1.3 Max or Min of a Quadratic Function – Worksheet

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SOLUTIONS

1) Determine the vertex for each quadratic function by completing the square. State if the vertex is a maximum or a minimum.

a) $f(x) = x^2 + 14x - 14$

$= (x^2 + 14x) - 14$

$= (x^2 + 14x + 49 - 49) - 14$

$= (x^2 + 14x + 49) - 49 - 14$

$= (x+7)^2 - 63$

vertex is $(-7, -63)$ and is a minimum.

b) $f(x) = x^2 - 6x + 17$

$= (x^2 - 6x) + 17$

$= (x^2 - 6x + 9 - 9) + 17$

$= (x^2 - 6x + 9) - 9 + 17$

$= (x-3)^2 + 8$

vertex is $(3, 8)$ and is a minimum.

c) $f(x) = x^2 + 7x + 11$

$= \left(x^2 + 7x + \frac{49}{4} - \frac{49}{4}\right) + 11$

$= \left(x^2 + 7x + \frac{49}{4}\right) - \frac{49}{4} + \frac{44}{4}$

$= \left(x + \frac{7}{2}\right)^2 - \frac{5}{4}$

vertex is $\left(-\frac{7}{2}, -\frac{5}{4}\right)$ and is a minimum

d) $f(x) = 2x^2 + 12x + 16$

$= (2x^2 + 12x) + 16$

$= 2(x^2 + 6x + 9 - 9) + 16$

$= 2(x^2 + 6x + 9) - 18 + 16$

$= 2(x+3)^2 - 2$

vertex is $(-3, -2)$ and is a minimum.

e) $f(x) = -3x^2 + 6x + 1$

$= (-3x^2 + 6x) + 1$

$= -3(x^2 - 2x + 1 - 1) + 1$

$= -3(x^2 - 2x + 1) + 3 + 1$

$= -3(x-1)^2 + 4$

Vertex is $(1, 4)$ and is a maximum.

f) $f(x) = -\frac{1}{2}x^2 - x + \frac{3}{2}$

$f(x) = \left(-\frac{1}{2}x^2 - 1x\right) + \frac{3}{2}$

$= -\frac{1}{2}(x^2 + 2x + 1 - 1) + \frac{3}{2}$

$= -\frac{1}{2}(x^2 + 2x + 1) + \frac{1}{2} + \frac{3}{2}$

$= -\frac{1}{2}(x+1)^2 + 2$

Vertex is $(-1, 2)$ and is a max

2) Use partial factoring to determine the vertex of each function. State if the vertex is a max or min.

a) $f(x) = 3x^2 - 6x + 11$

$11 = 3x^2 - 6x + 11$

$0 = 3x^2 - 6x$

$0 = 3x(x-2)$

$3x = 0 \qquad x - 2 = 0$

$x = 0 \qquad x = 2$

$x\text{-vertex} = \frac{0+2}{2}$

$= 1$

$y\text{-vertex} = 3(1)^2 - 6(1) + 11$

$= 8$

vertex is $(1, 8)$ and is a min.

b) $f(x) = -2x^2 + 8x - 3$

$-3 = -2x^2 + 8x - 3$

$0 = -2x^2 + 8x$

$0 = -2x(x-4)$

$-2x = 0 \qquad x - 4 = 0$

$x = 0 \qquad x = 4$

$x\text{-vertex} = \frac{0+4}{2}$

$= 2$

$y\text{-vertex} = -2(2)^2 + 8(2) - 3$

$= 5$

vertex is $(2, 5)$ and is a MAX.

c) $h(x) = -x^2 + 2x + 4$

$4 = -x^2 + 2x + 4$

$0 = -x^2 + 2x$

$0 = -1x(x-2)$

$-1x = 0 \quad x - 2 = 0$

$x = 0 \quad x = 2$

$x\text{-vertex} = \frac{0+2}{2}$

$= 1$

$y\text{-vertex} = -(1)^2 + 2(1) + 4$

$= 5$

vertex is $(1,5)$ and is a MAX

d) $f(x) = 2x^2 + 12x + 17$

$17 = 2x^2 + 12x + 17$

$0 = 2x^2 + 12x$

$0 = 2x(x+6)$

$2x = 0 \quad x + 6 = 0$

$x = 0 \quad x = -6$

$x\text{-vertex} = \frac{0+(-6)}{2}$

$= -3$

$y\text{-vertex} = 2(-3)^2 + 12(-3) + 17$

$= -1$

vertex is $(-3,-1)$ and is a MIN.

