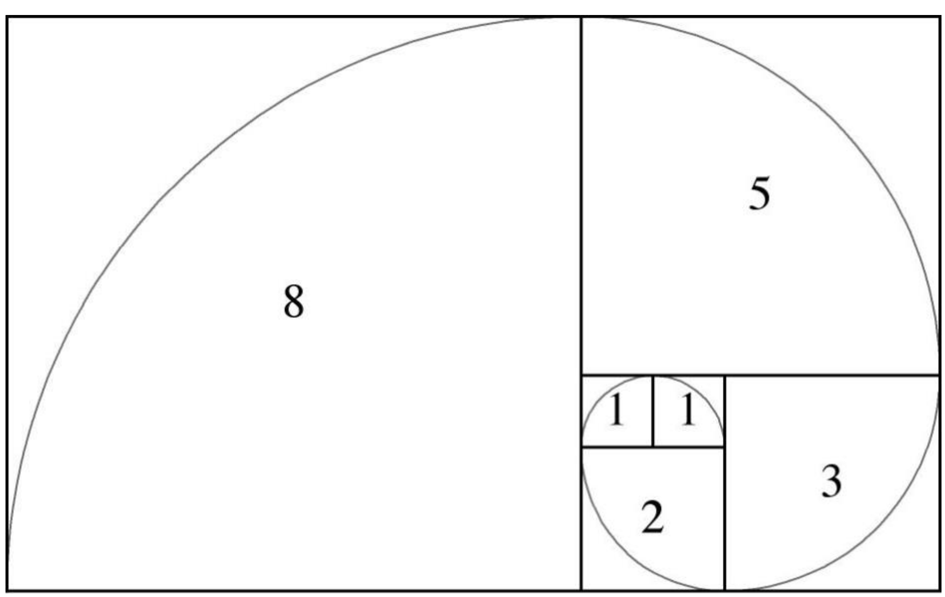
Chapter 6- Discrete Functions

Lesson Package

MCR3U



Chapter 6 Outline

Unit Goal: Be able to demonstrate an understanding of the relationships involved in arithmetic sequences and series, and recursive functions.

Section	Subject	Learning Goals	Curriculum Expectations
L1	Sequences	- Identify sequences as arithmetic, geometric or neither. Determine the equation for the general term of an arithmetic or geometric sequence.	C1.1, C1.3, C2.1, C2.2
L2	Series	- Determine the sum of an arithmetic or geometric series	C2.3
L3	More Sequences	- Solve problems involving arithmetic and geometric sequences	C2.4
L4	More Series	- Solve problems involving arithmetic and geometric series	C2.4
L5	Recursive Functions	- Represent a sequence using a recursion formula. Use a recursion formula to write the terms of a recursive function.	C1.2, C1.3, C1.4, C1.5
L6	Pascal's Triangle	- Expand binomials using Pascal's Triangle	C1.6

Assessments	**F/A/O**	**Ministry Code**	**P/O/C**	**KTAC**
Note Completion	A		P	
Practice Worksheet Completion	F/A		P	
PreTest Review	F/A		P	
Test – Trig Geometry	O	C1.1, C1.2, C1.3, C1.4, C1.5, C1.6, C2.1, C2.2, C2.3, C2.4	P	K(21%), T(34%), A(10%), C(34%)

Arithmetic and Geometric Sequences

DO IT NOW!

How much can you figure out about this list of numbers?

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233,...

https://www.youtube.com/watch?v=SjSHVDfXHQ4

Definitions

Formula for general term (explicit formula):

A formula that represents any term in a sequence relative to the term number (n)

Sequence:

an ordered list of numbers identified by a pattern or rule that may stop at some number of continue indefinitely

Ex.

3, 7, 11, 15

2, 6, 18, 54, … the three dots indicate that it continues forever

> **Note:** the terms of a sequence represent the range (*y*-values) of a function.

Example 1: Write the first three terms of each sequence, given the explicit formula for the *n*th term of the sequence.

a) $t_n = 3n^2 - 1$

$t_1 = 3(1)^1 - 1$
$= 2$

$t_2 = 3(2)^2 - 1$
$= 11$

$t_3 = 3(3)^2 - 1$
$= 26$

The first three terms are 2, 11, 26

b) $t_n = \frac{n-1}{n}$

$t_1 = \frac{1-1}{1}$
$= 0$

$t_2 = \frac{2-1}{2}$
$= \frac{1}{2}$

$t_3 = \frac{3-1}{3}$
$= \frac{2}{3}$

The First three terms are 0, ½, ⅔

Arithmetic Sequences

Examples of sequences:

+4 +4 +4

a) 14, 18, 22, 26,...

b) 7, 3, -1, -5,...

