Chapter 6- Discrete Functions

Lesson Package

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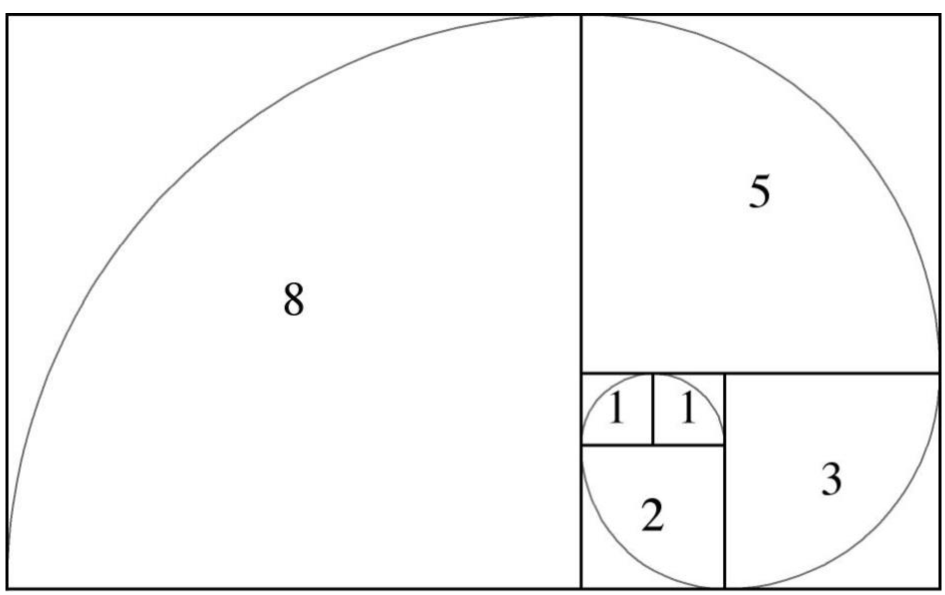

Chapter 6 Outline

Unit Goal: Be able to demonstrate an understanding of the relationships involved in arithmetic sequences and series, and recursive functions.

Section	Subject	Learning Goals	Curriculum Expectations
L1	Sequences	- Identify sequences as arithmetic, geometric or neither. Determine the equation for the general term of an arithmetic or geometric sequence.	C1.1, C1.3, C2.1, C2.2
L2	Series	- Determine the sum of an arithmetic or geometric series	C2.3
L3	More Sequences	- Solve problems involving arithmetic and geometric sequences	C2.4
L4	More Series	- Solve problems involving arithmetic and geometric series	C2.4
L5	Recursive Functions	- Represent a sequence using a recursion formula. Use a recursion formula to write the terms of a recursive function.	C1.2, C1.3, C1.4, C1.5
L6	Pascal's Triangle	- Expand binomials using Pascal's Triangle	C1.6

Assessments	**F/A/O**	**Ministry Code**	**P/O/C**	**KTAC**
Note Completion	A		P	
Practice Worksheet Completion	F/A		P	
PreTest Review	F/A		P	
Test – Trig Geometry	O	C1.1, C1.2, C1.3, C1.4, C1.5, C1.6, C2.1, C2.2, C2.3, C2.4	P	K(21%), T(34%), A(10%), C(34%)

L1 - Sequences (Part 1)
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DO IT NOW!

How much can you figure out about this list of numbers?

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233,...

Definitions

Formula for general term (explicit formula):

Sequence:

Example 1: Write the first three terms of each sequence, given the explicit formula for the nth term of the sequence.

a) $t_n = 3n^2 - 1$

b) $t_n = \frac{n-1}{n}$

Arithmetic Sequences

Examples of sequences:

a) 14, 18, 22, 26,...

b) 7, 3, -1, -5,...

These are called ***arithmetic*** sequences because they increase by a constant difference (+ or -)

Formula for the general term of an arithmetic sequence

$\boldsymbol{n}$**:**

$\boldsymbol{a}$**:**

$\boldsymbol{t_n}$**:**

$\boldsymbol{d}$**:**

Example 2

a) Determine a formula for the general term of the following arithmetic sequence.

14, 18, 22, 26,...

b) What is the value of t_{30}

Example 3

a) Determine a formula for the general term of the following arithmetic sequence.

7, 3, -1, -5,…

b) What is the value of t_{41}

Geometric Sequences

Examples of sequences:

a) 2, 6, 18, 54,…

b) 80, 40, 20, 10,…

These are called ***geometric*** sequences because the ratio of consecutive terms is constant.

Formula for the General Term of a Geometric Sequence

$\boldsymbol{n}$**:**

$\boldsymbol{a}$**:**

$\boldsymbol{t_n}$**:**

$\boldsymbol{r}$**:**

Example 4:

a) Determine a formula for the general term of the following geometric sequence.

2, 6, 18, 54, 162,...

b) What is the value of t_9

Example 5:

a) Determine a formula for the general term of the following geometric sequence.

