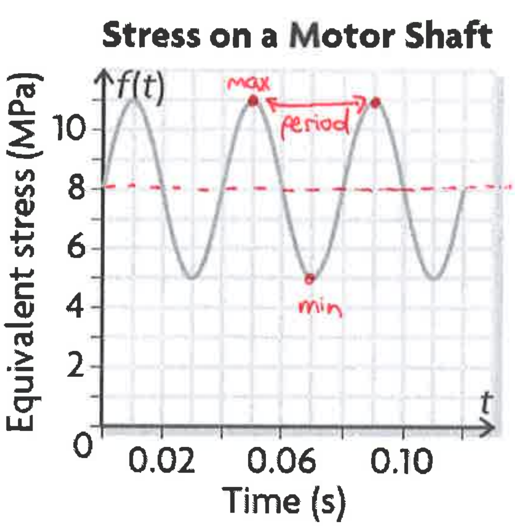
$k = \frac{360}{period} = \frac{360}{150-60} = \frac{360}{90} = 4$

$c = max - |a| = 3 - 1 = 2$

$d_{cos} = d_{sin} + \frac{90}{k} = 60 + \frac{90}{4} = 82.5$

$$y = \cos[4(x - 82.5)] + 2$$

4) The relationship between the stress on the shaft of an electric motor and time can be modelled with a sinusoidal function. Determine an equation of a function that describes stress in terms of time.



$a = \frac{max - min}{2} = \frac{11-5}{2} = 3$

$k = \frac{360}{period} = \frac{360}{0.04} = 9000$

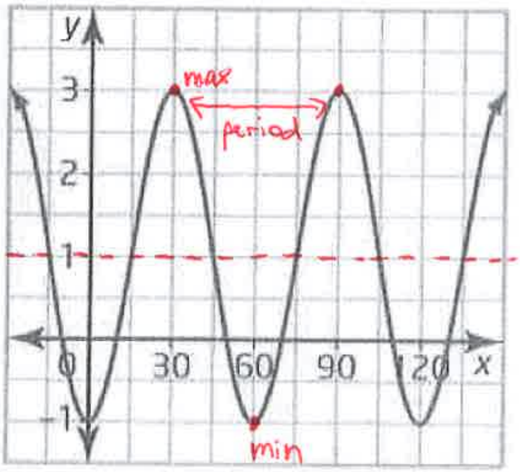
$c = max - |a| = 11 - 3 = 8$

$d_{sin} = 0$

$d_{cos} = 0.01$

$y = 3\sin[9000x] + 8$ OR $y = 3\cos[9000(x - 0.01)] + 8$

5) Determine the equation of the sine function shown.



$a = \frac{\text{max} - \text{min}}{2} = \frac{3-(-1)}{2} = 2$

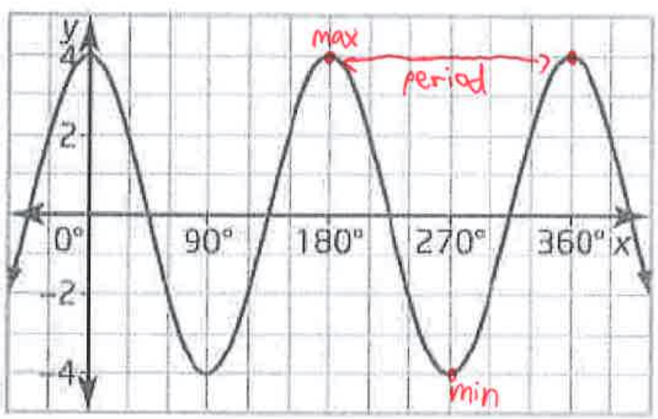
$k = \frac{360}{\text{period}} = \frac{360}{60} = 6$

$c = \text{max} - |a| = 3 - 2 = 1$

$d_{sin} = 15$

$y = 2\sin[6(x-15)] + 1$

6) Represent the graph of the following functions using a sine and cosine function.



