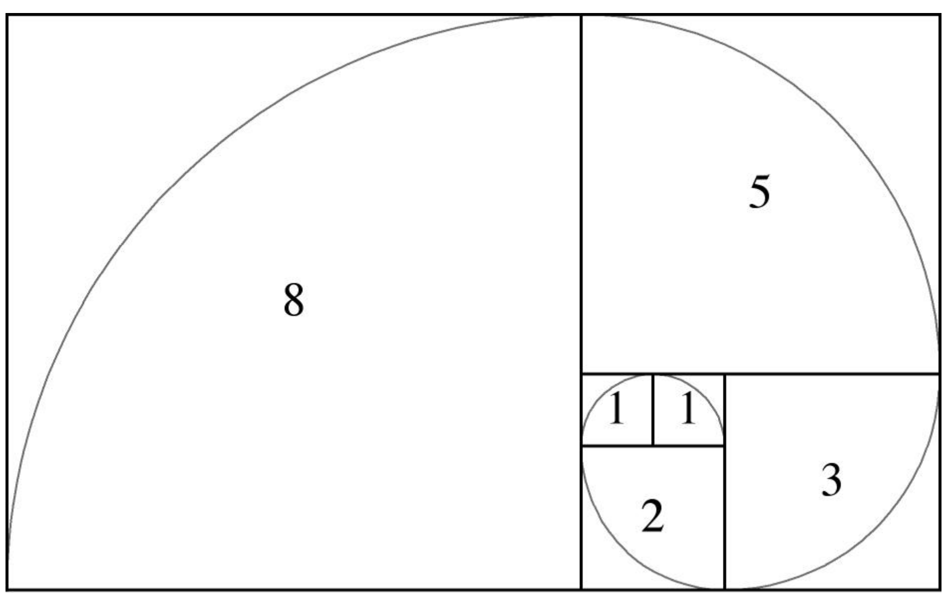
Chapter 6- Discrete Functions

WORKBOOK

MCR3U



Chapter 6 Workbook Checklist

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Sequences (Part 1) – Worksheet
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SOLUTIONS

General formula for an Arithmetic Sequence:

$t_n = a + (n-1)(d)$

General formula for a Geometric Sequence:

$t_n = a \cdot r^{n-1}$

1) Find the next three terms of each arithmetic sequence.

a) 3, 7, 11, 15, 19, 23, 27 ($+4$)

b) -13, -11, -9, -7, -5, -3, -1 ($+2$)

c) 22, 20, 18, 16, 14, 12, 10 (-2)

d) -2, -5, -8, -11, -14, -17, -20 (-3)

2) Find the next three terms of each geometric sequence.

a) 4, 8, 16, 32, 64, 128 ($\times 2$)

b) 1, -6, 36, -216, 1296, -7776 ($\times -6$)

c) 486, 162, 54, 18, 6, 2 ($\times \frac{1}{3}$)

d) 3, 15, 75, 375, 1875, 9375 ($\times 5$)

3) Determine whether each sequence is an arithmetic sequence, a geometric sequence or neither. If it is an arithmetic or geometric sequence, determine a formula to represent the sequence.

a) 4, 7, 9, 12, …

neither

b) 15, 13, 11, 9, … (-2)

Arithmetic

$t_n = 15 + (n-1)(-2)$

c) 4, 12, 36, 108, … ($\times 3$)

Geometric

$t_n = 4(3)^{n-1}$

d) 5, 10, 15, 20, … ($+5$)

Arithmetic

$t_n = 5 + (n-1)(5)$

e) 7, 10, 13, 16, … ($+3$)

Arithmetic

$t_n = 7 + (n-1)(3)$

f) 120, -60, 30, -15, .. ($\times -\frac{1}{2}$)

Geometric

$t_n = 120\left(-\frac{1}{2}\right)^{n-1}$

g) -6, -5, -3, -1, …

Neither

h) -13, -6, 1, 8, … ($+7$)

Arithmetic

$t_n = -13 + (n-1)(7)$

i) 625, 125, 25, 5, … ($\times \frac{1}{5}$)

Geometric

$t_n = 625\left(\frac{1}{5}\right)^{n-1}$

4) Charlie deposited $115 in a savings account. Each week thereafter, he deposits $35 into the account.

a) Write a formula to represent this sequence.

$t_n = 115 + (n-1)(35)$

b) How much total money has Charlie deposited after 30 weeks?

$t_{30} = 115 + (30-1)(35)$

$= 115 + 1015$

$= \$1130$

5) A ball is dropped from a height of 500 meters. The table shows the height of each bounce.

BOUNCE #	HEIGHT (m)
1	400
2	320
3	256

$\times \frac{4}{5}$ OR 0.8

$\times \frac{4}{5}$ OR 0.8

a) Write an equation to represent the height of the ball after each bounce.

$t_n = 400(0.8)^{n-1}$

b) How high does the ball bounce on the 6th bounce?

