

GRADE 11 MATH (mcr3u)

FINAL EXAM

Instructions: There is a 2 hour minimum, 3 hour maximum time limit for writing this exam.

Note: Number of Pages (including cover): 12

Total Marks: 123

Give exact answers where possible. Round approximate answers to 2 decimal places unless otherwise indicated.

Please check over your solutions!

Do your best! Do the questions in any order. Don't get stuck on one. Leave it and come back to it later.

Topic of Study	Points
Multiple Choice	16
Unit 1: Functions	24
Unit 2: Rational Expressions	8
Unit 3: Transformations	15
Unit 4: Exponential Functions	15
Unit 5/6: Trigonometry	29
Unit 7: Discrete Functions	16
Total	123

KTAC breakdown	Knowledge	Application	Thinking	Communication
% of exam	36%	31%	24%	12%

Section 1: Multiple Choice [16 marks]

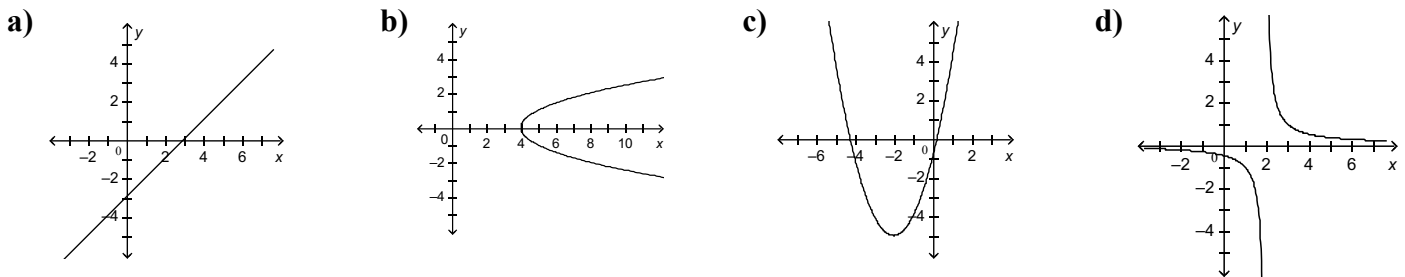
___ 1) For the function $f(x) = 3x^2 - 4x + 1$, what is the value of $f(-1)$?

- a) 0
- b) 2
- c) 8
- d) -6

___ 2) The range of the function $f(x) = -5x^2 + 4$ is

- a) $\{X \in \mathbb{R}\}$
- b) $\{Y \in \mathbb{R}\}$
- c) $\{Y \in \mathbb{R} | y \leq 4\}$
- d) $\{Y \in \mathbb{R} | y \geq -5\}$

___ 3) Which of the relations is NOT a function?



___ 4) The curve $y = 3x^2 - 4x + 8$ and the line $y = 3x + 5$ intersect at

- a) No point
- b) One point
- c) 2 points
- d) An infinite number of points

___ 5) For the function $f(x) = 2x^2 - 5x + 1$, if $f(a) = 4$, what is the value of a ?

- a) 4
- b) 3
- c) -3
- d) -4

___ 6) The restrictions on the expression $\frac{2x^2+5x+3}{4x^2-9}$ is (are)

- a) $x \neq 0$
- b) $x \neq \frac{3}{2}$
- c) $x \neq \frac{3}{2}, x \neq -\frac{3}{2}$
- d) $x \neq \frac{3}{2}, x \neq -\frac{3}{2}, x \neq -1$

___ 7) When the graph of $g(x) = 3f(2x - 2) - 1$ is graphed based on $f(x)$, the horizontal translation is

- a) 2 units to the right
- b) 2 units to the left
- c) 1 unit to the right
- d) 1 unit to the left

___ 8) A co-terminal angle for 240° is

- a) 60°
- b) -120°
- c) 120°
- d) Two of the above answers are correct

___ 9) The related acute (reference) angle for 210° is

- a) 30°
- b) 60°
- c) 570°
- d) -150°

___ 10) The graph of the cosine function is the same as the graph of a sine function translated

- a) 90° to the right
- b) 90° to the left
- c) 180° to the right
- d) 180° to the left

___ 11) The maximum value of $y = \sin x - 6$ is

- a) 1
- b) -5
- c) -6
- d) 6

___ 12) Determine how many x -intercepts the function $y = 3 \cos x + 5$ has between 0° and 360°

- a) 0
- b) 2
- c) 4
- d) 6

___ 13) Given the general term $t_n = 3n + 5$, which is the first four terms of the sequence?

- a) 3,5,7,9
- b) 1,3,5,7
- c) 8,11,14,17
- d) 5,8,11,14

___ 14) Which sequence is geometric?

- a) 1, 2, 3, 4, 5
- b) 2, 4, 6, 8
- c) 2, 4, 8, 16
- d) 1, 2, 3, 5, 8

___ 15) Which is the correct expression for the amount a \$1000 investment is worth after 3 years if interest is paid at 4%, compounded semi-annually?