$a = \frac{\text{max} - \text{min}}{2} = \frac{4-(-4)}{2} = 4$

$k = \frac{360}{\text{period}} = \frac{360}{180} = 2$

$c = \text{max} - |a| = 4 - 4 = 0$

$d_{cos} = 0$

$d_{sin} = d_{cos} - \frac{90}{k} = 0 - \frac{90}{2} = -45$

$y = 4\cos(2x)$

$y = 4\sin[2(x+45)]$

Answers

1) a) $y = 5\sin[3(x + 30°)] - 2$ **b)** $y = 5\cos 3x - 2$

2) a) $y = \frac{1}{2}\sin\left[\frac{1}{2}(x + 180°)\right] + 1$ **b)** $y = \frac{1}{2}\cos\frac{1}{2}x + 1$

3) $y = \cos[4(x - 82.5°)] + 2$

4) $y = 3\sin(9000x) + 8$ OR $y = 3\cos[9000(x - 0.01)] + 8$

5) a) $y = 2\sin[6(x - 15°)] + 1$
b) If the maximum values are half as far apart, the period of the function is reduced by one-half to 30°. The value of k doubles from 6 to 12. The equation for the new function is $y = 2\sin[12(x - 15°)] + 1$.

6) $y = 4\cos 2x$ and $y = 4\sin[2(x + 45°)]$.

5.5/5.6 Applications of Sine and Cosine Functions Worksheet #1

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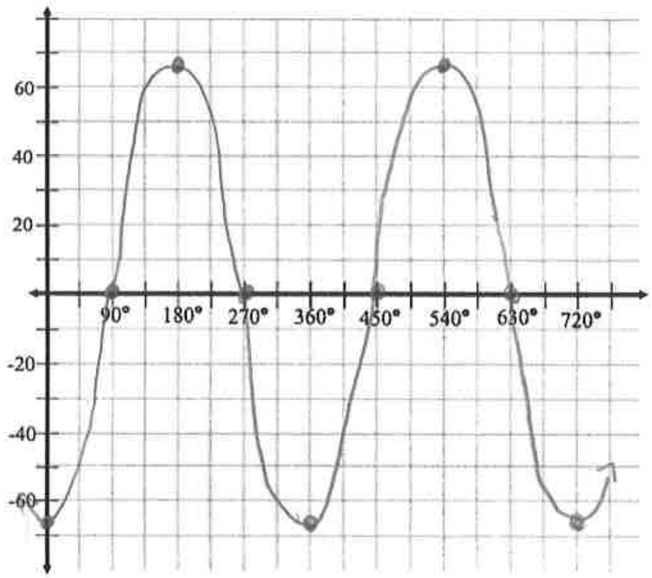
1) At a maximum height of 135 m, the Millennium Wheel, in London, England, is the largest cantilevered structure in the world. It moves so slowly that there is usually no need to stop the wheel to let people on or off. Let the origin be the center of the wheel.

a) Start a sketch of the **vertical displacement** from the center of the wheel of a car on the wheel as a function of the angle through which the wheel rotates, using the bottom of the wheel as the starting point of the trip. Sketch two cycles.



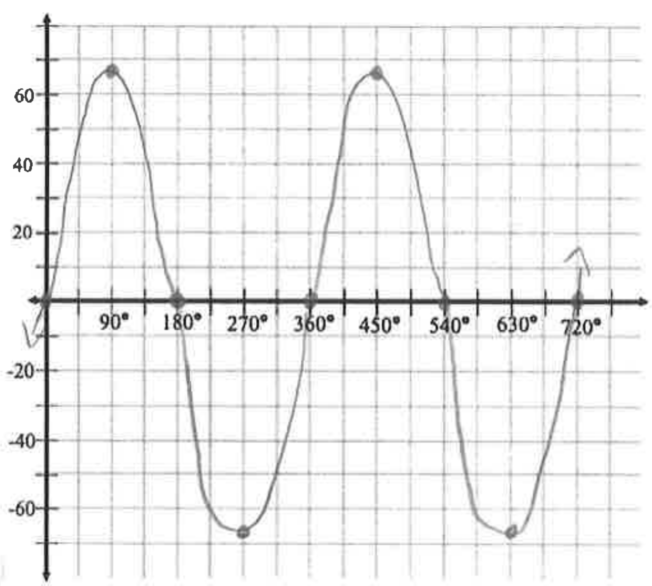
b) Determine the amplitude and period of the function.

$$\text{amplitude} = \frac{67.5-(-67.5)}{2}$$

$$= 67.5$$

period = 360°

2) a) Repeat question 1, except this time graph horizontal displacement instead of vertical displacement.