$t_6 = 400(0.8)^{6-1}$

$= 131.072\text{ m}$

Answers

1 a) 3, 7, 11, 15, 19, 23, 27,… **b)** -13, -11, -9, -7, -5, -3, -1,…
c) 22, 20, 18, 16, 14, 12, 10,… **d)** -2, -5, -8, -11, -14, -17, -20,…

2) a) 4, 8, 16, 32, 64, 128,… **b)** 1, -6, 36, -216, 1296, -7776,…
c) 486, 162, 54, 27, 9, 3,… **d)** 3, 15, 75, 375, 1875, 9375,…

3) a) 4, 7, 9, 12, … neither
b) 15, 13, 11, 9, … arithmetic $t_n = 15 + (n-1)(-2)$
c) 4, 12, 36, 108, … geometric $t_n = 4(3)^{n-1}$
d) 5, 10, 15, 20, … arithmetic $t_n = 5 + (n-1)(5)$
e) 7, 10, 13, 16, … arithmetic $t_n = 7 + (n-1)(3)$
f) 120, -60, 30, -15, ... geometric $t_n = 120(-1/2)^{n-1}$
g) -6, -5, -3, -1, … neither
h) -13, -6, 1, 8, … arithmetic $t_n = -13 + (n-1)(7)$
i) 625, 125, 25, 5, … geometric $t_n = 625(1/5)^{n-1}$

4) a) $t_n = 115 + (n-1)(35)$ **b)** $t_{30} = 1130$

5) a) $t_n = 400(0.8)^{n-1}$ **b)** $t_6 = 131.072$

Arithmetic and Geometric Series – Worksheet
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SOLUTIONS

General formula for an arithmetic series:

$$S_n = \frac{n}{2}[2a+(n-1)d] \quad \underline{OR} \quad S_n = \frac{n}{2}(a+t_n)$$

General formula for a geometric series:

