

1.7 Solve Linear-Quadratic Systems - Worksheet

MCR3U

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SOLUTIONS

1) Determine if each quadratic function will intersect once, twice, or not at all with the given linear function.

a) $y = 2x^2 - 2x + 1$ and $y = 3x - 5$

$$3x - 5 = 2x^2 - 2x + 1$$

$$0 = 2x^2 - 5x + 6$$

$$b^2 - 4ac = (-5)^2 - 4(2)(6)$$

$$= -23$$

∴ They DON'T intersect

b) $y = -x^2 + 3x - 5$ and $y = -x - 1$

$$-x - 1 = -x^2 + 3x - 5$$

$$0 = -x^2 + 4x - 4$$

$$b^2 - 4ac = (4)^2 - 4(-1)(-4)$$

$$= 0$$

∴ 1 PoI

c) $y = \frac{1}{2}x^2 + 4x - 2$ and $y = x + 3$

$$x + 3 = \frac{1}{2}x^2 + 4x - 2$$

$$0 = \frac{1}{2}x^2 + 3x - 5$$

$$b^2 - 4ac = (3)^2 - 4\left(\frac{1}{2}\right)(-5)$$

$$= 19$$

∴ 2 PoI's.

d) $y = -\frac{2}{3}x^2 + x + 3$ and $y = x$

$$x = -\frac{2}{3}x^2 + x + 3$$

$$0 = -\frac{2}{3}x^2 + 3$$

$$b^2 - 4ac = (0)^2 - 4\left(-\frac{2}{3}\right)(3)$$

$$= 8$$

∴ 2 PoI's.

2) Determine the coordinates of the point(s) of intersection of each linear-quadratic system.

a) $y = x^2 - 7x + 15$ and $y = 2x - 5$

$$2x - 5 = x^2 - 7x + 15$$

$$0 = x^2 - 9x + 20$$

$$0 = (x - 4)(x - 5)$$

$$x - 4 = 0 \quad x - 5 = 0$$

$$x = 4 \quad x = 5$$

PoI #1

$$f(x) = 2x - 5$$

$$f(4) = 2(4) - 5$$

$$f(4) = 3$$

$$(4, 3)$$

PoI #2

$$f(x) = 2x - 5$$

$$f(5) = 2(5) - 5$$

$$f(5) = 5$$

$$(5, 5)$$

b) $y = 3x^2 - 16x + 37$ and $y = 8x + 1$

$$8x + 1 = 3x^2 - 16x + 37$$

$$0 = 3x^2 - 24x + 36$$

$$0 = 3(x^2 - 8x + 12)$$

$$0 = x^2 - 8x + 12$$

$$0 = (x-2)(x-6)$$

$$x-2=0 \quad x-6=0$$

$$x=2 \quad x=6$$

POI #1

$$f(x) = 8x + 1$$

$$f(2) = 8(2) + 1$$

$$f(2) = 17$$

$$(2, 17)$$

POI #2

$$f(x) = 8x + 1$$

$$f(6) = 8(6) + 1$$

$$f(6) = 49$$

$$(6, 49)$$

c) $y = \frac{1}{2}x^2 - 2x - 3$ and $y = -3x + 1$

$$-3x + 1 = \frac{1}{2}x^2 - 2x - 3$$

$$0 = \frac{1}{2}x^2 + 1x - 4$$

$$0 = \frac{1}{2}(x^2 + 2x - 8)$$

$$0 = x^2 + 2x - 8$$

$$0 = (x+4)(x-2)$$

$$x+4=0 \quad x-2=0$$

$$x=-4 \quad x=2$$

POI #1

$$f(x) = -3x + 1$$

$$f(-4) = -3(-4) + 1$$

$$f(-4) = 13$$

$$(-4, 13)$$

POI #2

$$f(x) = -3x + 1$$

$$f(2) = -3(2) + 1$$

$$f(2) = -5$$

$$(2, -5)$$

3) Determine the value of the y-intercept of a line with the given slope that is a tangent line to the given curve.

a) $y = -2x^2 + 5x + 4$ and a line with a slope of 1 $y = 1x + b$

$$1x + b = -2x^2 + 5x + 4$$

$$0 = -2x^2 + 4x + (4-b)$$

*Set $b^2 - 4ac = 0$ *

$$(4)^2 - 4(-2)(4-b) = 0$$

$$16 - 4(-8 + 2b) = 0$$

$$16 + 32 - 8b = 0$$

$$-8b = -48$$

$$b = 6$$

b) $y = -x^2 - 5x - 5$ and a line with a slope of -3 $y = -3x + b$

$$-3x + b = -x^2 - 5x - 5$$

$$0 = -1x^2 - 2x + (-5 - b)$$

* Set $b^2 - 4ac = 0$ *

$$(-2)^2 - 4(-1)(-5 - b) = 0$$

$$4 - 4(5 + b) = 0$$

$$4 - 20 - 4b = 0$$

$$-4b = 16$$

$$b = -4$$

4) The path of an underground stream is given by the function $y = 4x^2 + 17x - 32$. Two new houses need wells to be dug. On the area plan, these houses lie on a line defined by the equation $y = -15x + 100$. Determine the coordinates where the two new wells should be dug.

$$-15x + 100 = 4x^2 + 17x - 32$$

$$0 = 4x^2 + 32x - 132$$

$$0 = 4(x^2 + 8x - 33)$$

$$0 = x^2 + 8x - 33$$

$$0 = (x + 11)(x - 3)$$

$$x + 11 = 0 \quad x - 3 = 0$$

$$x = -11 \quad x = 3$$

POI #1

$$f(x) = -15x + 100$$

$$f(-11) = -15(-11) + 100$$

$$f(-11) = 265$$

$$(-11, 265)$$

POI #2

$$f(3) = -15(3) + 100$$

$$f(3) = 55$$

$$(3, 55)$$

Answers

1) a) do not intersect b) once c) twice d) twice

2) a) (4, 3), (5, 5) b) (2, 17), (6, 49) c) (-4, 13), (2, -5)

3) a) 6 b) -4

4) (-11, 265), (3, 55)