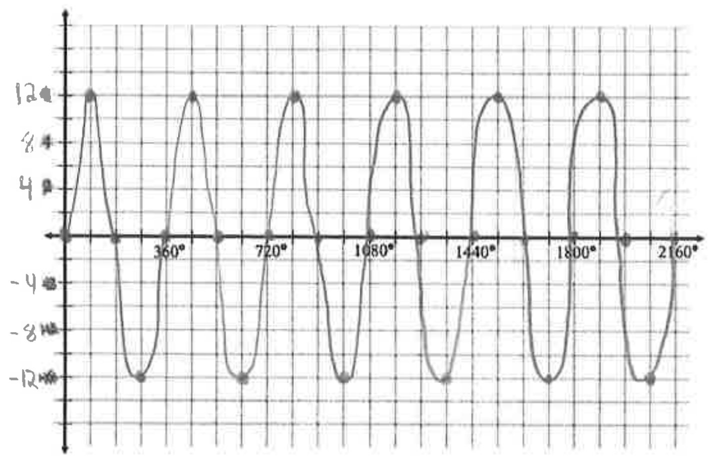
b) Determine the amplitude and period of the function.

a = 67.5

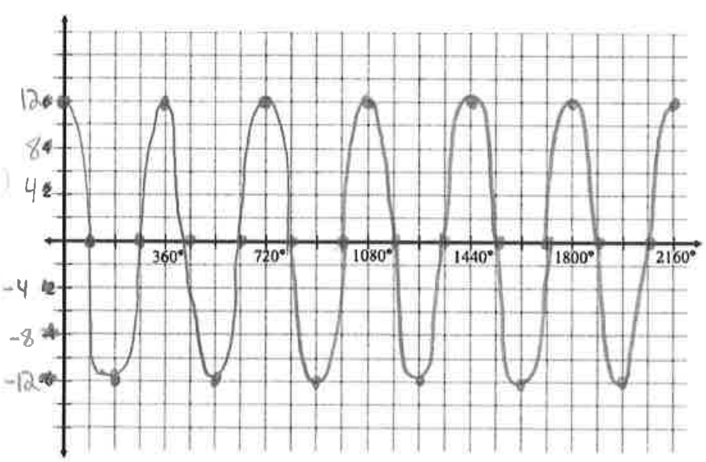
period = 360°

3) The hour hand on a clock has a length of 12cm.

a) Sketch the graph of the vertical position of the tip of the hour hand from the center of the clock versus the angle through which the hand turns for a time period of 72 h. Assume that the hour hand starts at 9.



b) Sketch the graph of the horizontal position of the tip of the hour hand versus the angle through which the hand turns for a time period of 72 h. Assume that the hour hand starts at 3.



c) How many cycles appear in the graph in part a) and b)?

$$\# \text{ of cycles} = \frac{72 \text{ hours}}{\# \text{of hours per cycle}} = \frac{72}{12} = 6 \text{ cycles.}$$

4) A Ferris wheel has a diameter of 20 m and is 4 m above ground level at its lowest point. Assume that a rider enters a car from a platform that is located 30° around the rim before the car reaches its lowest point.

a) Model the rider's height above the ground versus angle using a transformed sine function

$a = \frac{20}{2} = 10$ $\quad k = \frac{360}{360} = 1$ $\quad$ d-value: must rotate $30+90=120°$ before reaching rising midline; $\therefore d = 120$

$c = \text{max} - \text{amp}$
$= 24 - 10$
$= 14$

$$y = 10\sin(x - 120°) + 14$$

b) Model the rider's height above the ground versus angle using a transformed cosine function.

shift cosine function 90° to right to be equal to sine function.

$y = \sin x = \cos(x - 90)$

$\therefore y = 10\sin(x-120°)+14 = 10\cos(x-210°)+14$

OR

$a = 10$
$c = 14$
$k = 1$

$$y = 10\cos(x - 210) + 14$$

d-value: must rotate $30+180 = 210°$ to get to max height. $\therefore d = 210$

c) Suppose that the platform is moved to 60° around the rim from the lowest position of the car. How will the equations in parts a) and b) change? Write the new equations.

