Unit 3 - Factoring

Lessons

MPM2D

$3x^2 - 5x - 2$

$\underline{-6} + \underline{1} = -5$

$\underline{-6} \times \underline{1} = 3(-2) = -6$

$= 3x^2 - 6x + 1x - 2$

$= (3x^2 - 6x) + (x - 2)$

$= 3x(\underline{x-2}) + 1(\underline{x-2})$ common factor the common binomial

$= (x-2)(3x+1)$

Unit 3 Outline

Unit Goal: By the end of this unit, you will be able to factor polynomial expressions involving common factors, trinomials, and difference of squares

Section	Subject	Learning Goals	Curriculum Expectations
L1	Multiplying Binomials	- expand and simplify second degree polynomial expressions	A3.1
L2	Common Factoring	- factor polynomial expressions involving common factors	A3.2
L3	Factoring Quadratics 1	- factor polynomial expressions involving quadratic trinomials where the leading coefficient is 1	A3.2
L4	Factoring Quadratics 2	- factor polynomial expressions involving quadratic trinomials where the leading coefficient is NOT 1	A3.2
L5	Special Products	- factor polynomial expressions involving difference of squares and perfect square trinomials	A3.2

Assessments	F/A/O	Ministry Code	P/O/C	KTAC
Note Completion	A		P	
Practice Worksheet Completion	F/A		P	
Quiz – Factoring	F		P	
PreTest Review	F/A		P	
Test – Factoring	O	B1.1, B1.2	P	K(30%), T(30%), A(30%), C(10%)

L1 – Multiplying Binomials Unit 3
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Part 1: Review of Operations with Polynomials

__________________ are terms that have the same __________________ factors. You can simplify expressions containing like terms by adding their coefficients and keeping the variable factors the same.

Example 1: Simplify the following expressions

a) $3x + 4x + 2y$

b) $(x^2 + 4x - 2) + (2x^2 - 6x + 9)$

c) $2(6m^2 - mn + 4) - (7m^2 + 4mn - 2)$

When multiplying/dividing monomials, you must multiply/divide the numerical coefficients and then multiply/divide the variables using exponent rules:

PRODUCT RULE: $(x^a)(x^b) =$

QUOTIENT RULE: $\frac{x^a}{x^b} =$

Example 2: Simplify the following expressions

a) $3x(2x^2 - 5) + 4x$

b) $5y(2x + y) - 4(2xy - 3y)$

Part 2: Multiplying Binomials

When multiplying binomials, you must multiply each term in the first binomial by each term in the second binomial. The acronym FOIL can be used to remember all four products you need to calculate. FOIL stands for First, Outside, Inside, Last. After multiplying (expanding), make sure to simplify by collecting like terms.

How it works:

$$\overset{\text{Outside}}{\overset{\text{First}}{\underset{\text{Last}}{\underset{\text{Inside}}{(x+2)(x+3)}}}}$$

$$= \underbrace{x^2}_{\text{First}} + \underbrace{3x}_{\text{Outside}} + \underbrace{2x}_{\text{Inside}} + \underbrace{6}_{\text{Last}}$$

$$= x^2 + 5x + 6$$

Example 3: Expand and simplify each of the following

a) $(x+4)(x-5)$

b) $(3x+1)(2x+7)$

c) $3(2x-1)(2x+5)$

d) $2(3y+2)(y-1)-(y-2)(2y+1)$

e) $4(x-2)(x-6)+3(x+3)(3x-2)$

f) $2(3x+4)^2-(4x+5)(4x-5)$

Part 3: Multiplying Polynomials

The general rule when multiplying polynomials is that each term in one of the polynomials must be multiplied by each term in the other polynomial.

Example 4: Expand and simplify each of the following

a) $(x + 2y)(3x - 4y + 5)$

b) $(x^2 - 2x + 1)(x^2 + 5x + 6)$

c) $(4x - 3)^3$

L2 – Common Factoring Unit 3
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__________________ a polynomial is the OPPOSITE of expanding

Expanding → Multiplying

Factoring → Dividing

To factor a polynomial, remove the greatest common factor as the first factor, then __________________ each term by the greatest common factor to obtain the second factor.

