Graphs of Common Functions

and

Intro to Transformations

In this chapter you will learn about transformations of functions. There are three main functions that we will use to learn about transformations:

1. $f(x) = x^2$ (quadratic functions)

2. $f(x) = \sqrt{x}$ (radical or square root functions)

3. $f(x) = \dfrac{1}{x}$ (rational functions)

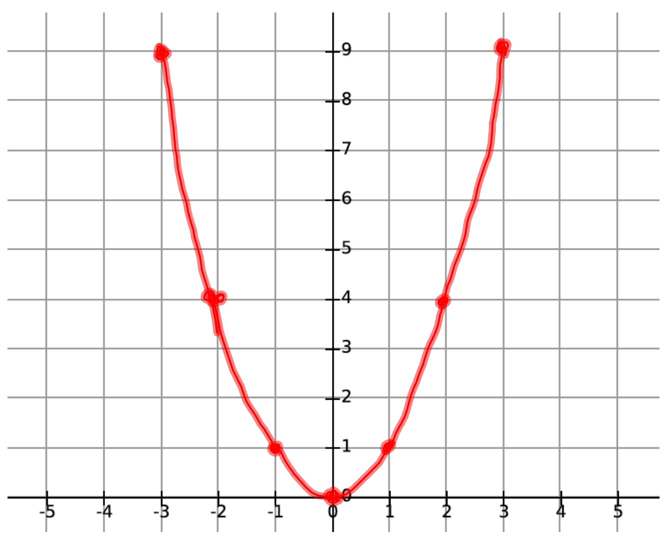
Note: *the equations given for each type of function are considered the base or parent functions of their respective families of functions. All transformations of these functions will be compared to these base functions.*

Before learning about transformations, you must understand what the base functions look like and be able to generate the key points for the graph of each function.

Quadratic Functions

Base Function: $f(x) = x^2$

Graph of Base Function



Key Points:

x	y $f(x)$
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

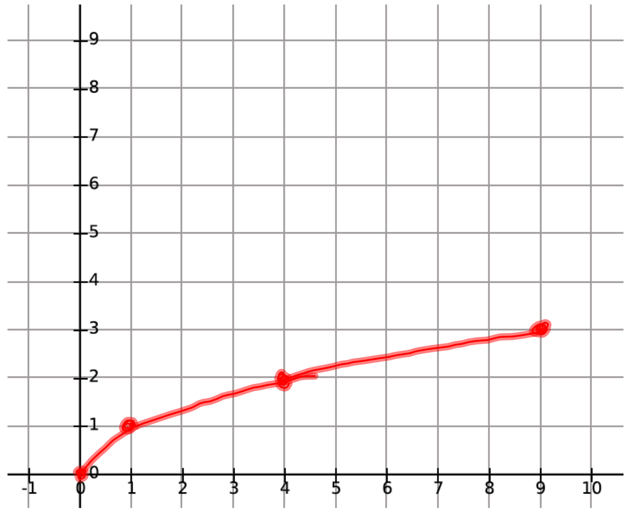
Radical (square root) Functions

Base Function: $f(x) = \sqrt{x}$

Graph of Base Function

Key Points:

x	y
0	0
1	1
4	2
9	3



Rational Functions

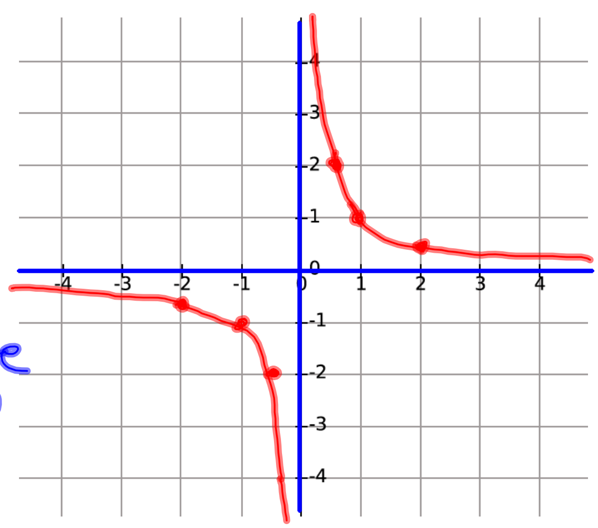
Base Function: $f(x) = \frac{1}{x}$

Graph of Base Function

Key Points:

x	y
-2	-½
-1	-1
-½	-2
0	undefined
½	2
1	1
2	½

vertical asymptote at $x=0$



Asymptotes

Asymptote: *a line that a curve approaches more and more closely but never touches.*

The function $f(x) = \frac{1}{x}$ has two asymptotes:

Vertical Asymptote: Division by zero is undefined. Therefore the expression in the denominator of the function can not be zero. Therefore $x \neq 0$. This is why the vertical line $x = 0$ is an asymptote for this function.