-4 -4 -4

These are called ***arithmetic*** sequences because they increase by a constant difference (+ or -)

Arithmetic Sequences

Formula for General Term of an Arithmetic Sequence

$$t_n = a + (n-1)d$$

n: *the term number*

a: *the value of the first term*

t_n: *a term in the sequence*

d: *the common difference*

Example 2

a) Determine a formula for the general term of the following arithmetic sequence.

$$\overset{+4}{14, 18, 22, 26,...}$$

$a = 14$
$d = 4$

$t_n = a + (n-1)d$

$t_n = 14 + (n-1)(4)$

b) What is the value of t_{30}

$t_{30} = 14 + (30-1)(4)$
$= 14 + 116$
$= 130$

Example 3:

a) Determine a formula for the general term of the following arithmetic sequence.

$$\underset{-4}{7, 3, -1, -5, \ldots}$$

$t_n = a + (n-1)d$

$t_n = 7 + (n-1)(-4)$

$a = 7$
$d = -4$

b) What is the value of t_{41}

$t_{41} = 7 + (41-1)(-4)$
$= 7 - 160$
$= -153$

Geometric Sequences

Examples of sequences:

$\times 3$

a) 2, 6, 18, 54,…

b) 80, 40, 20, 10,… * No division is used *

$\times \frac{1}{2}$

These are called ***geometric*** sequences because the ratio of consecutive terms is constant.

Increase/decrease by a constant multiple

Geometric Sequences

Formula for the General Term of a Geometric Sequence

$$t_n = a \cdot r^{n-1}$$

n: *term number*

a: *the first term*

t_n: *a term in the sequence*

r: *the constant multiple*

Note: no division is used $\rightarrow \div 2 = \times \frac{1}{2}$

Example 4: Determine a formula for the general term of the following geometric sequence.

$$\times 3$$

2, 6, 18, 54, 162,...

$a=2$

$r=3$

$t_n = a \cdot r^{n-1}$

$= 2(3)^{n-1}$

b) What is the value of t_9

$t_9 = 2(3)^{9-1}$

$= 13\,122$

Example 5: Determine a formula for the general term of the following geometric sequence.

270, 90, 30, 10,...

$\times \frac{1}{3}$

$t_n = a \cdot r^{n-1}$

$t_n = 270\left(\frac{1}{3}\right)^{n-1}$

$a=270$

$r=\frac{1}{3}$

b) What is the value of t_9

$t_9 = 270\left(\frac{1}{3}\right)^{9-1}$

$= 270\left(\frac{1}{3^8}\right)$

$= 270\left(\frac{1}{6561}\right)$

$= \frac{10}{243}$

Arithmetic and Geometric Series

Definitions

Formula for general term (explicit formula):

A formula that represents any term in a sequence relative to the term number (n)

Sequence:

an ordered list of numbers identified by a pattern or rule that may stop at some number of continue indefinitely

Arithmetic Sequence:

sequence in which the difference between consecutive terms is a constant

Geometric Sequence:

sequence in which the ratio of consecutive terms is constant

Series:

the indicated **sum** of the terms of a sequence

Example 1: Find S_4 of the sequence represented by:

sum of the first 4 terms of the sequence

$S_4 = t_1 + t_2 + t_3 + t_4$

$t_n = 1+(n-1)3$

$t_1 = 1+(1-1)(3)$
$= 1$

$t_2 = 1+(2-1)(3)$
$= 4$

$t_3 = 1+(3-1)(3)$
$= 7$

$t_4 = 1+(4-1)(3)$
$= 10$

$S_4 = 1+4+7+10$
$= 22$

Arithmetic Series

general form

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

because the general form for an arithmetic sequence is

$$tn = a+(n-1)d$$

OR we can rewrite S_n as:

$$S_n = \frac{n}{2}[a+t_n]$$

use this version when given the value of the last term of the series (t_n)

Example 2: For the series $1+3+5+7+\ldots$ find S_{23}

+2

$a=1$
$d=2$
$n=23$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{23} = \frac{23}{2}[2(1)+(23-1)(2)]$

$= \frac{23}{2}(46)$

$= 529$

Example 3: An arithmetic series with 52 terms starts with -7 and ends with 102. Find the sum of the series.

n a t_{52}

Note: Since we know t_{52}, it would be easier to use this version of the formula...

$$S_n = \frac{n}{2}(a+t_n)$$

$S_{52} = \frac{52}{2}(-7+102)$

$= 26(95)$

$= 2470$

Geometric Series

general form

$$\boxed{S_n = \frac{a(r^n - 1)}{r - 1}}$$

Example 4: For the geometric sequence $-1 + 2 - 4 + 8 - 16, \ldots$

$\times(-2)$

$a = -1$

$r = -2$

a) Find S_5.