e) $f(x) = 4x^2 + 64x + 156$

$156 = 4x^2 + 64x + 156$

$0 = 4x^2 + 64x$

$0 = 4x(x+16)$

$4x = 0 \quad x + 16 = 0$

$x = 0 \quad x = -16$

$x\text{-vertex} = \frac{0+(-16)}{2}$

$= -8$

$y\text{-vertex} = 4(-8)^2 + 64(-8) + 156$

$= -100$

Vertex is $(-8,-100)$ and is a MIN.

f) $f(x) = \frac{1}{2}x^2 - 3x + 8$

$8 = \frac{1}{2}x^2 - 3x + 8$

$0 = \frac{1}{2}x^2 - 3x$

$0 = \frac{1}{2}x(x-6)$

$\frac{1}{2}x = 0 \quad x - 6 = 0$

$x = 0 \quad x = 6$

$x\text{-vertex} = \frac{0+6}{2}$

$= 3$

$y\text{-vertex} = \frac{1}{2}(3)^2 - 3(3) + 8$

$= \frac{9}{2} - \frac{18}{2} + \frac{16}{2}$

$= \frac{7}{2}$

vertex is $(3, 7/2) \rightarrow$ MIN.

3) An electronics store sells an average of 60 entertainment systems per month at an average of $800 more than the cost price. For every $20 increase in the selling price, the store sells one fewer system. What amount over the cost price will maximize revenue?

Revenue = (cost)(# sold)

$0 = (800 + 20x)(60 - x)$

$800 + 20x = 0 \quad 60 - x = 0$

$20x = -800 \quad x = 60$

$x = -40$

$x\text{-vertex} = \frac{-40+60}{2}$

$= 10$

$y\text{-vertex} = [800 + 20(10)][60 - 10]$

$= (1000)(50)$

$= 50\,000$

vertex is $(10, 50\,000)$

$\uparrow$ # of increases $\quad \uparrow$ revenue

$\boxed{\text{cost} = 800 + 20(10) = \$1000}$

4) Last year, a banquet hall charged $30 per person, and 60 people attended the hockey banquet dinner. This year, the hall's manager has said that for every 10 extra people that attend the banquet, they will decrease the price by $1.50 per person. What size group would maximize the profit for the hall this year and what would the maximum profit be?

Profit = (cost)(# of people)

$0 = (30 - 1.50x)(60 + 10x)$

$30 - 1.5x = 0 \quad 60 + 10x = 0$

$30 = 1.5x \quad x = -6$

$x = 20$

$x\text{-vertex} = \frac{20+(-6)}{2}$

$= 7$

$y\text{-vertex} = [30 - 1.5(7)][60 + 10(7)]$

$= \underbrace{(19.5)}_{\text{cost}}\underbrace{(130)}_{\text{people}}$

$= 2535$

vertex is $(7, 2535)$

$\uparrow$ # of decreases $\quad \uparrow$ profit

A group of 130 people would give a max profit of $2535

5) The path of a rocket is given by the function, $h(t) = -3t^2 + 30t + 73$, where 'h' is the height in meters and 't' is the time in seconds.

) What is the maximum height of the rocket?

$h(t) = -3(t^2 - 10t) + 73$

$= -3(t^2 - 10t + 25 - 25) + 73$

$= -3(t^2 - 10t + 25) + 75 + 73$

$= -3(t-5)^2 + 148$

vertex is (5, 148)

time height

Max height is 148 m.

b) At what time does the rocket reach its maximum height?

5 seconds.

Answers

1) a) (-7,-63) min **b)** (3,8) min **c)** $\left(\frac{-7}{2}, \frac{-5}{4}\right)$ min **d)** (-3, -2) min **e)** (1, 4) max **f)** (-1, 2) max

2) a) (1, 8) min **b)** (2, 5) max **c)** (1, 5) max **d)** (-3, -1) min **e)** (-8, -100) min **f)** (3, $\frac{7}{2}$) min

3) \$1000

) A group of 130 would give a max profit of \$2535

5) a) 148 m **b)** 5 seconds