Initial position is 30° sooner; $\therefore$ phase shift must increase by 30° for each function

$$y = 10\sin(x - 150) + 14$$

$$y = 10\cos(x - 240) + 14$$

5) Suppose that the center of the Ferris wheel in the previous question is moved upward 2 m, but the platform is left in place at a point 30° before the car reaches its lowest point. How do the equations in parts a) and b) change? Write the new equations.

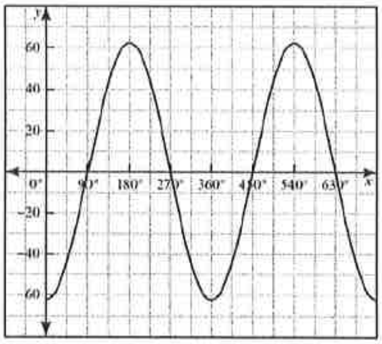
Vertical shift increases by 2; $\therefore c = 16$

$$y = 10\sin(x - 120) + 16$$

$$y = 10\cos(x - 210) + 16$$

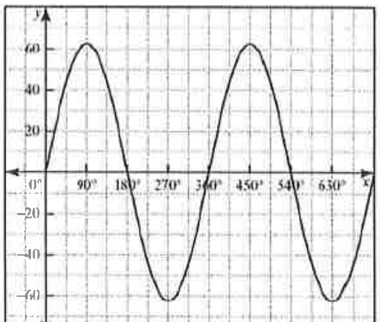
Answers

1) a)



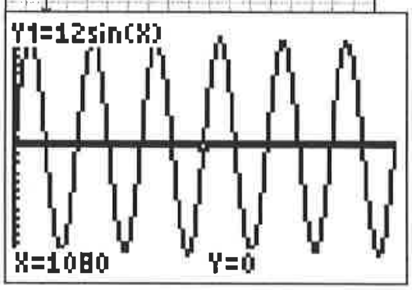
b) amplitude 67.5; period 360°

2) a)

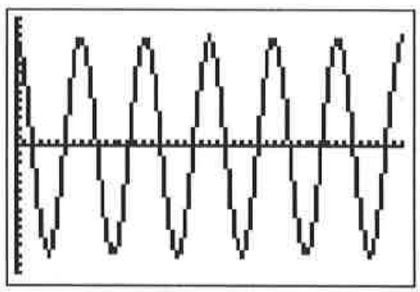


b) amplitude 67.5; period 360°

3) a)



b)



c) The graphs in part a) and b) both show 6 full cycles because there are six 12-h periods in 72 h.

4) a) The amplitude is 10 m and the midline is at 14 m. If the rider begins her ride 30° before the minimum, then she will reach the rising midline point after a rotation of 120° for a phase shift of 120° to the right. The period is 360°. An equation that models the rider's height versus the rotation angle is
$y = 10 \sin (x - 120°) + 14$.

b) For a cosine function, the rider must rotate 210° to reach the first maximum point. This requires a phase shift of 210° to the right. The other parameters remain the same. An equation that models the rider's height versus the rotation angle is $y = 10 \cos (x - 210°) + 14$.

c) If the initial position is placed 30° sooner, then the phase shift of both curves must increase by 30°.
A new sine equation that models the rider's height versus the rotation angle is $y = 10 \sin (x - 150°) + 14$.
A new cosine equation that models the rider's height versus the rotation angle is
$y = 10 \cos (x - 240°) + 14$.

5) If the centre of the Ferris wheel is raised by 2 m, then the vertical shift also increase by 2 from 14 to 16. The relative position of the platform does not change, so the phase shift is not affected.

a) A new sine equation that models the rider's height versus the rotation angle is $y = 10 \sin (x - 120°) + 16$.

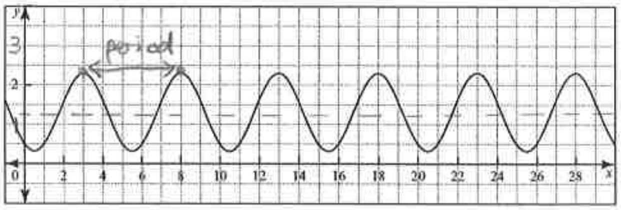
b) A new cosine equation that models the rider's height versus the rotation angle is
$y = 10 \cos (x - 210°) + 16$.