$S_5 = -1 + 2 - 4 + 8 - 16$

$= -11$

b) Find S_{13}

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{13} = \frac{-1[(-2)^{13} - 1]}{-2 - 1}$$

$$= \frac{8193}{-3}$$

$$= -2731$$

Example 5: A student is offered a job with a math teacher that will last 20 hours. It pays \$4.75 for the first hour, \$5 for the second hour, \$5.25 for the next hour, and so on. How much will the student earn in total?

$a = 4.75$

$d = 0.25$

$n = 20$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{20} = \frac{20}{2}\left[2(4.75) + (20-1)(0.25)\right]$$

$$S_{20} = 10(14.25)$$

$$= \$142.50$$

The student will be paid \$142.50 for 20 hours of work.

Sequences (part 2)

Sequences Questions

What is the difference between a sequence and a series?

Sequence *- a list of numbers that change by a constant value*

Series *- the sum of values in a sequence*

What is the difference between Arithmetic and Geometric?

arithmetic: *+ or - to get future terms*

geometric: *× to get future terms*

Formulas for general terms of a sequence

Arithmetic Sequence	$t_n = a + (n-1)d$

Geometric Sequence	$t_n = a \cdot r^{n-1}$

Example 1: $-10, \underbrace{-4,}_{+6} 2, \ldots$

a) Determine whether the sequence is arithmetic or geometric.

Arithmetic

b) Determine an equation for the sequence.

$$t_n = a + (n-1)d$$
$$= -10 + (n-1)(6)$$

c) Find the value of t_{21}

$$t_{21} = -10 + (21-1)(6)$$
$$= 110$$

Example 2: Insert two numbers between 8 and 32 so the four numbers form an arithmetic sequence.

$$8 \xrightarrow{+8} \underline{16} \xrightarrow{+8} \underline{24} \xrightarrow{+8} 32$$

$$\underbrace{8, \; 16, \; 24, \; 32}_{+24}$$

$$24/3 = 8$$

Example 3: An arithmetic sequence is 8, 14, 20, 26,.... Which term has the value 92? Prove mathematically.

$+6$

t_n

$a = 8$
$d = 6$

$$t_n = a + (n-1)d$$

$$92 = 8 + (n-1)(6)$$

$$84 = (n-1)(6)$$

$$\frac{84}{6} = n-1$$

$$14 = n-1$$

$$\boxed{15 = n}$$

$$\boxed{\therefore t_{15} = 92}$$

Example 4: 200, -100, 50,...

$\times\left(\frac{-1}{2}\right)$

a) Is the sequence arithmetic or geometric?

geometric

b) Find an equation to represent the sequence.

$t_n = a \cdot r^{n-1}$

$t_n = 200\left(\frac{-1}{2}\right)^{n-1}$

c) Find t_{14}.

$t_{14} = 200\left(\frac{-1}{2}\right)^{14-1}$

$= 200\left(\frac{-1}{8192}\right)$

$= \frac{-25}{1024}$

Example 5: Complete the geometric sequence:

$\times \frac{1}{2}$

320, 160, 80, 40, 20, 10

Example 6: The 50th term of an arithmetic sequence is 238 and the 93rd term is 539. Find a general equation to represent the sequence.

$t_{50} = a + (50-1)d$ $\qquad$ $t_{93} = a + (93-1)d$

① $238 = a + 49d$ $\qquad$ ② $539 = a + 92d$

* solve the system of equations *

② $539 = a + 92d$
① $238 = a + 49d$ $-$

$301 = 43d$
$7 = d$

sub $d=7$ into ①

$238 = a + 49(7)$
$238 - 343 = a$
$a = -105$

$\boxed{\therefore \text{ the general equation is } t_n = -105 + (n-1)(7)}$

Example 7: Determine the number of terms in the geometric sequence: 5, -10, 20, ..., -10 240

$t_n = -10\,240$
$n = ?$
$a = 5$
$r = -2$

$t_n = a \cdot r^{n-1}$

$-10\,240 = 5(-2)^{n-1}$

write as a power w/ base (-2). $\rightarrow$ $-2048 = (-2)^{n-1}$

$(-2)^{11} = (-2)^{n-1}$

$\therefore$ $11 = n-1$
$12 = n$

$\therefore$ there are 12 terms in the sequence.

