

# Unit 3 - Factoring

## Lessons

### MPM2D

$$\begin{aligned} & 3x^2 - 5x - 2 \quad \underline{-6} + \underline{1} = -5 \\ & = 3x^2 - 6x + 1x - 2 \quad \underline{-6} \times \underline{1} = 3(-2) = -6 \\ & = (3x^2 - 6x) + (x - 2) \\ & = 3x(x - 2) + 1(x - 2) \quad \text{Common factor} \\ & = (x - 2)(3x + 1) \quad \text{the common binomial} \end{aligned}$$



**W1 – Multiplying Binomials**

Unit 3

MPM2D

Jensen

1) Simplify the following expressions

a)  $(3x + 4) + (5x + 2)$

$$= 3x + 4 + 5x + 2$$

$$= 3x + 5x + 4 + 2$$

$$= 8x + 6$$

b)  $(2 - 3yz) + (7 + 6yz)$

$$= 2 - 3yz + 7 + 6yz$$

$$= -3yz + 6yz + 2 + 7$$

$$= 3yz + 9$$

c)  $(5x + 7) - (2x - 1)$

$$= 5x + 7 - 2x + 1$$

$$= 5x - 2x + 7 + 1$$

$$= 3x + 8$$

2) Multiply

a)  $(4x)(7x^2)$

$$= 28x^3$$

b)  $(-6m^2n^3)(-7mn^2)$

$$= 42m^3n^5$$

c)  $(2xy)(-3x^2y^3)(-3x^2)$

$$= 18x^5y^4$$

3) Expand and simplify

a)  $2(x - 4) + 5(x + 3)$

$$= 2x - 8 + 5x + 15$$

$$= 2x + 5x - 8 + 15$$

$$= 7x + 7$$

b)  $4(2x - 7) - 5(4x + 9)$

$$= 8x - 28 - 20x - 45$$

$$= 8x - 20x - 28 - 45$$

$$= -12x - 73$$

c)  $4x + 3(2x - 5) + 6(1 - 5x)$

$$= 4x + 6x - 15 + 6 - 30x$$

$$= 4x + 6x - 30x - 15 + 6$$

$$= -20x - 9$$

4) Expand and simplify

a)  $(x + 1)(x + 5)$

$$= x^2 + 5x + x + 5$$

$$= x^2 + 6x + 5$$

b)  $(x - 4)(x - 3)$

$$= x^2 - 3x - 4x + 12$$

$$= x^2 - 7x + 12$$

c)  $(c + 2)(c - 8)$

$$= c^2 - 8c + 2c - 16$$

$$= c^2 - 6c - 16$$

$$\text{d) } (a-3)(2a-5)$$

$$= 2a^2 - 5a - 6a + 15$$

$$= 2a^2 - 11a + 15$$

$$\text{e) } (x-5)(4x+3)$$

$$= 4x^2 + 3x - 20x - 15$$

$$= 4x^2 - 17x - 15$$

$$\text{f) } (3a-5)(3a+5)$$

$$= 9a^2 + 15a - 15a - 25$$

$$= 9a^2 - 25$$

$$\text{g) } 2(x+3)(x+5)$$

$$= (2x+6)(x+5)$$

$$= 2x^2 + 10x + 6x + 30$$

$$= 2x^2 + 16x + 30$$

$$\text{h) } -2(4y+1)(y-3)$$

$$= -2(4y^2 - 12y + y - 3)$$

$$= -2(4y^2 - 11y - 3)$$

$$= -8y^2 + 22y + 6$$

$$\text{i) } (3x+y)(x+4y)$$

$$= 3x^2 + 12xy + xy + 4y^2$$

$$= 3x^2 + 13xy + 4y^2$$

