

3.3 Rational Exponents - Worksheet

MCR3U

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SOLUTIONS

1) Evaluate each cube root.

a) $\sqrt[3]{64}$

$$= 4$$

b) $(-1000)^{\frac{1}{3}}$

$$= -10$$

c) $\sqrt[3]{\frac{1}{8}}$

$$= \frac{1}{2}$$

d) $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

$$= \frac{2}{3}$$

2) Evaluate each root.

a) $81^{\frac{1}{4}}$

$$= 3$$

b) $\sqrt[4]{\frac{16}{625}}$

$$= \frac{2}{5}$$

c) $64^{\frac{1}{6}}$

$$= 2$$

d) $\sqrt[5]{-100\,000}$

$$= -10$$

3) Evaluate.

a) $8^{\frac{2}{3}}$

$$= \left(8^{\frac{1}{3}}\right)^2$$

$$= 4$$

b) $32^{\frac{4}{5}}$

$$= \left(32^{\frac{1}{5}}\right)^4$$

$$= 16$$

c) $(-64)^{\frac{5}{3}}$

$$= \left[(-64)^{\frac{1}{3}}\right]^5$$

$$= -1024$$

d) $\left(\frac{1}{10\,000}\right)^{\frac{3}{4}}$

$$= \frac{1}{1000}$$

4) Evaluate.

a) $16^{-\frac{1}{4}}$

$$= \frac{1}{16^{\frac{1}{4}}}$$

$$= \frac{1}{2}$$

b) $25^{-\frac{3}{2}}$

$$= \frac{1}{25^{\frac{3}{2}}}$$

$$= \frac{1}{125}$$

c) $\left(\frac{1}{8}\right)^{-\frac{7}{3}}$

$$= \left(\frac{8}{1}\right)^{\frac{7}{3}}$$

$$= 128$$

d) $\left(-\frac{1}{32}\right)^{-\frac{2}{5}}$

$$= \left(-\frac{32}{1}\right)^{\frac{2}{5}}$$

$$= 4$$

e) $\left(\frac{10\,000}{81}\right)^{-\frac{3}{4}}$

$$= \left(\frac{81}{10\,000}\right)^{\frac{3}{4}}$$

$$= \frac{27}{1000}$$

f) $\left(-\frac{8}{27}\right)^{-\frac{2}{3}}$

$$= \left(-\frac{27}{8}\right)^{\frac{2}{3}}$$

$$= \frac{9}{4}$$

5) Simplify. Express your answers using only positive exponents.

$$\text{a) } x^{\frac{1}{4}} \cdot x^{\frac{1}{4}}$$

$$= x^{\frac{1}{2}}$$

$$\text{b) } (m^{\frac{1}{3}})(m^{\frac{3}{4}})$$

$$= m^{\frac{4}{12}} \cdot m^{\frac{9}{12}}$$

$$= m^{\frac{13}{12}}$$

$$\text{c) } \frac{w^{\frac{1}{2}}}{w^{\frac{1}{3}}} = \frac{w^{\frac{3}{6}}}{w^{\frac{2}{6}}}$$

$$= w^{\frac{1}{6}}$$

$$\text{d) } \frac{ab^2}{a^{\frac{1}{2}}b^{\frac{1}{3}}} = \frac{a^{\frac{2}{2}}b^{\frac{6}{3}}}{a^{\frac{1}{2}}b^{\frac{1}{3}}}$$

$$= a^{\frac{3}{2}}b^{\frac{5}{3}}$$

$$\text{e) } (y^{\frac{1}{2}})^{\frac{2}{3}}$$

$$= y^{\frac{2}{6}}$$

$$= y^{\frac{1}{3}}$$

$$\text{f) } (u^{\frac{3}{4}}v^{\frac{1}{2}})^{\frac{2}{9}} = u^{\frac{6}{36}}v^{\frac{2}{18}}$$

$$= u^{\frac{1}{6}}v^{\frac{1}{9}}$$

6) Simplify. Express your answers using only positive exponents.

$$\text{a) } k^{\frac{3}{4}} \div k^{-\frac{1}{4}}$$

$$= k^{\frac{4}{4}}$$

$$= k$$

$$\text{b) } \frac{p^{\frac{2}{3}}}{p^{\frac{5}{6}}} = \frac{p^{\frac{4}{6}}}{p^{\frac{5}{6}}}$$

$$= p^{-\frac{1}{6}}$$

$$= \frac{1}{p^{\frac{1}{6}}}$$

$$\text{c) } (y^{\frac{2}{3}})^{-3}$$

$$= y^{-\frac{6}{3}}$$

$$= y^{-2}$$

$$= \frac{1}{y^2}$$

$$\text{d) } (w^{-\frac{8}{9}})^{\frac{3}{4}}$$

$$= w^{\frac{24}{36}}$$

$$= w^{\frac{2}{3}}$$

$$\text{e) } (8x)^{\frac{2}{3}}(27x)^{-\frac{1}{3}}$$

$$= \frac{4x^{\frac{2}{3}}}{(27x)^{\frac{1}{3}}}$$

$$= \frac{4x^{\frac{2}{3}}}{3x^{\frac{1}{3}}}$$

$$= \frac{4x^{\frac{1}{3}}}{3}$$

$$\text{f) } 5(7y^{-\frac{2}{3}})^{-2}$$

$$= 5(7^{-2})(y^{\frac{4}{3}})$$

$$= \frac{5y^{\frac{4}{3}}}{49}$$

7) The surface area, S, of a sphere can be expressed in terms of its volume, V, using the formula

$S(V) = (4\pi)^{\frac{1}{3}}(3V)^{\frac{2}{3}}$. A beach ball has a volume of 24 000 cm³. Find its surface area, to the nearest hundred square centimeters.

$$S(24000) = (4\pi)^{\frac{1}{3}} [3(24000)]^{\frac{2}{3}}$$

$$= (4\pi)^{\frac{1}{3}} (72000)^{\frac{2}{3}}$$

$$= 4023.7 \text{ cm}^2$$

$$\approx 4000 \text{ cm}^2$$

Answers

1. a) 4 b) -10 c) $\frac{1}{2}$ d) $\frac{2}{3}$
2. a) 3 b) $\frac{2}{5}$ c) 2 d) -10
3. a) 4 b) 16 c) -1024 d) $\frac{1}{1000}$
4. a) $\frac{1}{2}$ b) $\frac{1}{125}$ c) 128
d) 4 e) $\frac{27}{1000}$ f) $\frac{9}{4}$
5. a) $x^{\frac{1}{2}}$ b) $m^{\frac{13}{12}}$ c) $w^{\frac{1}{6}}$
d) $a^{\frac{1}{2}}b^{\frac{5}{3}}$ e) $y^{\frac{1}{3}}$ f) $u^{\frac{1}{8}}v^{\frac{1}{9}}$
6. a) k b) $\frac{1}{p^{\frac{3}{2}}}$ c) $\frac{1}{y^{\frac{1}{2}}}$
d) $w^{\frac{2}{3}}$ e) $\frac{4}{3}x^{\frac{1}{3}}$ f) $\frac{5}{49}y^{\frac{4}{3}}$
7. 4000 cm²