- a) $\$1000(1.04)^3$
- b) $\$1000(1.02)^6$
- c) $\$1000(1.04)^6$
- d) $\$1000(1.08)^3$

___ 16) A colony of ants has an initial population of 200 and doubles every day. A function that can be used to model the ant population, p , after t days is

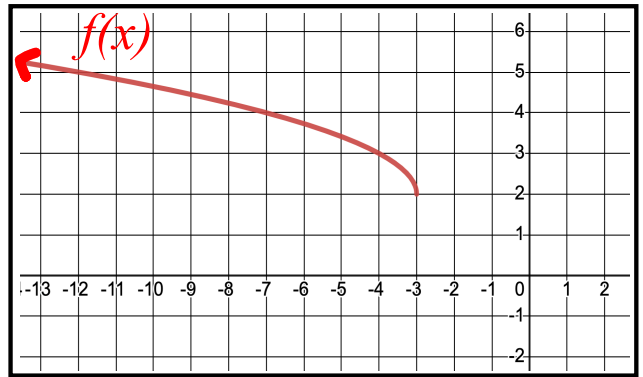
- a) $p(t) = 200(2)^t$
- b) $p(t) = 2(200)^t$
- c) $p(t) = 200\left(\frac{1}{2}\right)^t$
- d) $p(t) = \frac{1}{2}(200)^t$

Section 2: Functions [24 marks]

17) Using the graph of the relationship $f(x)$ shown the right...

[5 marks]

a) State the domain and range of the relationship



b) Is the relation a function? Justify your answer.

c) What is the value of $f(-7)$?

18) Solve the following quadratic equations using the most appropriate method (solve by factoring where possible). Give simplified EXACT answers only. [6 marks]

a) $2 = 2x^2 - 10x + 14$

b) $0 = 3x^2 - 10x + 5$

19) Fully simplify each expression. Show your work so it is evident you simplified and not just input on your calculator. [5 marks]

a) $\sqrt{150}$

b) $(2\sqrt{10})(5\sqrt{2})$

c) $3\sqrt{48} - 5\sqrt{27} + 2\sqrt{3} - \sqrt{32}$

20) Determine the vertex of the quadratic function $f(x) = -5x^2 + 30x - 37$ by completing the square. Verify your answer using partial factoring. Then state if the vertex is a max or min point. [5]

Completing the Square

Partial Factoring

Vertex: _____
Max or Min? _____

21) For what value of k will the line $y = -2x + k$ be tangent to the parabola $y = 3x^2 + 4x - 1$ [3]

Section 3: Rational Expressions [8 marks]

22) Fully simplify and state the restrictions of the following expressions [8 marks]

a) $\frac{9x}{x^2-5x+6} - \frac{4x}{x^2+x-12}$

b) $\frac{6x^2-11x-35}{2x^2-x-21} \div \frac{3x^2+2x-5}{x^2-2x-15}$

Simplified Expression:

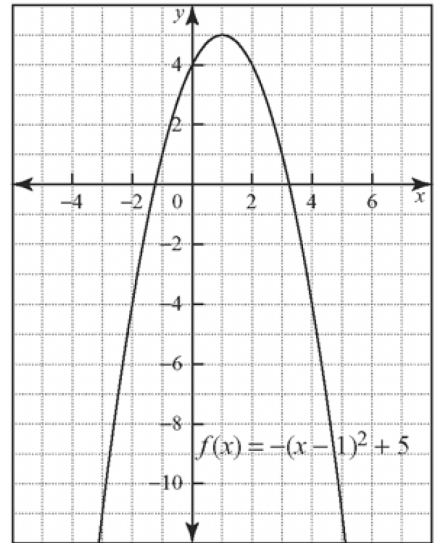
Restrictions:

Simplified Expression:

Restrictions:

Section 4: Transformations [15 marks]

