1.5 Solving Quadratic Equations – Part 2: Solve Using the Q.F. - Worksheet

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SOLUTIONS

1) Use the discriminant to determine the number of roots for each quadratic equation.

a) $x^2 - 10x + 25 = 0$

$b^2 - 4ac = (-10)^2 - 4(1)(25)$

$= 0$

∴ 1 root

b) $3x^2 + 4x + \frac{4}{3} = 0$

$b^2 - 4ac = (4)^2 - 4(3)(\frac{4}{3})$

$= 0$

∴ 1 root

c) $2x^2 - 8x + 9 = 0$

$b^2 - 4ac = (-8)^2 - 4(2)(9)$

$= -8$

∴ No Roots

d) $-2x^2 + 0.75x + 5 = 0$

$b^2 - 4ac = (0.75)^2 - 4(-2)(5)$

$= 40.5625$

∴ 2 Roots

2) Solve each quadratic using the quadratic formula. Give exact answers.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a) $x^2 + 4x - 3 = 0$

$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-3)}}{2(1)}$

$x = \frac{-4 \pm \sqrt{28}}{2}$

$x = \frac{-4 \pm 2\sqrt{7}}{2}$

$x = \frac{2(-2 \pm \sqrt{7})}{2}$

$x_1 = -2 + \sqrt{7}$

and

$x_2 = -2 - \sqrt{7}$

b) $-x^2 + 12 = 9x$

$-x^2 - 9x + 12 = 0$

$-1(x^2 + 9x - 12) = 0$

$x^2 + 9x - 12 = 0$

$x = \frac{-9 \pm \sqrt{(9)^2 - 4(1)(-12)}}{2(1)}$

$x = \frac{-9 \pm \sqrt{129}}{2}$

$x_1 = \frac{-9 + \sqrt{129}}{2}$ or $x_2 = \frac{-9 - \sqrt{129}}{2}$

c) $x^2 = -5x + 2$

$x^2 + 5x - 2 = 0$

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-2)}}{2(1)}$

$x = \frac{-5 \pm \sqrt{33}}{2}$

$x_1 = \frac{-5 + \sqrt{33}}{2}$ $x_2 = \frac{-5 - \sqrt{33}}{2}$

d) $x^2 - 3x + 1 = 6$

$x^2 - 3x - 5 = 0$

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-5)}}{2(1)}$

$x = \frac{3 \pm \sqrt{29}}{2}$

$x_1 = \frac{3 + \sqrt{29}}{2}$ $x_2 = \frac{3 - \sqrt{29}}{2}$

e) $x^2 + 6x + 9 = 0$

$x = \dfrac{-6 \pm \sqrt{(6)^2 - 4(1)(9)}}{2(1)}$

$x = \dfrac{-6 \pm \sqrt{0}}{2}$

$\boxed{x = -3}$

f) $4x^2 - 6x - 1 = 0$

$x = \dfrac{6 \pm \sqrt{(-6)^2 - 4(4)(-1)}}{2(4)}$ $\qquad \rightarrow x = \dfrac{2(3 \pm \sqrt{13})}{8 \cdot 4}$

$x = \dfrac{6 \pm \sqrt{52}}{8}$ $\qquad\qquad x = \dfrac{3 \pm \sqrt{13}}{4}$

$x = \dfrac{6 \pm 2\sqrt{13}}{8}$ $\qquad \boxed{x_1 = \dfrac{3 + \sqrt{13}}{4} \qquad x_2 = \dfrac{3 - \sqrt{13}}{4}}$

g) $5x^2 - 3x - 1 = 0$

$x = \dfrac{3 \pm \sqrt{(-3)^2 - 4(5)(-1)}}{2(5)}$

$x = \dfrac{3 \pm \sqrt{29}}{10}$

$\boxed{x_1 = \dfrac{3 + \sqrt{29}}{10} \qquad x_2 = \dfrac{3 - \sqrt{29}}{10}}$

h) $-x^2 + 7x - 18 = 0$

$x = \dfrac{-7 \pm \sqrt{(7)^2 - 4(-1)(-18)}}{2(-1)}$

$x = \dfrac{-7 \pm \sqrt{-23}}{-2}$

$\boxed{\therefore \text{No Roots}}$

i) $4x^2 - 25 = 0$

$4x^2 + 0x - 25 = 0$

$x = \dfrac{0 \pm \sqrt{(0)^2 - 4(4)(-25)}}{2(4)}$

$x = \dfrac{\pm 20}{8}$

$x = \dfrac{5}{2} \qquad x = \dfrac{-5}{2}$

j) $3x^2 - 7x - 4 = x^2 - 4x$

$2x^2 - 3x - 4 = 0$

$x = \dfrac{3 \pm \sqrt{(-3)^2 - 4(2)(-4)}}{2(2)}$

$x = \dfrac{3 \pm \sqrt{41}}{4}$

$\boxed{x_1 = \dfrac{3 + \sqrt{41}}{4} \qquad x_2 = \dfrac{3 - \sqrt{41}}{4}}$

k) $8x^2 + 4x - 5 = 0$

$x = \dfrac{-4 \pm \sqrt{(4)^2 - 4(8)(-5)}}{2(8)}$ $\qquad x = \dfrac{4(-1 \pm \sqrt{11})}{16 \cdot 4}$