270, 90, 30, 10,...

b) What is the value of t_9

L2 - Arithmetic and Geometric Series
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Definitions

Formula for general term (explicit formula):

Sequence:

Arithmetic Sequence:

Geometric Sequence:

Series:

Example 1: Find S_4 of the sequence represented by $t_n = 1 + (n - 1)3$

Arithmetic Series

General Form		
	OR	

Example 2: For the series $1 + 3 + 5 + 7 + \cdots$ find S_{23}

Example 3: An arithmetic series with 52 terms starts with -7 and ends with 102. Find the sum of the series.

> **Note:** Since we know t_{52}, it would be easier to use this version of the formula...
>
> $$S_n = \frac{n}{2}(a + t_n)$$

<u>Geometric Series</u>

General form

Example 4: For the geometric sequence $-1+2-4+8-16\ldots$

a) Find S_5

b) Find S_{13}

Example 5: A student is offered a job with a math teacher that will last 20 hours. It pays $4.75 for the first hour, $5 for the second hour, $5.25 for the next hour, and so on. How much will the student earn in total?

L3 - Arithmetic and Geometric Sequences
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Sequences Questions

What is the difference between a sequence and a series?

What is the difference between Arithmetic and Geometric?

Formulas for general terms of a sequence

Example 1: $-10, -4, 2, \ldots$

a) Determine whether the sequence is arithmetic or geometric.

b) Determine an equation for the sequence.

c) Find the value of t_{21}

Example 2: Insert two numbers between 8 and 32 so the four numbers form an arithmetic sequence.

Example 3: An arithmetic sequence is 8, 14, 20, 26, ... Which term has the value 92? Prove mathematically.

Example 4: $200, -100, 50, \ldots$

a) Is the sequence arithmetic or geometric?

b) Find an equation to represent the sequence.

c) Find t_{14}

Example 5: Complete the geometric sequence

$$\_\_\_\_, 160, \_\_\_\_, \_\_\_\_, \_\_\_\_, 10$$

Example 6: The 50th term of an arithmetic sequence is 238 and the 93rd term is 539. Find a general equation to represent the sequence.

Example 7: Determine the number of terms in the geometric sequence: 5, -10, 20, ... , -10 240

L4 - Arithmetic and Geometric Series
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DO IT NOW!

In an arithmetic sequence, $t_3 = 25$ and $t_9 = 43$. Determine the formula for the general term of this sequence.

Arithmetic

Sequence:

$$t_n = a + (n-1)d$$

Series:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{n}{2}(a + t_n)$$

Geometric

Sequence:

$$t_n = a \cdot r^{n-1}$$

Series:

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Example 1: In an amphitheater, seats are arranged in 50 semicircular rows facing a domed stage. The first row contains 23 seats, and each row contains 4 more seats than the previous row. How many seats are in the amphitheater?

> ***Note:*** *use formula for arithmetic series because difference between consecutive rows is a constant.*

Example 2: Determine the sum of -31 -35 -39-.....-403

Example 3: Determine the sum of the first 20 terms of the arithmetic series in which the 15th term is 107 and the terms decrease by 3.

Example 4: The 10th term of an arithmetic series is 34, and the sum of the first 20 terms is 710. Determine the 25th term.

Example 5: Determine the sum of the first seven terms of the geometric series in which $t_5 = 5$ and $t_8 = -40$.

Method 1:

Method 2:

Example 6: Calculate the sum of the geometric series, 960 + 480 + 240 +....+15

Method 1: write out full series

Method 2: Solve using logarithms

Figure out how many terms are in the series by solving for *n* in the formula:

Method 3: Solve using powers with the same base

Example 7: A tennis tournament has 128 entrants. A player is dropped from the competition after losing one match. Winning players go on to another match. What is the total number of matches that will be played in this tournament?

Note: The first term is 128/2 = 64 because 2 players participate in one match. The last term is 1 but we don't know what term number it is.

L5 - Recursive Procedures

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In earlier sections we used function notation to write an explicit formula to determine the value of any term in a sequence. Sometimes it is easier to calculate one term in a sequence using the previous terms.

Recursion Formula:

Recursive Functions

Functions that get new terms in the sequence by using earlier terms.

Example 1: Write the first four terms of the sequence.

a) $t_n = t_{n-1} - 2$ where $t_1 = 7$

b) $t_n = 2t_{n-1} + 4$ where $t_1 = 5$

You may also see questions asked in function notation.

Example 2: Find the first 4 terms.

$f(n)=2f(n\text{-}1) - 7$ where $f(1) = 2$

Example 3: Find the first seven terms of the sequence.

$t_n = t_{n-2} + t_{n-1}$ where $t_1 = 1$ and $t_2 = 1$

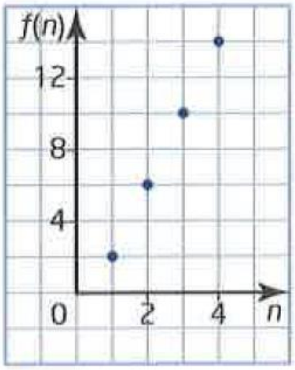
Example 4: Write a recursion formula for each sequence

a) -3, 6, -12, 24,....