More Arithmetic and Geometric Series Questions

DO IT NOW!

In an arithmetic sequence, $t_3 = 25$ and $t_9 = 43$. Determine the formula for the general term of this sequence.

$\boxed{t_3}$

$t_n = a + (n-1)d$

$25 = a + (3-1)d$

① $25 = a + 2d$

$\boxed{t_9}$

$43 = a + (9-1)d$

② $43 = a + 8d$

* solve using elimination

② $43 = a + 8d$

① $25 = a + 2d$ $-$

$18 = 6d$

$3 = d$

* sub $d = 3$ into ①

$25 = a + 2(3)$

$25 = a + 6$

$19 = a$

$\boxed{\text{The general formula is } t_n = 19 + (n-1)(3)}$

Arithmetic

Sequence:

$$t_n = a + (n-1)d$$

Series:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{n}{2}(a + t_n)$$

Geometric

Sequence:

$$t_n = a \cdot r^{n-1}$$

Series:

$$S_n = \frac{a(r^n - 1)}{r-1}$$

Example 1: In an amphitheatre, seats are arranged in 50 semicircular rows facing a domed stage. The first row contains 23 seats, and each row contains 4 more seats than the previous row. How many seats are in the amphitheatre?

$a = 23 \quad d = 4 \quad n = 50 \quad S_{50} = ?$

Note: *use formula for arithmetic series because difference between consecutive rows is a constant.*

$$S_{50} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{50}{2}[2(23) + (50-1)(4)]$$

$$= 25(242)$$

$$= 6050$$

The amphitheatre has 6050 seats.

arithmetic series

Example 2: Determine the sum of -31 -35 -39-.....-403

-4 -4

last term. $n = ?$

Start by determining what term # the last term is.

$$t_n = a + (n-1)d$$
$$-403 = -31 + (n-1)(-4)$$
$$-372 = (n-1)(-4)$$
$$93 = n-1$$
$$94 = n$$

$\therefore t_{94} = -403$

Next, find S_{94}:

$$S_n = \frac{n}{2}(a + t_n)$$
$$S_{94} = \frac{94}{2}(-31 - 403)$$
$$= 47(-434)$$
$$= -20\,398$$

Example 3: Determine the sum of the first 20 terms of the arithmetic series in which the 15th term is 107 and the terms decrease by 3.

Start by finding the value of 'a' using $t_{15} = 107$

$$t_n = a + (n-1)d$$
$$107 = a + (15-1)(-3)$$
$$107 = a - 42$$
$$149 = a$$

Now find S_{20}:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{20}{2}[2(149) + (20-1)(-3)]$$
$$= 10(241)$$
$$= 2410$$

Example 4: The 10th term of an arithmetic series is 34, and the sum of the first 20 terms is 710. Determine the 25th term.

$t_{10} = 34 \quad S_{20} = 710 \quad t_{25} = ?$

$\underline{t_{10}}$

$t_n = a + (n-1)d$

$34 = a + (10-1)d$

① $34 = a + 9d$

$\underline{S_{20}}$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$710 = \frac{20}{2}[2a + (20-1)d]$

$710 = 10(2a + 19d)$

② $710 = 20a + 190d$

* solve using substitution or elimination.

② ⟶ $710 = 20a + 190d$

① ×20 ⟶ $680 = 20a + 180d$ −

$30 = 10d$

$3 = d$

* sub $d = 3$ into ①

$34 = a + 9(3)$

$34 = a + 27$

$7 = a$

* no solve for t_{25}: $t_n = a + (n-1)d$

$t_{25} = 7 + (25-1)(3)$

$t_{25} = 79$

Example 5: Determine the sum of the first seven terms of the geometric series in which $t_5 = 5$ and $t_8 = -40$.

method 1

$\frac{5}{16}, -\frac{5}{8}, \frac{5}{4}, -\frac{5}{2}, 5, -10, 20, -40$

1st, 2nd, 3rd, 4th, 5th, 6th, 7th, 8th

$\div(-2) \quad \times(-2)$

$\therefore r = -2$

and

$a = \frac{5}{16}$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_7 = \frac{\left(\frac{5}{16}\right)\left[(-2)^7 - 1\right]}{-2-1}$

$= \frac{\left(\frac{5}{16}\right)(-129)}{-3}$

$= \frac{5}{16}(43)$

$= \frac{215}{16}$

Example 5: Determine the sum of the first seven terms of the geometric series in which $t_5 = 5$ and $t_8 = -40$.