$x = \dfrac{-4 \pm \sqrt{176}}{16}$

$x = \dfrac{-4 \pm 4\sqrt{11}}{16}$ $\qquad \boxed{x_1 = \dfrac{-1 + \sqrt{11}}{4}}$

$\boxed{x_2 = \dfrac{-1 - \sqrt{11}}{4}}$

l) $4x^2 - 18x = 0$

$x = \dfrac{18 \pm \sqrt{(-18)^2 - 4(4)(0)}}{2(4)}$

$x = \dfrac{18 \pm 18}{8}$

$x = \dfrac{18 + 18}{8} \qquad x = \dfrac{18 - 18}{8}$

$\boxed{x_1 = \dfrac{9}{2} \qquad x_2 = 0}$

3) Solve each quadratic equation using any method

1) $3x^2 - 12x = 0$

$3x(x-4) = 0$

$3x = 6 \qquad x-4 = 0$

$\boxed{x_1 = 0} \qquad \boxed{x_2 = 4}$

b) $2x^2 + 4x - 6 = 0$

$5:4$
$p:-12$ $\qquad$ $\boxed{6 \text{ and } -2}$

$2x^2 + 6x - 2x - 6 = 0$

$(2x^2 + 6x) + (-2x-6) = 0$

$2x(x+3) - 2(x+3) = 0$

$(x+3)(2x-2) = 0$

$\boxed{x_1 = -3} \quad \boxed{x_2 = 1}$

c) $3x^2 + 5x - 2 = 0$ $\quad$ $5:5$ $\quad$ $\boxed{6 \text{ and } -1}$
$\qquad\qquad\qquad\qquad p:-6$

$3x^2 - 1x + 6x - 2 = 0$

$x(3x-1) + 2(3x-1) = 0$

$(3x-1)(x+2) = 0$

$\boxed{x_1 = \frac{1}{3} \quad x_2 = -2}$

d) $4x^2 - 11x - 8 = 0$

$x = \dfrac{11 \pm \sqrt{(-11)^2 - 4(4)(-8)}}{2(4)}$

$x = \dfrac{11 \pm \sqrt{249}}{8}$

$\boxed{x_1 = \dfrac{11+\sqrt{249}}{8} \qquad x_2 = \dfrac{11-\sqrt{249}}{8}}$

4) Three lengths of pipe measuring 24 cm, 31 cm, and 38 cm will be used to create a right triangle. The same length of pipe will be cut off each of the three pipes to allow a right triangle to be created. What is that length?

$(24-x)^2 + (31-x)^2 = (38-x)^2$

$576 - 48x + x^2 + 961 - 62x + x^2 = 1444 - 76x + x^2$

$x^2 - 34x + 93 = 0$

$(x-3)(x-31) = 0$

$\boxed{x=3} \quad x = 31$
$\qquad\qquad\uparrow$
$\qquad$ reject because can't
$\qquad$ cut 31 cm from 24 cm.

$\boxed{3 \text{ cm}}$

Answers

1) a) one **b)** one **c)** none **d)** two

2) a) $-2+\sqrt{7}, -2-\sqrt{7}$ **b)** $\dfrac{-9-\sqrt{129}}{2}, \dfrac{-9+\sqrt{129}}{2}$ **c)** $\dfrac{-5+\sqrt{33}}{2}, \dfrac{-5-\sqrt{33}}{2}$ **d)** $\dfrac{3+\sqrt{29}}{2}, \dfrac{3-\sqrt{29}}{2}$ **e)** -3 **f)** $\dfrac{3+\sqrt{13}}{4}, \dfrac{3-\sqrt{13}}{4}$

g) $\dfrac{3+\sqrt{29}}{10}, \dfrac{3-\sqrt{29}}{10}$ **h)** no roots **i)** $\dfrac{5}{2}, -\dfrac{5}{2}$ **j)** $\dfrac{3+\sqrt{41}}{4}, \dfrac{3-\sqrt{41}}{4}$ **k)** $\dfrac{-1+\sqrt{11}}{4}, \dfrac{-1-\sqrt{11}}{4}$ **l)** $\dfrac{9}{2}, 0$

3) a) $x=0$ and $x=4$ **b)** $x=1$ and $x=-3$ **c)** $x=\dfrac{1}{3}$ and $x=-2$ **d)** $x=\dfrac{11\pm\sqrt{249}}{8}$

4) 3 cm