$$S_n = \frac{a(r^n-1)}{r-1}$$

1) Find the designated sum of the arithmetic series

a) S_{14} of $3 \overset{+4}{+} 7 + 11 + 15 + \cdots$

$S_{14} = \frac{14}{2}[2(3)+(14-1)(4)]$

$= 7[6+13(4)]$

$= 406$

b) S_{11} of $-13 \overset{+2}{-} 11 - 9 - 7 - \cdots$

$S_{11} = \frac{11}{2}[2(-13)+(11-1)(2)]$

$= 5.5[-26+10(2)]$

$= -33$

) S_9 of $22 \overset{-2}{+} 20 + 18 + 16 + \cdots$

$S_9 = \frac{9}{2}[2(22)+(9-1)(-2)]$

$= 4.5[44+8(-2)]$

$= 126$

d) S_{35} of $-2 \overset{-3}{-} 5 - 8 - 11 - \cdots$

$S_{35} = \frac{35}{2}[2(-2)+(35-1)(-3)]$

$= 17.5[-4+34(-3)]$

$= -1855$

2) Determine the sum of each arithmetic series

a) $6 \overset{+7}{+} 13 + 20 + \cdots + 69$

$t_n = a+(n-1)d$

$69 = 6+(n-1)(7)$

$63 = (n-1)(7)$

$9 = n-1$

$n = 10$

$S_{10} = \frac{10}{2}(6+69)$

$= 5(75)$

$= 375$

b) $4 \overset{+11}{+} 15 + 26 + \cdots + 213$

$213 = 4+(n-1)(11)$

$209 = (n-1)(11)$

$19 = n-1$

$n = 20$

$S_{20} = \frac{20}{2}(4+213)$

$= 10(217)$

$= 2170$

c) $5 \overset{-13}{-} 8 - 21 - \cdots - 190$

$-190 = 5+(n-1)(-13)$

$-195 = (n-1)(-13)$

$15 = n-1$

$n = 16$

$S_{16} = \frac{16}{2}(5-190)$

$= 8(-185)$

$= -1480$

d) $100 \overset{-10}{+} 90 + 80 + \cdots - 100$

$-100 = 100+(n-1)(-10)$

$-200 = (n-1)(-10)$

$20 = n-1$

$n = 21$

$S_{21} = \frac{21}{2}(100-100)$

$= 0$

3) Find the designated sum of the geometric series

a) S_7 of $4 \overset{\times 2}{+} 8 + 16 + 32 + \cdots$

$S_7 = \frac{4(2^7 - 1)}{2-1}$

$= 508$

b) S_{13} of $1 \overset{\times -6}{-} 6 + 36 - 216 + \cdots$

$S_{13} = \frac{1[(-6)^{13} - 1]}{-6-1} = 1\,865\,813\,431$

c) S_{17} of $\underbrace{486 + 162}_{\times \frac{1}{3}} + 54 + 18 + \cdots$

$S_{17} = \frac{486[(\frac{1}{3})^{17} - 1]}{(\frac{1}{3}) - 1} = 729$

d) S_6 of $3 \overset{\times 5}{+} 15 + 75 + 375 + \cdots$

$S_6 = \frac{3[(5)^6 - 1]}{5-1} = 11718$

4) Determine S_n for each geometric series

a) $a = 6, r = 2, n = 9$

$S_9 = \frac{6[(2)^9 - 1]}{2-1} = 3066$

b) $f(1) = 2, r = -2, n = 12$

$S_{12} = \frac{2[(-2)^{12} - 1]}{-2-1} = -2730$

c) $f(1) = 729, r = -3, n = 15$

$S_{15} = \frac{729[(-3)^{15} - 1]}{-3-1} = 2\,615\,088\,443$

d) $f(1) = 2700, r = 10, n = 8$

$S_8 = \frac{2700((10)^8 - 1)}{10-1} \approx 2.999\,999\,97 \times 10^{10}$

5) If the first term of an arithmetic series is 2, the last term is 20, and the increase constant is +2 …

a) Determine the number of terms in the series

$t_n = a + (n-1)d$

$20 = 2 + (n-1)(2)$

$18 = (n-1)(2)$

$9 = n - 1$

$n = 10$

b) Determine the sum of all the terms in the series

$S_{10} = \frac{10}{2}[2 + 20]$

$= 5(22)$

$= 110$

6) A geometric series has a sum of 1365 (S_6). Each term increases by a factor of 4 (r). If there are 6 (n) terms, find the value of the first term.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$1365 = \frac{a[(4)^6 - 1]}{4 - 1}$$

$$1365(3) = a(4^6 - 1)$$

$$4095 = a(4095)$$

$$a = 1$$

Answers

1) a) 406 **b)** -33 **c)** 126 **d)** -1855

2) a) 375 **b)** 2170 **c)** -1480 **d)** 0

3) a) 508 **b)** 1 865 813 431 **c)** 729 **d)** 11 718

4) a) 3066 **b)** -2730 **c)** 2 615 088 483 **d)** $2.999\,999\,97 \times 10^{10}$

5) a) $n = 10$ **b)** $S_{10} = 110$

6) $t_1 = 1$

Arithmetic and Geometric Sequences – Worksheet #2

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SOLUTIONS

1) For each arithmetic sequence, determine the values of a and d. Then, write the next four terms.

a) $12, \overset{+3}{15}, 18, \ldots$

$a = 12$

$d = 3$

$12, 15, 18, 21, 24, 27, 30$

b) $\frac{1}{2}, \overset{+\frac{1}{2}}{1}, \frac{3}{2}, \ldots$

$a = \frac{1}{2}$

$d = \frac{1}{2}$

$\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}$

2) Given the values of a and d, write the first three terms of the arithmetic sequence. Then, write the formula for the general term.

a) $a = 5, d = 2$

$5, 7, 9$

$t_n = 5 + (n-1)(2)$

b) $a = \frac{3}{4}, d = \frac{1}{2}$

$\frac{3}{4}, \frac{5}{4}, \frac{7}{4}$

$t_n = \frac{3}{4} + (n-1)\left(\frac{1}{2}\right)$

3) Given the formula for the general term of an arithmetic sequence, determine t_{12}.

a) $t_n = 1 - 4n$

$t_{12} = 1 - 4(12)$

$= 1 - 48$

$= -47$

b) $t_n = \frac{1}{2}n + \frac{3}{2}$

$t_{12} = \frac{1}{2}(12) + \frac{3}{2}$

$= \frac{12}{2} + \frac{3}{2}$

$= \frac{15}{2}$

4) Which term in the arithmetic sequence $9, \overset{-5}{4}, -1, \ldots$ has the value -146?

$t_n = a + (n-1)d$

$-146 = 9 + (n-1)(-5)$

$-155 = (n-1)(-5)$

$31 = n - 1$

$n = 32$

5) Determine the number of terms in each arithmetic sequence

a) $38, 36, 34, \dots, -20$

-2

$t_n = a + (n-1)d$

$-20 = 38 + (n-1)(-2)$

$-58 = (n-1)(-2)$

$29 = n - 1$

$n = 30$

b) -5, -8, -11, …, -269

-3

$-269 = -5 + (n-1)(-3)$

$-264 = (n-1)(-3)$

$88 = n - 1$

$n = 89$

6) Determine a and d and then write the formula for the n^{th} term of each arithmetic sequence with the given terms.

