L3 – Compound Interest / Initial Amount

MCR3U

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General Properties of Exponential Decay (or growth)

Equation: $y = a(b)^x$

$a =$ initial amount

$b =$ growth ($b > 1$) or decay ($0 < b < 1$) factor

$y =$ future amount

$x =$ # of growth/decay periods

To calculate x, use the equation:

$$x = \frac{\text{total time}}{\text{time of 1 growth/decay period}}$$

Finding Initial Amount

Example 1: You are going to ship some U-239 which has a half-life of 2 years. There must be 500g upon arrival. If shipping will take 4 months, how much should you package initially?

24 months

$y = 500$

$a = ?$

$b = \frac{1}{2}$

$x = \frac{4}{24} = \frac{1}{6}$

$y = a(b)^x$

$\frac{500}{\left(\frac{1}{2}\right)^{1/6}} = \frac{a\left(\frac{1}{2}\right)^{1/6}}{\left(\frac{1}{2}\right)^{1/6}}$

$a \approx 561.23\ g$

Example 2: We (as a class) have been hired by a surgeon to grow a skin graft. It takes 3 days for the amount of skin to double. If we need 2kg of skin in one week, how much should we start with?

$y = 2$

$a = ?$

$b = 2$

$x = \frac{t}{3} = \frac{7}{3}$

$y = a(b)^x$

$\frac{2^{3/3}}{2^{7/3}} = \frac{a(2)^{7/3}}{(2)^{7/3}}$

$\frac{1}{2^{4/3}} = a$

$a \approx 0.4\ kg$

Compound Interest

Formula: $A = P\left(1 + \frac{i}{n}\right)^{nt}$

A: Future amount

P: Principle amount

i: interest rate (decimal)

n: number of times interest is compounded each year

t: number of years

Example 3: You have just passed GO and you receive $200. You decide to invest it for 4 years in an account that pays 5% interest per year. How much will you have after 10 years if…

a) the interest is compounded annually?

$A = P\left(1 + \frac{i}{n}\right)^{nt}$

$A = 200\left(1 + \frac{0.05}{1}\right)^{1(4)}$

$A = 200(1.05)^{4}$

$A = \$243.10$

b) the interest is compounded semi-annually?

$A = 200\left(1 + \frac{0.05}{2}\right)^{2(4)}$

$A = 200(1.025)^{8}$

$A = \$243.68$

c) the interest is compounded monthly?

$A = 200\left(1 + \frac{0.05}{12}\right)^{12(4)}$

$A = 200\left(1 + \frac{0.05}{12}\right)^{48}$

$A = \$244.18$

Example 4: You are about to go to University. When you are done in 4 years, you want to buy a new car. The one you are looking at costs $16,000. If you can find an investment that pays 10.9% interest per year, compounded annually, how much should you invest now?

$$A = P\left(1 + \frac{i}{n}\right)^{nt}$$

$$16000 = P\left(1 + \frac{0.109}{1}\right)^{1(4)}$$

$$\frac{16000}{1.109^4} = \frac{P(1.109)^4}{1.109^4}$$

$$P \simeq \$10577.76$$