5.5/5.6 Application of Sine and Cosine Functions Worksheet #2

MCR3U
Jensen

SOLUTIONS

1) A motion sensor recorded the motion of a child on a swing. The data was graphed, as shown.



a) Find the max and min values.

max = 2.25 min = 0.25

b) Find amplitude $a = \frac{2.25 - 0.25}{2} = 1$

c) Determine the vertical shift of the function.

$C = \text{max} - \text{amp} = 2.25 - 1 = 1.25$

d) Find the period of the function

period = 8 - 3 = 5

e) Determine the phase shift, if the motion were to be modelled using a sine function.

rising midline is $\frac{90}{k} = \frac{90}{72} = 1.25$ to the left of the max.

$\therefore d = 3 - 1.25 = 1.75$

2) The height of the blade of a wind turbine as it turns through an angle of θ is given by the function $h(\theta) = 8.5 \sin(\theta + 180°) + 40$, with height measured in metres.

a) Find the maximum and minimum positions of the blade.

max = 40 + 8.5 = 48.5 m   min = 40 - 8.5 = 31.5 m

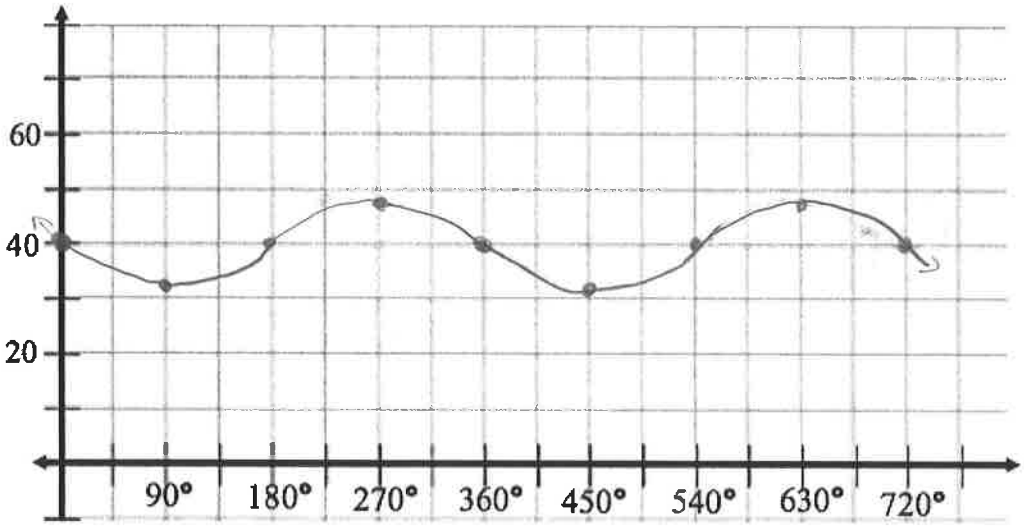
b) Explain what the value of 40 in the equation represents.

Vertical shift; OR height of the center of the turbine.

c) Explain what the value of the amplitude represents.

Length of the blade.

d) Sketch the function over two cycles.



3) The height, h, in meters, of the tide in a given location on a given day at t hours after midnight can be modeled using the sinusoidal function $h(t) = 5\sin[30(t-5)] + 7$.

a) Find the max and min values for the depth of water.

$\max = 7+5 = 12$ m

$\min = 7-5 = 2$ m

b) What time is high tide? What time is low tide?

rising midline for sine function is moved 5 units right.

$\text{Max} = 5 + \frac{90}{k} = 5 + \frac{90}{30} = 8$ (8:00 am)

$\text{Min} = 5 - \frac{90}{k} = 5 - \frac{90}{30} = 2$ (2:00 am)

Because of the 12 hour period, there is also a max at 8pm and min at 8am.

c) What is the depth of the water at 9:00 am?

$h(9) = 5\sin[30(9-5)] + 7$

$= 5\sin(120) + 7$

$= 11.3$ m

d) Find all the times during a 24-h period when the depth of the water is 3 meters.

$3 = 5\sin[30(t-5)] + 7$

$\frac{-4}{5} = \sin[30(t-5)]$

$30(t-5) = \sin^{-1}\left(\frac{-4}{5}\right)$

$t - 5 = \frac{-53.13}{30}$

$t - 5 = -1.77$

$t = 3.23$

about 3 hours 14 minutes

This is 1 hour 14 mins after the min; so the height will be the same 1 hour and 14 mins before that; at 12:46 am.