$\boxed{\text{method 2}}$

$\underline{t_5}$

$5 = a(r)^{5-1}$

$5 = a(r)^4$

$a = \frac{5}{r^4}$

$\underline{t_8}$

$-40 = a(r)^{8-1}$

$-40 = a(r)^7$

* solve using substitution

$-40 = \left(\frac{5}{r^4}\right)(r^7)$

$-40 = 5r^3$

$-8 = r^3$

$\sqrt[3]{(-8)} = r$

$-2 = r$

$a = \frac{5}{r^4}$

$a = \frac{5}{(-2)^4}$

$a = \frac{5}{16}$

* geo. series formula

$S_7 = \frac{a(r^n - 1)}{r-1}$

$S_7 = \frac{\left(\frac{5}{16}\right)[(-2)^7 - 1]}{-2-1}$

$S_7 = \frac{215}{16}$

Example 6: Calculate the sum of the geometric series, 960 + 480 + 240 +....+15

$\times\frac{1}{2}$

Method 1: write out full series

$= 960 + 480 + 240 + 120 + 60 + 30 + 15$

$= 1905$

Method 2: Solve using logarithms

Figure out how many terms are in the series by solving for n in the formula: $t_n = a \cdot r^{n-1}$

$a = 960 \quad r = \frac{1}{2}$

$t_n = a(r)^{n-1}$

$15 = 960\left(\frac{1}{2}\right)^{n-1}$ last term $= 15$

$\frac{15}{960} = \left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{64} = \left(\frac{1}{2}\right)^{n-1}$

$\log\left(\frac{1}{64}\right) = \log\left(\frac{1}{2}\right)^{n-1}$

$\log\left(\frac{1}{64}\right) = (n-1)\cdot\log\left(\frac{1}{2}\right)$

$\frac{\log\left(\frac{1}{64}\right)}{\log\left(\frac{1}{2}\right)} = n-1$

$6 = n-1$

$7 = n$

Now solve S_7:

$S_n = \frac{a(r^n-1)}{r-1}$

$S_7 = \frac{960\left[\left(\frac{1}{2}\right)^7-1\right]}{\left(\frac{1}{2}\right)-1}$

$= \frac{-952.5}{-0.5}$

$= 1905$

Method 3: Solve using powers with the same base

$a = 960 \quad r = \frac{1}{2}$

$t_n = a(r)^{n-1}$

$15 = 960\left(\frac{1}{2}\right)^{n-1}$ last term $= 15$

$\frac{15}{960} = \left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{64} = \left(\frac{1}{2}\right)^{n-1}$

$\left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^{n-1}$

$6 = n-1$

$7 = n$

Now solve S_7:

$S_n = \frac{a(r^n-1)}{r-1}$

$S_7 = \frac{960\left[\left(\frac{1}{2}\right)^7-1\right]}{\left(\frac{1}{2}\right)-1}$

$= \frac{-952.5}{-0.5}$

$= 1905$

Example 7: A tennis tournament has 128 entrants. A player is dropped from the competition after losing one match. Winning players go on to another match. What is the total number of matches that will be played in this tournament?

$a = 64 \quad r = \frac{1}{2}$ last term is 1.

Note: The first term is 128/2 = 64 because 2 players participate in one match. The last term is 1 but we don't know what term number it is.

Start by determining the # of terms in the series

$$t_n = a(r)^{n-1}$$

$$1 = 64\left(\tfrac{1}{2}\right)^{n-1}$$

$$\frac{1}{64} = \left(\tfrac{1}{2}\right)^{n-1}$$

$$\left(\tfrac{1}{2}\right)^6 = \left(\tfrac{1}{2}\right)^{n-1}$$

$$6 = n-1$$

$$7 = n$$

Next, calculate S_7

$$S_n = \frac{a(r^n - 1)}{r-1}$$

$$S_7 = \frac{64\left[\left(\tfrac{1}{2}\right)^7 - 1\right]}{\tfrac{1}{2} - 1}$$

$$= \frac{-63.5}{-0.5}$$

$$= 127$$

$\therefore$ 127 matches will be played in the tournament.