Horizontal Asymptote: For the range, there can never be a situation where the result of the division is zero. Therefore the line $y = 0$ is a horizontal asymptote. For all functions where the denominator is a higher degree than the numerator, there will by a horizontal asymptote at $y = 0$.

Transformations of Functions

Transformation:

A change made to a figure or a relation such that the figure or the graph of the relation is shifted or changed in shape.

Translations, stretches, and reflections are types of transformations.

The general function:

$f(x)$ parent function you are transforming

$$g(x) = af[k(x - d)] + c$$

a transformed function

takes f(x) and performs transformations to it

Changes to the *y*-coordinates (vertical changes)

c: vertical translation $g(x) = f(x) + c$

The graph of $g(x) = f(x) + c$ is a vertical translation of the graph of $f(x)$ by c units.

If c > 0, the graph shifts **up**
If c < 0, the graph shifts **down**

a: vertical stretch/compression $g(x) = af(x)$

The graph of $g(x) = af(x)$ is a vertical stretch or compression of the graph of $f(x)$ by a factor of a.

If $a > 1$ or $a < -1$, **vertical stretch** by a factor of a.
If $-1 < a < 1$, **vertical compression** by a factor of a.
If $a < 0$, **vertical reflection** (reflection over the x-axis)

Note: a vertical stretch or compression means that distance from the x-axis of each point of the parent function changes by a factor of a.

Note: for a vertical reflection, the point (x, y) becomes point $(x, -y)$

Changes to the *x*-coordinates (horizontal changes)

d: horizontal translation $g(x) = f(x - d)$

The graph of $g(x) = f(x - d)$ is a horizontal translation of the graph of $f(x)$ by d units.

If d > 0, the graph shifts **right**
If c < 0, the graph shifts **left**

k: horizontal stretch/compression

The graph of $g(x) = f(kx)$ is a horizontal stretch or compression of the graph of $f(x)$ by a factor of $\frac{1}{k}$

If $k > 1$ or $k < -1$, **compressed horizontally** by a factor of $\frac{1}{k}$
If $-1 < k < 1$, **stretched horizontally** by a factor of $\frac{1}{k}$
If $k < 0$, **horizontal reflection** (reflection in the y-axis)

Note: a vertical stretch or compression means that distance from the y-axis of each point of the parent function changes by a factor of $1/k$.

Note: for a horizontal reflection, the point (x, y) becomes point $(-x, y)$

Order of Transformations

1. stretches, compressions, reflections

2. translations

$a \rightarrow k \rightarrow d \rightarrow c$

Example 1: List the transformations and the order in which they should be done to a function $f(x)$.

a) $g(x)=-f(x)$ $a=-1$

vertical reflection (change sign of all y-values)

b) $g(x)= 2f(1/3x)$ $a=2$ $k=\frac{1}{3}$

vertical stretch by a factor of 2 (multiply y-coordinates by 2)

horizontal stretch by a factor of 3 (multiply x-coordinates by 3)

c) $g(x)=3f(x+2)-1$ $a=3$ $d=-2$ $c=-1$

vertical stretch by a factor of 3 (multiply y-coordinates by 3)

shift left 2 units (x-coordinates - 2)

shift down 1 unit (y-coordinates - 1)

d) $g(x)=1/4\, f\,[2(x-1)]$ $a=\frac{1}{4}$ $k=2$ $d=1$

vertical compression by a factor of 1/4 (divide y-coordinates by 4)

horizontal compression by a factor of 1/2 (divide x-coordinates by 2)

shift right 1 unit (x-coordinates plus 1)

e) $g(x)= -5f\,[-1/4(x+2)]+7$ $a=-5$ $k=-\frac{1}{4}$

$d=-2$ $c=7$

vertical stretch by a factor of 5 (multiply y-coordinates by 5)

vertical reflection (change sign of y-coordinates)

horizontal stretch by a factor of 4 (multiply x-coordinates by 4)

horizontal reflection (change sign of x-coordinates)