a) $t_{10} = 50$ and $t_{27} = 152$

$t_{10} = a + (10-1)d$ $\qquad$ $t_{27} = a + (27-1)d$

① $50 = a + 9d$ $\qquad$ ② $152 = a + 26d$

② $152 = a + 26d$

① $50 = a + 9d$ −

$102 = 17d$

$d = 6$

$50 = a + 9(6)$

$50 = a + 54$

$a = -4$

$\boxed{t_n = -4 + (n-1)(6)}$

b) $t_5 = -20$ and $t_{18} = -59$

$t_5 = a + (5-1)d$ $\qquad$ $t_{18} = a + (18-1)d$

① $-20 = a + 4d$ $\qquad$ ② $-59 = a + 17d$

① $-20 = a + 4d$

② $-59 = a + 17d$ −

$39 = -13d$

$d = -3$

$-20 = a + 4(-3)$

$-20 = a - 12$

$-8 = a$

$\boxed{t_n = -8 + (n-1)(-3)}$

7) In a lottery, the owner of the first ticket drawn receives \$10 000. Each successive winner receives \$500 less than the previous winner.

a) How much does the 10th winner receive?

$t_n = 10\,000 + (n-1)(-500)$

$t_{10} = 10\,000 + (10-1)(-500)$

$= \$5\,500$

b) How many winners are there in total?

$500 = 10\,000 + (n-1)(-500)$

$-9500 = (n-1)(-500)$

$19 = n - 1$

$n = 20$

∴ 20 winners.

8) At the end of the second week after opening, a new fitness club as 870 members. At the end of the seventh week, there are 1110 members. If the increase is arithmetic, how many members were there in the first week?

$t_2 = a + (2-1)d$

① $870 = a + d$

$t_7 = a + (7-1)d$

② $1110 = a + 6d$

② $1110 = a + 6d$
① $870 = a + d$ −

$240 = 5d$

$d = 48$

$870 = a + 48$

$a = 822$

∴ There were 822 members in the first week.

9) State the common ratio for each geometric sequence and write the next three terms.

a) 1, 2, 4, 8, ... ($\times 2$)

$r = 2$

1, 2, 4, 8, 16, 32, 64

b) -3, 9, -27, 81, ...

$r = -3$

-3, 9, -27, 81, -243, 729, -2187

c) $\frac{2}{3}, -\frac{2}{3}, \frac{2}{3}, -\frac{2}{3}, \ldots$

$r = -1$

$\frac{2}{3}, -\frac{2}{3}, \frac{2}{3}, -\frac{2}{3}, \frac{2}{3}, -\frac{2}{3}, \frac{2}{3}$

d) 600, -300, 150, -75, ... ($\times -\frac{1}{2}$)

$r = -\frac{1}{2}$

600, -300, 150, -75, 37.5, -18.75, 9.375

10) For the geometric sequence 54, 18, 6, ... determine the formula for the general term and then find t_9.

$a = 54$

$r = \frac{1}{3}$

$t_n = 54\left(\frac{1}{3}\right)^{n-1}$

$t_9 = 54\left(\frac{1}{3}\right)^{9-1}$

$t_9 = \frac{54}{3^8}$

$t_9 = \frac{54}{6561}$

$t_9 = \frac{2}{243}$

11) Write the first four terms of each geometric sequence.

a) $t_n = 5(2)^{n-1}$

5, 10, 20, 40

b) $a = -1, r = \frac{1}{5}$

$-1, -\frac{1}{5}, -\frac{1}{25}, -\frac{1}{125}$

12) Determine the number of terms in the geometric sequence 6, 18, 54, ... , 4374. ($\times 3$)

$a = 6$

$r = 3$

$t_n = 6(3)^{n-1}$

$4374 = 6(3)^{n-1}$

$729 = 3^{n-1}$

$3^6 = 3^{n-1}$

$6 = n - 1$

$n = 7$

13) Which term of the geometric sequence 1, 3, 9, ... has a value of 19 683?

$\times 3$

$19\,683 = 1(3)^{n-1}$

$19\,683 = 3^{n-1}$

$\log 19683 = \log 3^{n-1}$

$\log 19683 = (n-1)\log 3$

$\frac{\log 19683}{\log 3} = n-1$

$9 = n-1$

$n = 10$

Answers

1) a) $a = 12, d = 3;\ \ 12, 15, 18, 21, 24, 27, 30$ **b)** $a = \frac{1}{2}, d = \frac{1}{2};\ \ \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}$

2) a) $5, 7, 9;\ t_n = 5 + (n-1)(2)$ **b)** $\frac{3}{4}, \frac{5}{4}, \frac{7}{4};\ t_n = \frac{3}{4} + (n-1)\left(\frac{1}{2}\right)$

3) a) -47 **b)** $\frac{15}{2}$

4) $n = 32$

5) a) 30 **b)** 89

6) a) $t_n = -4 + (n-1)(6)$ **b)** $t_n = -8 + (n-1)(-3)$

7) a) \$5500 **b)** 20 winners

8) 822 members

9) a) $r = 2$ **b)** $r = -3$ **c)** $r = -1$ **d)** $-\frac{1}{2}$

10) $\frac{2}{243}$

11) a) 5, 10, 20, 40 **b)** -1, $-\frac{1}{5}, -\frac{1}{25}, -\frac{1}{125}$

12) 7 terms

13) 10th term

Arithmetic and Geometric Series – Worksheet #2

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SOLUTIONS

1) The first and last terms in each arithmetic series are given. Determine the sum of the series.