6.2 Recursive Functions

In earlier sections we used function notation to write an explicit formula to determine the value of any term in a sequence. Sometimes it is easier to calculate one term in a sequence using the previous terms.

Recursion formula:

a formula by which each term of a sequence is generated from the preceding term or terms.

Recursive Functions

Functions that get new terms in the sequence by using earlier terms.

t_n = the value of term 'n'
t_{n-1} = the value before t_n

Example 1: Write the first 4 terms of the sequence.

a) $t_n = t_{n-1} - 2$ where $t_1 = 7$

$t_2 = t_{2-1} - 2$
$= t_1 - 2$
$= 7 - 2$
$= 5$

$t_3 = t_2 - 2$
$= 5 - 2$
$= 3$

$t_4 = t_3 - 2$
$= 3 - 2$
$= 1$

The first four terms of the sequence are 7, 5, 3, 1.

b) $t_n = 2t_{n-1} + 4$ where $t_1 = 5$

$t_2 = 2t_1 + 4$
$= 2(5) + 4$
$= 14$

$t_3 = 2t_2 + 4$
$= 2(14) + 4$
$= 32$

$t_4 = 2t_3 + 4$
$= 2(32) + 4$
$= 68$

The first four terms of the sequence are 5, 14, 32, 68.

You may also see questions asked in function notation.

Example 2: Find the first 4 terms.

$f(n)=2f(n-1) - 7$ where $f(1) = 2$

$f(2) = 2f(1) - 7$
$= 2(2) - 7$
$= -3$

$f(3) = 2 \cdot f(2) - 7$
$= 2(-3) - 7$
$= -13$

$f(4) = 2 \cdot f(3) - 7$
$= 2(-13) - 7$
$= -33$

The first four terms of the sequence are 2, -3, -13, -33.

Example 3: Find the first 7 terms of the sequence.

$t_n = t_{n-2} + t_{n-1}$ where $t_1 = 1$ where $t_2 = 1$

$t_3 = t_{3-2} + t_{3-1}$
$= t_1 + t_2$
$= 1 + 1$
$= 2$

$t_4 = t_2 + t_3$
$= 1 + 2$
$= 3$

$t_5 = t_3 + t_4$
$= 2 + 3$
$= 5$

> ***Note:*** *each term in the sequence is the sum of the previous two terms. This is the Fibonacci sequence!*

The first seven terms in this sequence are 1, 1, 2, 3, 5, 8, 13.

Example 4:

Write a recursion formula for each sequence

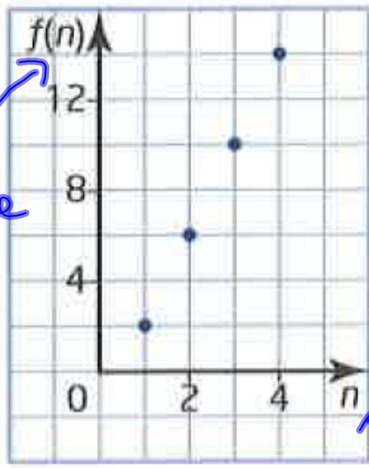
$\times(-2)$

a) -3, 6, -12, 24,....

$t_n = -2 \cdot t_{n-1}$

> Look for a pattern in the terms:
>
> $t_1 = -3$
> $t_2 = t_1 \times (-2)$
> $t_3 = t_2 \times (-2)$
> $t_4 = t_3 \times (-2)$

b)