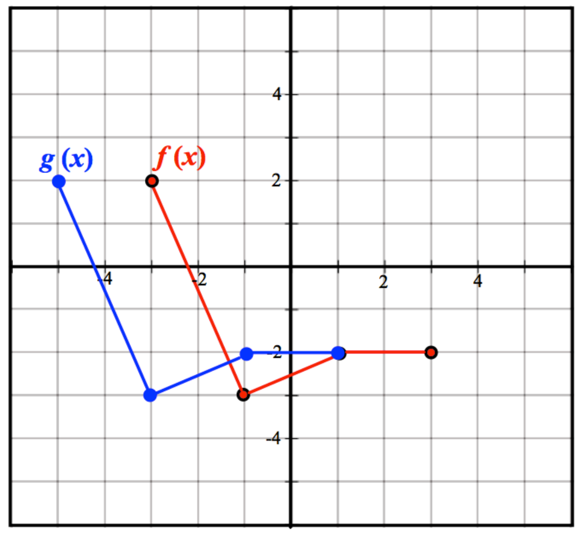
shift left two units (x-coordinates - 2)

shift up 7 units (y-coordinates + 7)

Example 2: List the transformations and the order in which they should be done to the function $f(x)$. Use the given graph of $f(x)$ to sketch the graph of $g(x)$

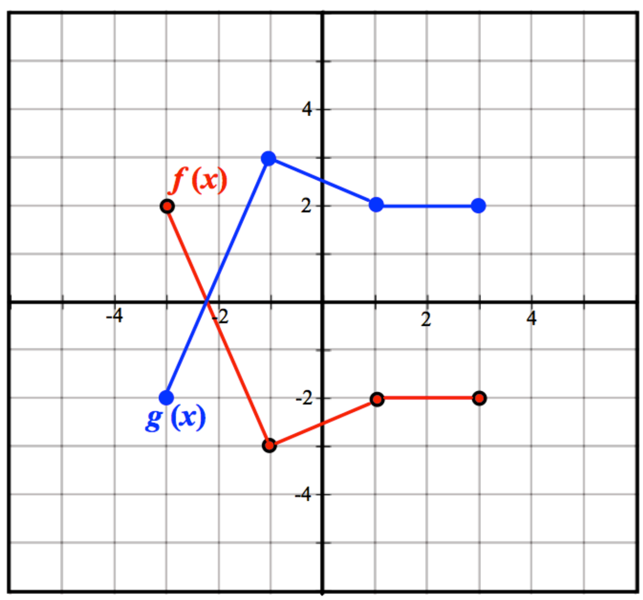
a) $g(x) = f(x+2)$

shift left 2 units



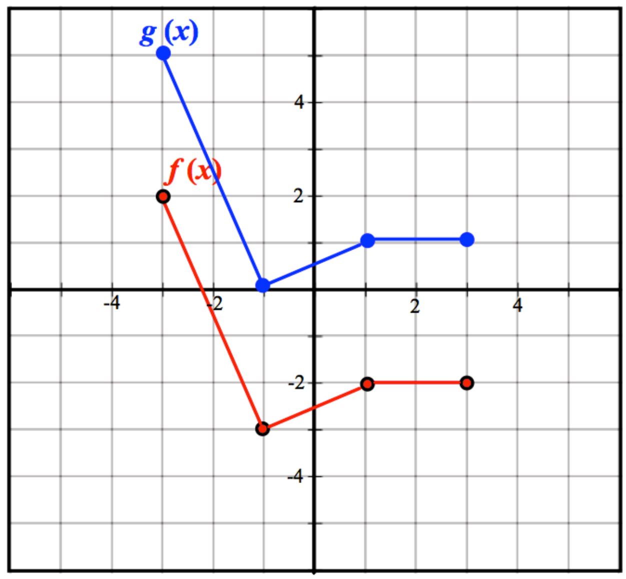
b) $g(x) = -f(x)$

vertical reflection over the x-axis



c) $g(x) = f(x) + 3$

shift up 3 units



d) $g(x) = f(2x) - 1$

It may help to make a list of ***image points*** *(any point that has been transformed from a point on the original figure or graph)*

horizontal compression by a factor of 1/2 (x/2)

shift down 1 unit (y - 1)

$\boxed{f(x)} \longrightarrow \boxed{g(x)}$

x	y
-3	2
-1	-3
1	-2
3	-2

$\frac{x}{2}$	$y-1$
-1.5	1
-0.5	-4
0.5	-3
1.5	-3

↑

image points