a) $a = \frac{1}{2}$ and $t_8 = 4$

$S_n = \frac{n}{2}(a + t_n)$

$S_8 = \frac{8}{2}(\frac{1}{2} + 4)$

$S_8 = 18$

b) $a = 11$ and $t_{20} = 101$

$S_{20} = \frac{20}{2}(11 + 101)$

$S_{20} = 1120$

2) Determine the sum of the arithmetic series $-1 \overset{+3}{+} 2+5+\ldots+164$. $\leftarrow t_{56}$

$t_n = a + (n-1)d$

$164 = -1 + (n-1)(3)$

$165 = (n-1)(3)$

$55 = n - 1$

$n = 56$

$S_{56} = \frac{56}{2}(-1 + 164)$

$= 4564$

3) The 15th term in an arithmetic sequence is 43 and the sum of the first 15 terms of the series is 120. Determine the first three terms of the sequence.

$t_n = a + (n-1)d$

$43 = a + (15-1)d$

① $43 = a + 14d$

$S_{15} = \frac{15}{2}[2a + (15-1)d]$

$120 = 7.5(2a + 14d)$

② $120 = 15a + 105d$

$t_n = -27 + (n-1)(5)$

15×① $645 = 15a + 210d$

② $120 = 15a + 105d$

$525 = 105d$

$d = 5$

$43 = a + 14d$

$43 = a + 14(5)$

$43 - 70 = a$

$a = -27$

∴ first three terms are: $-27, -22, -17$

4) A toy car is rolling down an inclined track and picking up speed as it goes. The car travels 4 cm in the first second, 8 cm in the second second, 12 cm in the next second, and so on. Determine the total distance travelled by the car in 30 seconds.

$a = 4$
$d = 4$

$S_{30} = \frac{30}{2}[2(4) + (30-1)(4)]$

$= 15[8 + 29(4)]$

$= 1860$

∴ The car travels 1860 cm in 30 seconds.

5) For each geometric series, determine the values of a and r. Then, determine the indicated sum.

a) S_8 for $2 + 6 + 18 + \cdots$

$a = 2$
$r = 3$

$$S_8 = \frac{2(3^8 - 1)}{3 - 1}$$

$$= \frac{2(6560)}{2}$$

$$= 6560$$

b) S_{10} for $24 - 12 + 6 - \cdots$

$a = 24$
$r = -\frac{1}{2}$

$$S_{10} = \frac{24\left[\left(-\frac{1}{2}\right)^{10} - 1\right]}{-\frac{1}{2} - 1}$$

$$= \frac{24\left(\frac{1}{1024} - \frac{1024}{1024}\right)}{-\frac{3}{2}}$$

$$= \frac{-24552}{1024} \cdot -\frac{2}{3}$$

$$= \frac{49104}{3072}$$

$$= \frac{1023}{64}$$

6) Determine the sum of the geometric series $\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \frac{8}{81} + \cdots + \frac{128}{6561}$

$a = \frac{1}{3}$
$r = \frac{2}{3}$

$$t_n = ar^{n-1}$$

$$\frac{128}{6561} = \frac{1}{3}\left(\frac{2}{3}\right)^{n-1}$$

$$\frac{384}{6561} = \left(\frac{2}{3}\right)^{n-1}$$

$$\frac{128}{2187} = \left(\frac{2}{3}\right)^{n-1}$$

$$\left(\frac{2}{3}\right)^7 = \left(\frac{2}{3}\right)^{n-1}$$

$$7 = n - 1$$

$$n = 8$$

$$S_8 = \frac{\left(\frac{1}{3}\right)\left[\left(\frac{2}{3}\right)^8 - 1\right]}{\frac{2}{3} - 1}$$

$$= \frac{\frac{1}{3}\left(\frac{256}{6561} - \frac{6561}{6561}\right)}{-\frac{1}{3}}$$

$$= -1\left(\frac{-6305}{6561}\right)$$

$$= \frac{6305}{6561}$$

7) Determine the sum of the geometric series $5 - 15 + 45 - \cdots + 3645$

$a = 5$
$r = -3$

$$t_n = ar^{n-1}$$

$$3645 = 5(-3)^{n-1}$$

$$729 = (-3)^{n-1}$$

$$(-3)^6 = (-3)^{n-1}$$

$$6 = n - 1$$

$$n = 7$$

$$S_7 = \frac{5\left[(-3)^7 - 1\right]}{-3 - 1}$$

$$= \frac{5(-2188)}{-4}$$

$$= 2735$$

8) The sum of $\overset{\times 3}{4 + 12} + 36 + 108 + \cdots + t_n$ is 4372. How many terms are in this series?