term value

term #

$2 \xrightarrow{+4} 6 \rightarrow 10 \rightarrow 14$

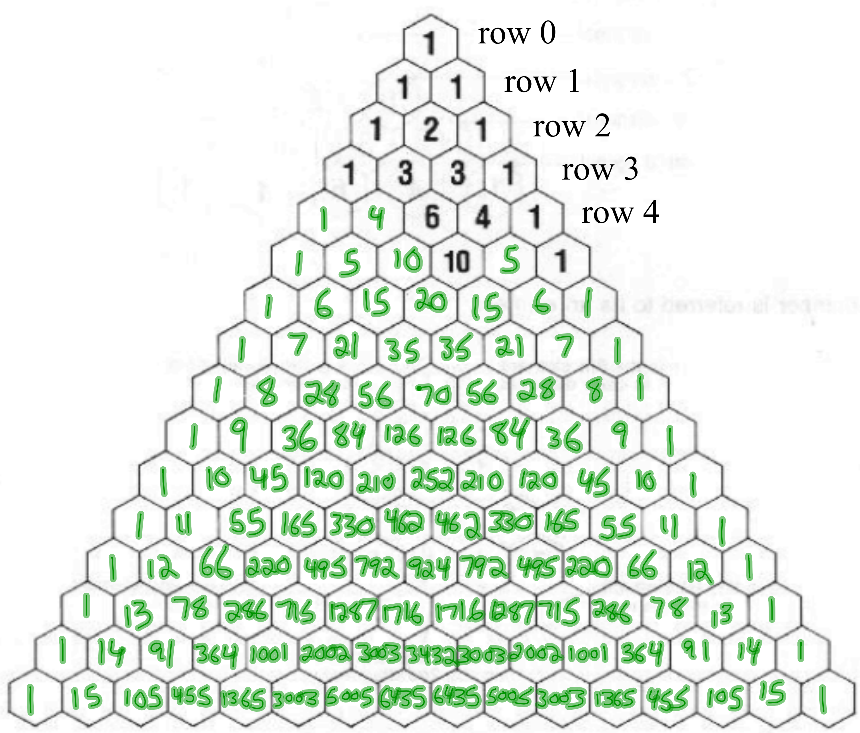
$$\boxed{t_n = t_{n-1} + 4}$$

c) $3 \xrightarrow{+2} 5 \xrightarrow{+3} 8 \xrightarrow{+4} 12, \ldots$

$$\boxed{t_n = t_{n-1} + n}$$

Pascal's Triangle

a) Complete Pascal's Triangle



b) What patterns do you notice in Pascal's Triangle?

Main Pattern:

Each term in Pascal's Triangle is the sum of the two terms directly above it. The first and last terms in each row are 1 since the only term immediately above them is always a 1.

Other Patterns:

- sum of each row is a power of 2 (sum of nth row is 2n, begin count at 0)
- symmetrical down the middle

c) Expand each of the following binomials

$(a+b)^0 = 1$

$(a+b)^1 = 1a + 1b$

$(a+b)^2 = (a+b)(a+b)$

$= 1a^2 + ab + ab + 1b^2$

$= 1a^2 + 2ab + 1b^2$

$(a+b)^3 = (a+b)(a+b)(a+b)$

$= (a+b)(a^2+2ab+b^2)$

$= a^3+2a^2b+ab^2+a^2b+2ab^2+b^3$

$= 1a^3+3a^2b+3ab^2+1b^3$

$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$

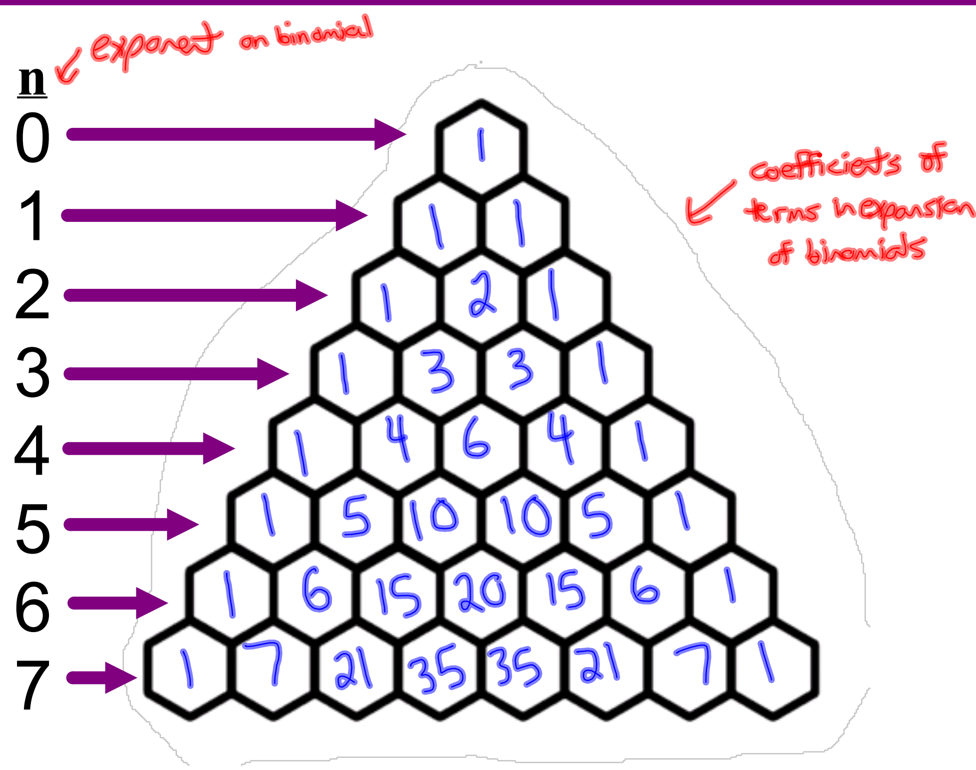
Blaise Pascal (French Mathematician) discovered a pattern in the expansion of $(a+b)^n$.... which patterns do you notice?