23) Given the graph of $f(x) = -(x - 1)^2 + 5$, determine the equation of $f^{-1}(x)$ and graph it. Also state if $f^{-1}(x)$ is a function or not. [5 marks]



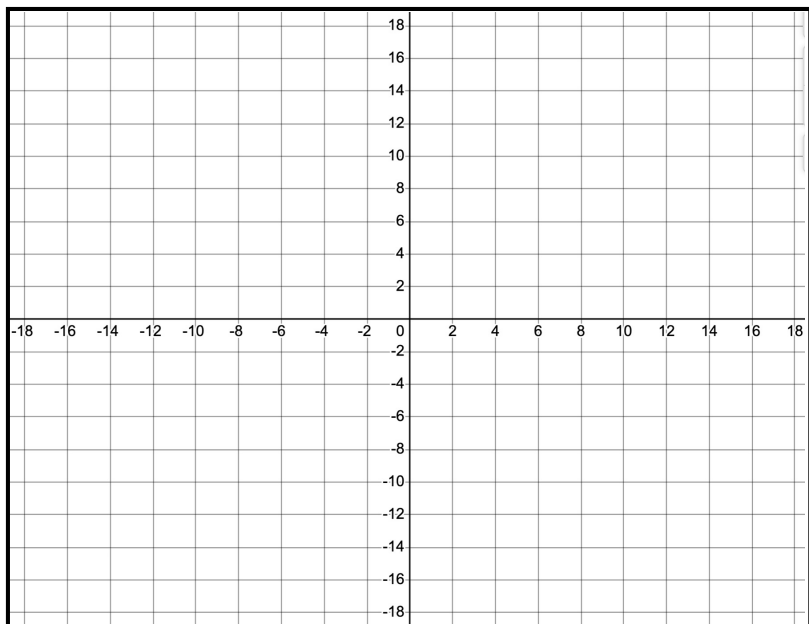
24) The base function $f(x) = \sqrt{x}$ is transformed by a vertical reflection in the x -axis, followed by a vertical stretch by a factor of 2, a horizontal compression by a factor of $\frac{1}{3}$, a horizontal translation of 2 units to the left, and a vertical translation of 1 unit down. [6 marks]

a) Determine the equation of the transformed function $g(x)$.

b) Create a table of values and sketch the graph of the base function and the transformed function.

$f(x)$	
x	y

$g(x)$	

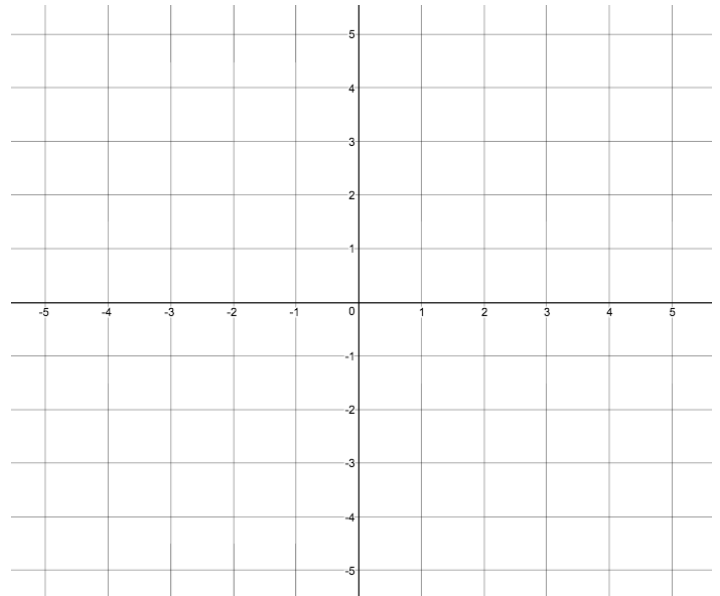


25) $f(x) = \frac{1}{x}$. Graph $g(x) = f(x + 1) + 2$. Also write the transformed equation. [4 marks]

- i) create a table of values of points for $f(x)$ and $g(x)$
- ii) graph the transformed function ONLY
- iii) write the equation of the transformed function

$f(x)$	
x	y

$g(x)$	



Transformed Function: $g(x) =$

Section 5: Exponential Functions

26) David bought a new car for \$30 000. Its resale value will depreciate by 22% of its current value every year. What will his car be worth in 5 years?