$a = 4$

$r = 3$

$4372 = \frac{4(3^n - 1)}{3 - 1}$

$8744 = 4(3^n - 1)$

$2186 = 3^n - 1$

$2187 = 3^n$

$3^7 = 3^n$

$n = 7$

9) The third term of a geometric series is 24 and the fourth term is 36. Determine the sum of the first 10 terms.

$t_3 = ar^{3-1}$

$24 = ar^2$

$a = \frac{24}{r^2}$

$t_4 = ar^{4-1}$

$36 = ar^3$

$36 = \left(\frac{24}{r^2}\right)r^3$

$36 = 24r$

$r = \frac{36}{24}$

$r = \frac{3}{2}$

$a = \frac{24}{\left(\frac{3}{2}\right)^2}$

$= \frac{24}{\left(\frac{9}{4}\right)}$

$= 24 \cdot \frac{4}{9}$

$= \frac{32}{3}$

$S_{10} = \frac{\frac{32}{3}\left[\left(\frac{3}{2}\right)^{10} - 1\right]}{\frac{3}{2} - 1}$

$= \frac{\frac{32}{3}\left[\frac{59049}{1024} - \frac{1024}{1024}\right]}{\frac{1}{2}}$

$= \frac{\overset{1}{\cancel{64}}}{3}\left(\frac{58025}{\underset{16}{\cancel{1024}}}\right)$

$= \frac{58025}{48}$

Answers

1) a) 18 **b)** 1120

2) 4564

3) -27, -22, -17

4) 1860 cm

5) a) $S_8 = 6560$ **b)** $S_{10} = \frac{1023}{64}$

6) $S_8 = \frac{6305}{6561}$

7) $S_7 = 2735$

) 7 terms

9) $S_{10} = \frac{58025}{48}$

6.2 Recursive Procedures – Worksheet

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SOLUTIONS

1) Write the first four terms of each sequence.

a) $t_1 = 4,\ t_n = t_{n-1} + 3$

$4, 7, 10, 13$

b) $t_1 = 50,\ t_n = \frac{t_{n-1}}{2}$

$50, 25, \frac{25}{2}, \frac{25}{4}$

c) $t_1 = 100,\ t_n = \frac{5t_{n-1}}{0.1} = 50 \cdot t_{n-1}$

$100, 5000, 250\,000, 12\,500\,000$

2) Write the first four terms of each sequence

a) $f(1) = 3, f(n) = \frac{f(n-1)}{n}$

$f(2) = \frac{f(1)}{2} = \frac{3}{2}$

$f(3) = \frac{f(2)}{3} = \frac{3/2}{3} = \frac{3}{6} = \frac{1}{2}$

$f(4) = \frac{f(3)}{4} = \frac{1/2}{4} = \frac{1}{8}$

$3, \frac{3}{2}, \frac{1}{2}, \frac{1}{8}$

b) $f(1) = 0.5),\ f(n) = -f(n-1)$

$f(1) = 0.5$

$f(2) = -f(1) = -0.5$

$f(3) = -f(2) = -(-0.5) = 0.5$

$f(4) = -f(3) = -0.5$

$0.5, -0.5, 0.5, -0.5$

3) Determine a recursion formula for each sequence.

a) $5, 11, 17, 23, \ldots$

$t_n = t_{n-1} + 6$

b) $4, 1, -2, -5, \ldots$

$t_n = t_{n-1} - 3$

c) $4, 8, 16, 32, \ldots$

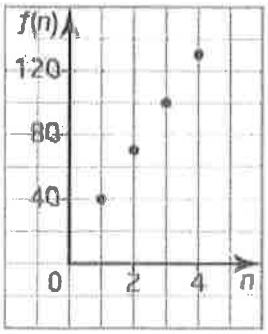
$t_n = 2t_{n-1}$

d) $-4, -2, -1, -\frac{1}{2}$

$t_n = \frac{t_{n-1}}{2}$

4) For each graph, write the sequence of terms and determine a recursion formula.

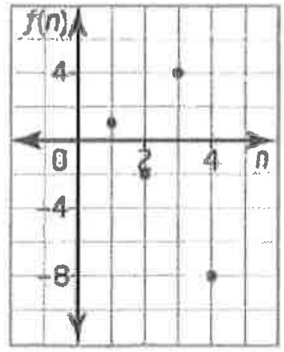
a)



40, 70, 100, 130

$t_n = t_{n-1} + 30$

b)



1, -2, 4, -8

$t_n = -2t_{n-1}$

5) A new theatre is being built for a youth orchestra. This theatre has 50 seats in the first row, 54 in the second row, 62 in the third row, 74 in the next row, and so on. Represent the number of seats in the rows as a sequence and then write a recursion formula to represent the number of seats in any row.

50, 54, 62, 74, ...