ON BACK $\longrightarrow$

h(3) at 12:46 am and p.m. (because of 12 hour period)

and

3:14 am and 3:14 p.m.

4) The population, P, of a lakeside town with a large number of seasonal residents can be modeled using the function $P(t) = 5000\sin[30(t-7)] + 8000$, where t is the number of months after New Year's Day.

a) Find the max and min values for the population over a whole year.

max = 8000 + 5000 = 13 000

min = 8000 - 5000 = 3 000

b) When is the population a maximum? When is it a minimum?

max of sine at $\frac{90}{k} = \frac{90}{30} = 3$ but you must shift 7 to the right.

min at $-\frac{90}{k} = \frac{-90}{30} = -3$

∴ max 10 months after New Years and min 4 months after New Years.

c) What is the population on September 30th?

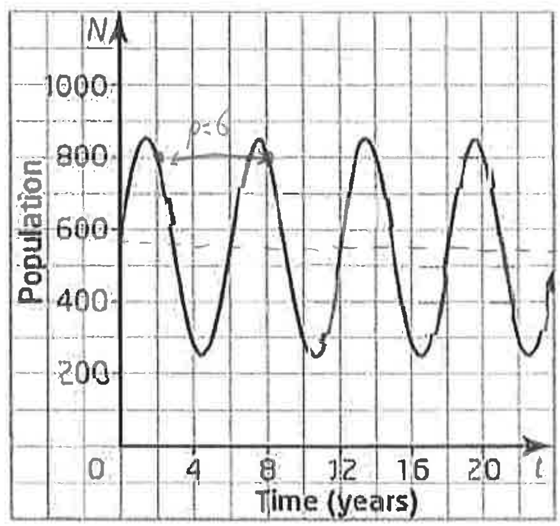
$P(9) = 5000\sin[30(9-7)] + 8000$

$= 5000\sin(60) + 8000$

$= 12\ 330$

5) The population of prey in a predator-prey relation is shown. Time is in years since 1985.

a) Determine the max and min values of the population, to the nearest 50. Use these to find the amplitude.



max = 850

min = 250

$a = \frac{850-250}{2}$

$a = 300$

b) Determine the vertical shift, c.

c = max - amp

c = 850 - 300

c = 550

c) Determine the phase shift, d.

d = 0

d) Determine the period. Use the period to determine the value of k.

Period = 8 - 2 = 6 $\qquad k = \frac{360}{6} = 60$

e) Model the population versus time with a sinusoidal function.

$P = 300\sin(60t) + 550$

6) The number of millions of visitors that a tourist attraction gets can be modeled using the equation $y = 2.3\sin[30(x+1)] + 4.1$, where $x = 1$ represents January, $x = 2$ represents February, and so on.

a) Determine the period of the function and explain its meaning.

Period $= \frac{360}{30} = 12$

12 months.

b) Graph the function for 12 months.