The coefficients in the expansion of $(a + b)^n$ can be found in row n of Pascal's triangle.

In each expansion, the exponents of a start at n and decrease by 1 down to zero, while the exponents of b start at zero and increase by 1 up to n.

In each term, the sum of the exponents of a and b is always n.

Pascal's Triangle



Example 1

Expand using the Binomial Theorem:

use row 6 of Pascal's triangle

a) $(a + b)^6$ ← n

$= 1a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + 1b^6$

b) $(2x - 3)^5$

$= 1(2x)^5 + 5(2x)^4(-3)^1 + 10(2x)^3(-3)^2 + 10(2x)^2(-3)^3$

$+ 5(2x)^1(-3)^4 + 1(-3)^5$

$= 32x^5 - 240x^4 + 720x^3 - 1080x^2 + 810x - 243$

c) $(2x + 3y^2)^5$

$$= 1(2x)^5(3y^2)^0 + 5(2x)^4(3y^2)^1 + 10(2x)^3(3y^2)^2$$
$$+ 10(2x)^2(3y^2)^3 + 5(2x)^1(3y^2)^4 + 1(2x)^0(3y^2)^5$$

$$= 1(32)(x^5)(1) + 5(16)(x^4)(3)(y^2) + 10(8)(x^3)(9)(y^4)$$
$$+ 10(4)(x^2)(27)(y^6) + 5(2)(x)(81)(y^8) + 1(1)(243)(y^{10})$$

$$= 32x^5 + 240x^4y^2 + 720x^3y^4 + 1080x^2y^6 + 810xy^8 + 243y^{10}$$

d) $\left(\frac{y}{2} - y^2\right)^4$ ← n

$$= 1\left(\frac{y}{2}\right)^4(-y^2)^0 + 4\left(\frac{y}{2}\right)^3(-y^2)^1 + 6\left(\frac{y}{2}\right)^2(-y^2)^2$$
$$+ 4\left(\frac{y}{2}\right)^1(-y^2)^3 + 1\left(\frac{y}{2}\right)^0(-y^2)^4$$

$$= \frac{y}{16} + 4\left(\frac{y^3}{8}\right)(-1)(y^2) + 6\left(\frac{y^2}{4}\right)(1)(y^4)$$
$$+ 4\left(\frac{y}{2}\right)(-1)(y^6) + 1(1)(1)(y^8)$$

$$= \frac{y}{16} - \frac{y^5}{2} + \frac{3y^6}{2} - 2y^7 + y^8$$

Example 2

How many terms will there be if you expand $(x + 2y)^{20}$?

number of terms in a binomial expansion is always equal to $n + 1$

$20 + 1 = 21$

$\boxed{21 \text{ terms}}$

Example 3

a) What is the 2nd term in the expansion of $(x+6)^7$

$= 7(x)^6(6)^1$

$= 42x^6$

b) What is the 5th term in the expansion of $(3y - 4)^8$

$= 70(3y)^4(-4)^4$

$= 70(81)(y^4)(256)$

$= 1451520y^4$

Example 4

a) What is the coefficient of x^3 in the expansion of $(x + 6)^6$

4^{th} term

$= 20(x)^3(6)^3$

$= 20x^3(216)$

$= 4320x^3$

b) What is the coefficient of y^4x^2 in the expansion of $(y + 3x)^6$

3^{rd} term

$= 15(y)^4(3x)^2$

$= 15(y^4)(9)(x^2)$

$= 135y^4x^2$

Homework: Worksheet Questions