$t_n = t_{n-1} + 4(n-1)$

6) Write the first four terms of each sequence.

a) $t_1 = 1,\ \ t_n = (t_{n-1})^2 + 3n$

$t_2 = (1)^2 + 3(2) = 7$

$t_3 = (7)^2 + 3(3) = 58$

$t_4 = (58)^2 + 3(4) = 3376$

b) $t_1 = \frac{1}{2},\ \ t_n = 4t_{n-1} + 2$

$t_2 = 4(\frac{1}{2}) + 2 = 4$

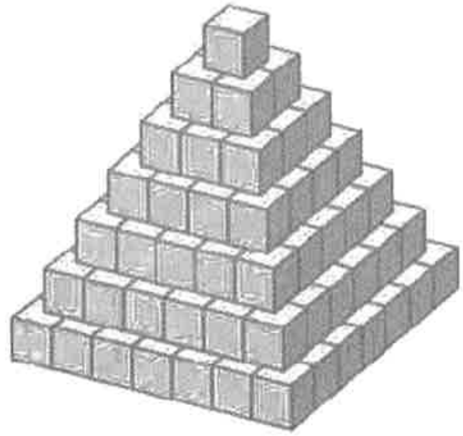
$t_3 = 4(4) + 2 = 18$

$t_4 = 4(18) + 2 = 74$

7) A square based pyramid with height 7 meters is constructed with cubic blocks measuring 1 m on each side. Write a recursion formula for the sequence that represents the number of blocks used at each level from top down.

n	1	2	3	4	5	6	7
t_n	1,	4,	9,	16,	25,	36,	49

$t_n = t_{n-1} + 2n - 1$



Answers

1) a) $4, 7, 10, 13$ **b)** $50, 25, \frac{25}{2}, \frac{25}{4}$ **c)** $100, 5\,000, 250\,000, 12\,500\,000$

2) a) $3, \frac{3}{2}, \frac{1}{2}, \frac{1}{8}$ **b)** 0.5, -0.5, 0.5, -0.5

3) a) $t_n = t_{n-1} + 6$ **b)** $t_n = t_{n-1} - 3$ **c)** $t_n = 2 \cdot t_{n-1}$ **d)** $t_n = \frac{t_{n-1}}{2}$

4) a) $t_n = t_{n-1} + 30$ **b)** $t_n = -2t_{n-1}$

5) $t_n = t_{n-1} + 4(n-1)$

)a) $1, 7, 58, 3376$ **b)** $\frac{1}{2}, 4, 18, 74$

7) $t_n = t_{n-1} + 2n - 1$

6.3 Pascal's Triangle – Worksheet #1

MCR3U
Jensen

SOLUTIONS.

1) Find each coefficient described.

a) coefficient of x^2 in the expansion of $(2+x)^5$

3rd term is $10(2)^3(x)^2 = 80x^2$

∴ the coefficient is 80.

b) coefficient of x^2 in the expansion of $(x+2)^5$

4th term is $10(x)^2(2)^3 = 80x^2$

∴ the coefficient is 80.

c) coefficient of x in the expansion of $(x+3)^5$

5th term is $5(x)^1(3)^4 = 405x$

∴ the coefficient is 405

d) coefficient of b in the expansion of $(3+b)^4$

2nd term is $4(3)^3(b) = 108b$

∴ the coefficient is 108.

e) coefficient of x^3y^2 in expansion of $(x-3y)^5$

3rd term is $10(x)^3(-3y)^2 = 90x^3y^2$

∴ the coefficient is 90

f) coefficient of a^2 in the expansion of $(2a+1)^5$

4th term is $10(2a)^2(1)^3 = 40a^2$

∴ the coefficient is 40.

2) Find each term described.

a) 2nd term in expansion of $(y-2x)^4$

$= 4(y)^3(-2x)^1$

$= -8y^3x$

b) 4th term in expansion of $(4y+x)^4$

$= 4(4y)(x)^3$

$= 16yx^3$

c) 1st term in expansion of $(a+b)^5$

$= 1(a)^5(b)^0$

$= a^5$

d) 2nd term in expansion of $(y-x)^4$

$= 4(y)^3(-x)^1$

$= -4y^3x$

3) Expand completely

a) $(2m-1)^4$

$= 1(2m)^4 + 4(2m)^3(-1) + 6(2m)^2(-1)^2 + 4(2m)(-1)^3 + 1(-1)^4$

$= 16m^4 - 32m^3 + 24m^2 - 8m + 1$

b) $(x-y)^3$

$= 1(x)^3 + 3(x)^2(-y) + 3(x)(-y)^2 + 1(-y)^3$

$= x^3 - 3x^2y + 3xy^2 - y^3$

c) $(x^4 - y)^5$

$= 1(x^4)^5 + 5(x^4)^4(-y) + 10(x^4)^3(-y)^2$
$+ 10(x^4)^2(-y)^3 + 5(x^4)(-y)^4 + 1(-y)^5$