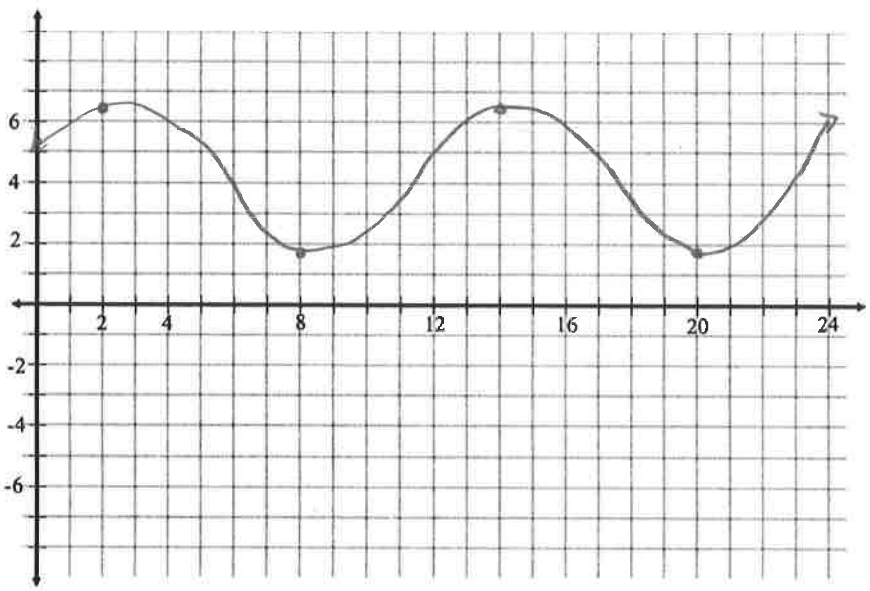
$\max = 4.1 + 2.3 = 6.4$

$\min = 4.1 - 2.3 = 1.8$

Rising midline at -1

First max at $-1 + \frac{90}{30} = -1 + 3$
$= 2$

first min is $\frac{180}{30} = 6$ after max.



c) Which month has the most visitors?

month 2: February

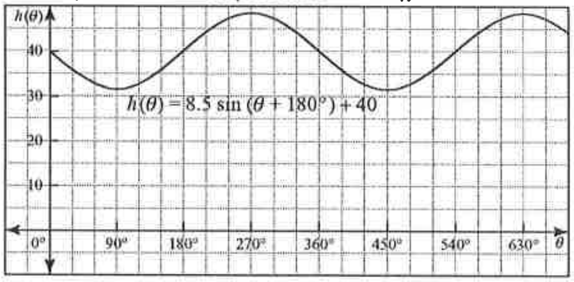
d) Which month has the least visitors?

Month 8: August

Answers

1) a) maximum 2.25, minimum 0.25
b) amplitude 1
c) vertical shift up 1.25
d) period 5
e) horizontal shift 1.75 to the right

2) a) maximum 48.5, minimum 31.5
b) The height of the center of the turbine
c) The amplitude of 8.5 represents the length of the blade.
d)



3) a) From the equation, $c = 7$ and $a = 5$, so the function has a midline value of 7 and an amplitude of 5. The maximum height is 12 m and the minimum height is 2 m.

b) From the equation, $k = 30$ and $d = 5$, so the period is 12 h and the phase shift is 5 h right. The first midline value occurs at 5:00 a.m. The first maximum occurs one-quarter period, or 3 h after this, at 8:00 a.m. The previous minimum is 3 h prior to 5:00 a.m., at 2:00 a.m. Because of the 12-h period, there will also be a maximum at 8:00 p.m. and a minimum at 2:00 p.m.

c) 11.3 m

d) The solution gives a time of approximately 3:14 a.m. This time is 1 h 14 min after the first minimum so the depth should also occur 1 h 14 min before 2:00 a.m, at 12:46 a.m. Because of the 12-h period, the depth will also occur at 12:46 p.m. and 3:14 p.m.

4) a) From the equation, $c = 8000$ and $a = 5000$, so the function has a midline value of 8000 and an amplitude of 5000. The maximum population is 13 000 and the minimum population is 3000.

b) From the equation, $k = 30$ and $d = 7$, so the period is 12 months and the phase shift is 7 months right. The initial midline value occurs at $t = 7$. The maximum occurs 3 months later at $t = 10$ (October) and the minimum 3 months earlier at $t = 4$ (April).

c) 12 330

5) a) From the graph the maximum population is approximately 850 animals and the minimum population is approximately 250 animals.The amplitude is approximately 300 animals, so $a = 300$.

b) The vertical shift is the maximum value minus the amplitude, so $c = 550$.

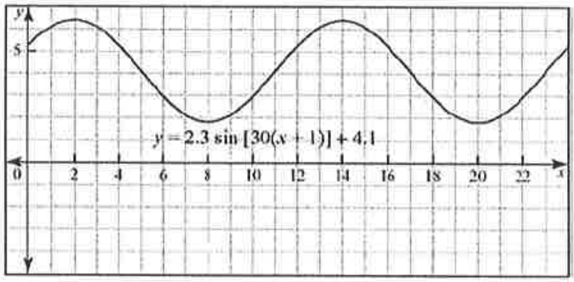
c) The midline intersects the graph at $t = 0$ so no horizontal shift is necessary, so $d = 0$.

d) The pattern repeats every 6 years, so the period is 6 years. k=*60*.

e) A sine function that models the population of prey, N, with respect to time, t, is $N = 300 \sin 60t + 550$.

6) a) 12 months

b)



c) February

d) August