A greatest common factor is the greatest number and/or variable that is a factor (divides evenly into) of all terms in a set.

Part 1: Monomial Common Factor

Example 1: Factor each of the following expressions

a) $-5x + 20$

b) $8x^2 - 7x$

c) $25x^6 + 15x^4$

d) $21x^4y^3 - 28x^2y^5 + 7xy^3$

e) $4x^2y^3 + 10x^4y^2 - 12x^3y^2$

f) $8x^3 - 6x^2y^2 + 4x^2y$

Part 2: Binomial Common Factor and Factoring by Grouping

A greatest common factor is not necessarily a monomial.

Example 2: Factor each of the following expressions

a) $3x(y+1)+7(y+1)$

b) $2x(x-3)-5(x-3)$

c) $5x(x^2+2x+7)-4(x^2+2x+7)$

Some polynomials do not have a common factor but can be factored by ____________________. When factoring by grouping:

1) group pairs of terms with a common factor (always separate groups with an addition sign)
2) remove a common factor from each group
3) factor the common binomial (or other type of polynomial) from the expression

Example 3: Factor each of the following expressions

a) $ac+bc+ad+bd$

b) $9x^2+15x+3x+5$

L3 – Factor $x^2 + bx + c$ Unit 3
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To learn how to factor trinomials of the form $x^2 + bx + c$, let's study the expansion of $(x+m)(x+n)$

$$\begin{aligned}(x+m)(x+n) &= x^2 + nx + mx + mn \\ &= x^2 + mx + nx + mn \\ &= x^2 + (m+n)x + mn\end{aligned}$$

Compare the result above to the general expression $x^2 + bx + c$

$$x^2 + (m+n)x + mn$$

$$x^2 + bx + c$$

So to factor $x^2 + bx + c$, you must find the numbers that add to b and multiply to c.

General Rule:

$$x^2 + bx + c = (x+m)(x+n)$$

Where $b = m + n$ and $c = mn$

Example 1: Factor each of the following

a) $x^2 + 7x + 12$

b) $x^2 + 8x + 15$

c) $x^2 - 29x + 28$

d) $x^2 + 3x - 18$

e) $2x^2 - 8x - 42$

f) $-2x^2 + 8x + 42$

g) $x^2 + 11xy + 24y^2$

h) $x^2 + 10x + 25$

i) $x^4 + 4x^2 + 3$

L4 – Factor $ax^2 + bx + c$ where $a \neq 1$ Unit 3

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Steps for factoring $ax^2 + bx + c$ when $a \neq 1$

1) Check for any common factors that can be factored out
2) Replace the middle term bx with two terms whose coefficients have a sum of b and a product of $a \times c$
3) Group pairs of terms and remove a common factor from each pair
4) Remove the common binomial factor

Example 1: Factor each of the following

a) $3x^2 - 5x - 2$

b) $2x^2 + 11x + 12$

c) $6x^2 + 13x - 5$

d) $4x^2 - 5xy - 6y^2$

e) $6x^2 + 14x + 4$

f) $16x^2 + 26x - 12$

L5 – Special Products Unit 3
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Difference of Squares:

A difference of squares is the difference of two perfect square terms

$$a^2 - b^2 =$$

Perfect Square Trinomial

The trinomial that results from squaring a binomial is called a perfect square trinomial. Notice the first and last terms are perfect squares, and the middle term is twice the product of the square roots of the first and last terms.

$$a^2 + 2ab + b^2 =$$

$$a^2 - 2ab + b^2 =$$

Example 1: Expand each of the following

a) $(x-3)(x+3)$

b) $(3x+1)(3x-1)$

c) $(4x^2-3y)(4x^2+3y)$

d) $(x+4)^2$

e) $(x-5)^2$

f) $(3x+2)^2$

Example 2: Factor each of the following

a) $x^2 - 36$

b) $x^2 + 14x + 49$

c) $16x^2 - 25$

d) $x^2 - 20x + 100$

e) $4x^2 - 9y^2$

f) $x^2 - 8xy + 16y^2$