$= x^{20} - 5x^{16}y + 10x^{12}y^2 - 10x^8y^3$
$+ 5x^4y^4 - y^5$

d) $(2x^3 + 1)^5$

$= 1(2x^3)^5 + 5(2x^3)^4(1) + 10(2x^3)^3(1)^2$
$+ 10(2x^3)^2(1)^3 + 5(2x^3)(1)^4 + 1(1)^5$

$= 32x^{15} + 80x^{12} + 80x^9 + 40x^6 + 10x^3 + 1$

e) $(y - x^2)^3$

$= 1(y)^3 + 3(y)^2(-x^2)^1 + 3(y)^1(-x^2)^2 + 1(-x^2)^3$

$= y^3 - 3y^2x^2 + 3yx^4 - x^6$

f) $(y^3 - 4x)^3$

$= 1(y^3)^3 + 3(y^3)^2(-4x)^1 + 3(y^3)^1(-4x)^2 + 1(-4x)^3$

$= y^9 - 12y^6x + 48y^3x^2 - 64x^3$

Answers

1) a) 80 **b)** 80 **c)** 405 **d)** 108 **e)** 90 **f)** 40

2) a) $-8y^3x$ **b)** $16yx^3$ **c)** a^5 **d)** $-4y^3x$

3) a) $16m^4 - 32m^3 + 24m^2 - 8m + 1$ **b)** $x^3 - 3x^2y + 3xy^2 - y^3$

c) $x^{20} - 5x^{16}y + 10x^{12}y^2 - 10x^8y^3 + 5x^4y^4 - y^5$

d) $32x^{15} + 80x^{12} + 80x^9 + 40x^6 + 10x^3 + 1$

e) $y^3 - 3y^2x^2 + 3yx^4 - x^6$ **f)** $y^9 - 12y^6x + 48y^3x^2 - 64x^3$

6.3 Pascal's Triangle – Worksheet #2

MCR3U

Jensen

SOLUTIONS

1) Expand each expression using Pascal's triangle

a) $(x+4)^3$

$= 1x^3 + 3x^2(4) + 3x(4^2) + 1(4^3)$

$= x^3 + 12x^2 + 48x + 64$

b) $(1-2x)^4$

$= 1(1^4) + 4(1^3)(-2x) + 6(1^2)(-2x)^2 + 4(1)(-2x)^3 + 1(-2x)^4$

$= 1 - 8x + 24x^2 - 32x^3 + 16x^4$

c) $(3x+y)^2$

$= 1(3x)^2 + 2(3x)(y) + 1(y)^2$

$= 9x^2 + 6xy + y^2$

d) $(x-5)^5$

$= 1(x)^5 + 5(x)^4(-5) + 10(x)^3(-5)^2 + 10(x)^2(-5)^3 + 5(x)(-5)^4 + 1(-5)^5$

$= x^5 - 25x^4 + 250x^3 - 1250x^2 + 3125x - 3125$

e) $(3+2n)^6$

$= 1(3)^6 + 6(3)^5(2n) + 15(3)^4(2n)^2 + 20(3)^3(2n)^3$
$+ 15(3)^2(2n)^4 + 6(3)(2n)^5 + 1(2n)^6$

$= 729 + 2916n + 4860n^2 + 4320n^3$
$+ 2160n^4 + 576n^5 + 64n^6$

f) $(x-7)^{11}$

skip.

g) $(2x + 5)^7$

$= 1(2x)^7 + 7(2x)^6(5) + 21(2x)^5(5)^2 + 35(2x)^4(5)^3 + 35(2x)^3(5)^4 + 21(2x)^2(5)^5 + 7(2x)(5)^6 + 1(5)^7$

$= 128x^7 + 2240x^6 + 16\,800x^5 + 70000x^4 + 175\,000x^3 + 262\,500x^2 + 218\,750x + 78\,125$

Answers

1. a) $x^3 + 12x^2 + 48x + 64$

b) $1 - 8x + 24x^2 - 32x^3 + 16x^4$

c) $9x^2 + 6xy + y^2$

d) $x^5 - 25x^4 + 250x^3 - 1250x^2 + 3125x - 3125$

e) $729 + 2916n + 4860n^2 + 4320n^3 + 2160n^4 + 576n^5 + 64n^6$

f) $x^{11} - 77x^{10} + 2695x^9 - 56\,595x^8 + 792\,330x^7 - 7\,764\,834x^6 + 54\,353\,838x^5 - 271\,769\,190x^4 + 951\,192\,165x^3 - 2\,219\,448\,285x^2 + 3\,107\,227\,739x - 1\,977\,326\,743$

g) $128x^7 + 2240x^6 + 16\,800x^5 + 70\,000x^4 + 175\,000x^3 + 262\,500x^2 + 218\,750x + 78\,125$