Look for a pattern in the terms:

$t_1 = -3$
$t_2 = t_1 \times (-2)$
$t_3 = t_2 \times (-2)$
$t_4 = t_3 \times (-2)$

b)



c) 3, 5, 8, 12, …

L6 - Pascal's Triangle
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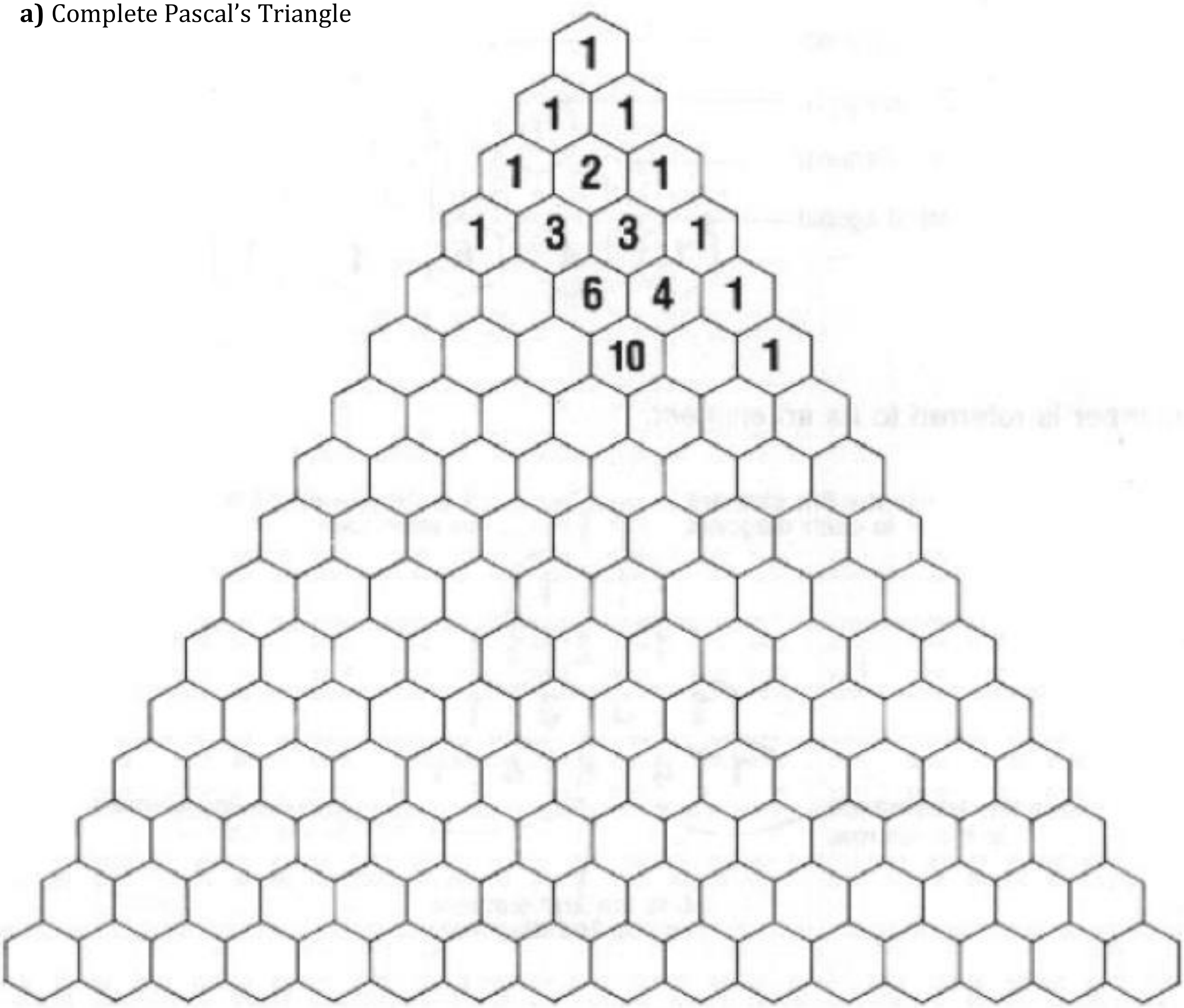
a) Complete Pascal's Triangle



b) What patterns do you notice in Pascal's Triangle?

c) Expand each of the following binomials.

$(a + b)^0 =$

$(a + b)^1 =$

$(a + b)^2 =$

$(a + b)^3 =$

$(a + b)^4 =$

Blaise Pascal (French Mathematician) discovered a pattern in the expansion of $(a+b)^n$.... which patterns do you notice?

Example 1: Expand each binomial using Pascal's Triangle

a) $(a+b)^6$

b) $(2x-3)^5$

c) $(2x+3y^2)^5$

d) $\left(\frac{y}{2}-y^2\right)^4$

Example 2: How many terms will there be if you expand $(x + 2y)^{20}$?

Example 3:

a) What is the second term in the expansion of $(x + 6)^7$

b) What is the 5th term in the expansion of $(3y - 4)^8$

Example 4:

a) What is the coefficient of x^3 in the expansion of $(x + 6)^6$

b) What is the coefficient of y^4x^2 in the expansion of $(y + 3x)^6$