Answer:

27) The population of a pod of whales has doubled in size recently over a 6 year period. They have a current population of 21 individuals. What might the population be after 15 more years

Answer:

28) A radioactive isotope decays rapidly with a half-life of 5 minutes. For a 25 minute test, technicians need at least 7 grams of the isotope to be there at the end of the test. How much must they start with?

Answer:

29) How long does it take \$100 to grow to \$500 if the amount is invested at a rate of 13.5% per annum, compounded semi-annually?

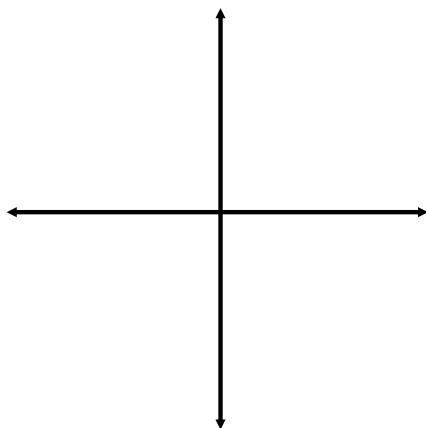
Answer:

Section 6: Trigonometry [29 marks]

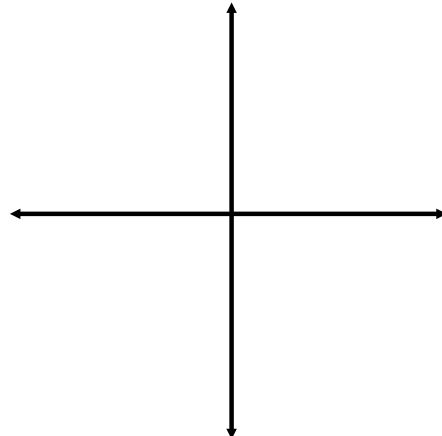
30) Draw both special triangles learned in this unit. Make sure to label all angles and side lengths. [2 marks]

31) Find the *exact* values of each of the following ratios. Show all work including a sketch of the terminal arm and its reference angle. [6 marks]

a) $\sin 225^\circ$



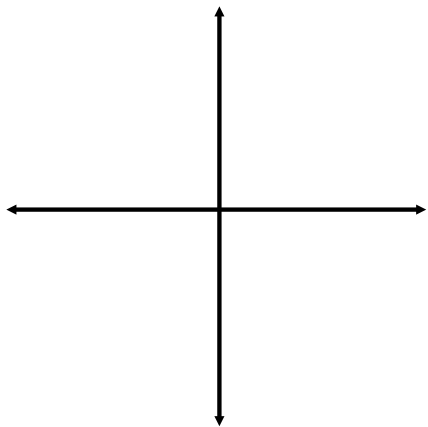
b) $\sec 300^\circ$



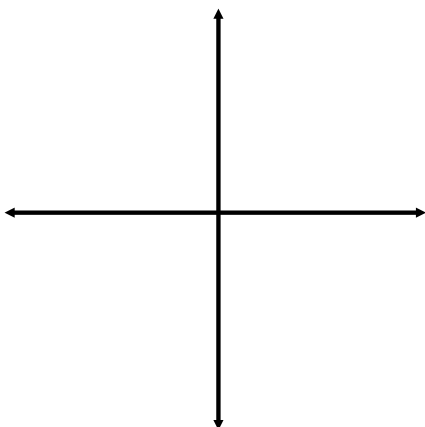
32) Solve the following equations for $0^\circ \leq \theta \leq 360^\circ$. Round answers to the nearest degree if necessary. Show all of your work.

a) $\sin \theta = \frac{3}{5}$

[6 marks]



b) $\sec \theta + 2 = 0$



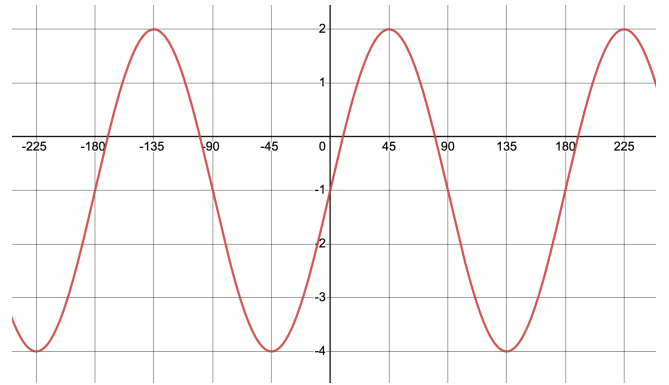
33) $\angle K$ is located in the third quadrant with a secant of -6 . Find exact expressions for the other five trigonometric ratios for $\angle K$.

[3 marks]

34) Prove the trig identity $\frac{\cos \theta}{\sec \theta} - \frac{\sin \theta}{\cot \theta} = \frac{\cot \theta \cos \theta - \tan \theta}{\csc \theta}$

[3 marks]

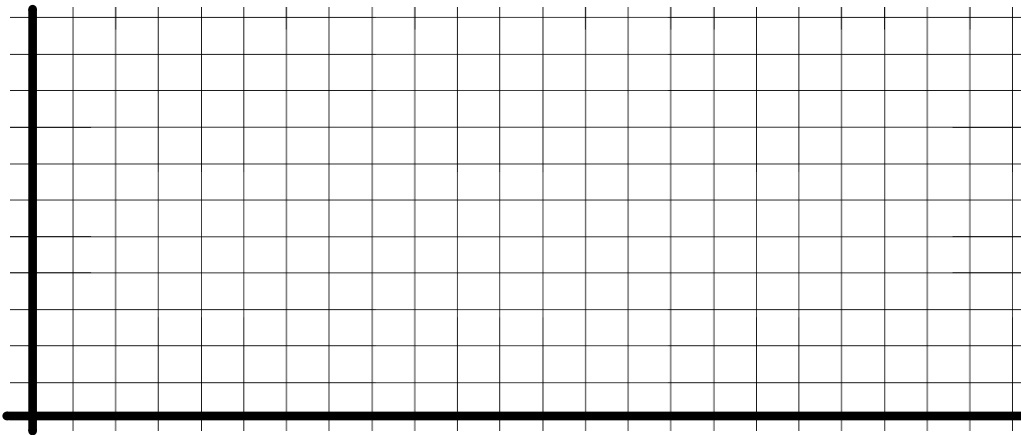
35) Determine the equation of both a sine and cosine function that models the function shown in the graph. [5 marks]



Equation 1: _____ Equation 2: _____

36) A Ferris wheel has a radius of 15 m and is 2 m above the ground at its lowest point. The rider enters the car from a platform located 20° around the rim before the car gets to its lowest point. It rotates counterclockwise.

a) Graph two complete rotations of the riders height vs. the angle of rotation. (adjust the x and y scales appropriately) [2 marks]



b) Model the rider's height above the ground versus the angle using a transformed cosine function. [2 marks]

37) For each of the following sequences...

[8 marks]

- i) state if it is arithmetic or geometric
- ii) write an explicit formula for the general term
- iii) calculate t_{10}
- iv) algebraically calculate the number of terms in the sequence

a) 16, 23, 30, ... , 583

b) $-36, -18, -9, \dots, \frac{-9}{128}$

38) A jogger runs 300 m in the first minute. The distance the jogger covers decreases by 20 m in each succeeding minute. What total distance does the jogger run during a 10 minute run? [2 marks]

39) Determine a and d and then write the formula for the n th term of the arithmetic sequence with $t_8 = 79$ and $t_{21} = 235$. [3 marks]

40) Use Pascal's triangle to expand the binomial $(2x - 3y)^4$

[3 marks]

Formula Sheet: Functions 11

Quadratics

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Transformations

$$g(x) = af[k(x - d)] + c$$

Exponential

$$y = a \cdot b^x \quad y = a(1 \pm r)^x \quad A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Discrete

Arithmetic

$$t_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_n = \frac{n}{2}(a + t_n)$$

Geometric

$$t_n = a(r)^{n-1}$$

$$S_n = \frac{a[(r)^n - 1]}{r - 1}$$

Trigonometry

Fundamental Trigonometric Identities		
Reciprocal Identities	Quotient Identities	Pythagorean Identities
$\csc \theta = \frac{1}{\sin \theta}$	$\frac{\sin \theta}{\cos \theta} = \tan \theta$	$\sin^2 \theta + \cos^2 \theta = 1$
$\sec \theta = \frac{1}{\cos \theta}$	$\frac{\cos \theta}{\sin \theta} = \cot \theta$	$\sin^2 \theta = 1 - \cos^2 \theta$
$\cot \theta = \frac{1}{\tan \theta}$		$\cos^2 \theta = 1 - \sin^2 \theta$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

$$\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$$