$$\text{j) } (-3a+4b)(2a+3b)$$

$$= -6a^2 - 9ab + 8ab + 12b^2$$

$$= -6a^2 - ab + 12b^2$$

$$\text{k) } (x+6)(x+4) + (x+2)(x+3)$$

$$= x^2 + 4x + 6x + 24 + x^2 + 3x + 2x + 6$$

$$= 2x^2 + 15x + 30$$

$$\text{l) } 2(3x+2)(3x+2) - 3(2x-1)(x+4)$$

$$= 2(9x^2 + 6x + 6x + 4) - 3(2x^2 + 8x - x - 4)$$

$$= 2(9x^2 + 12x + 4) - 3(2x^2 + 7x - 4)$$

$$= 18x^2 + 24x + 8 - 6x^2 - 21x + 12$$

$$= 12x^2 + 3x + 20$$

$$\text{m) } 12 - 2(3y-2)(3y+2) - (2y+5)(y-4)$$

$$= 12 - 2(9y^2 + 6y - 6y - 4) - (2y^2 - 8y + 5y - 20)$$

$$= 12 - 2(9y^2 - 4) - (2y^2 - 3y - 20)$$

$$= 12 - 18y^2 + 8 - 2y^2 + 3y + 20$$

$$= -20y^2 + 3y + 40$$

5)a)  $(2x + 3)^2$

$$= (2x+3)(2x+3)$$

$$= 4x^2 + 6x + 6x + 9$$

$$= 4x^2 + 12x + 9$$

b)  $(2x + 3)^3$

$$= (2x+3)(2x+3)(2x+3)$$

$$= (2x+3)(4x^2+12x+9)$$

$$= 8x^3 + 24x^2 + 18x + 12x^2 + 36x + 27$$

$$= 8x^3 + 36x^2 + 54x + 27$$

c)  $(x^2 + 3x - 2)(x + 4)$

$$= x^3 + 4x^2 + 3x^2 + 12x - 2x - 8$$

$$= x^3 + 7x^2 + 10x - 8$$

d)  $(x^2 - 4x + 1)(x^2 + 3x + 5)$

$$= x^4 + 3x^3 + 5x^2 - 4x^3 - 12x^2 - 20x + x^2 + 3x + 5$$

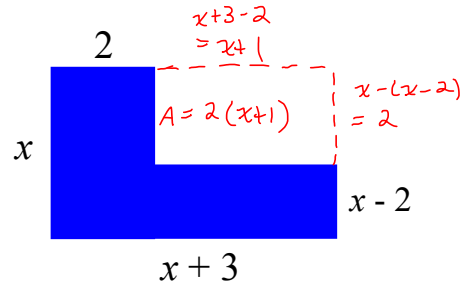
$$= x^4 - x^3 - 6x^2 - 17x + 5$$

6) Write and simplify an expression for the area of the following figure.

$$A = x(x+3) - 2(x+1)$$

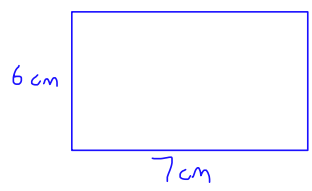
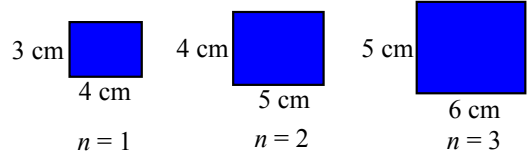
$$= x^2 + 3x - 2x - 2$$

$$= x^2 + x - 2$$



7) The diagrams show the first three rectangles in a pattern.

a) State the area of the 4<sup>th</sup> rectangle.



$n = 4$

$$A = 6(7)$$

$$= 42 \text{ cm}^2$$

b) Write a product of two binomials to represent the area of the  $n^{\text{th}}$  rectangle in terms of  $n$ .

$$A = (n+2)(n+3)$$

c) State the area of the 28<sup>th</sup> rectangle

$$\begin{aligned} A &= (28+2)(28+3) \\ &= (30)(31) \\ &= 930 \text{ cm}^2 \end{aligned}$$

### Answers

- 1)a)  $8x + 6$  b)  $3yz + 9$  c)  $3x + 8$   
2)a)  $28x^3$  b)  $42m^3n^5$  c)  $18x^5y^4$   
3)a)  $7x + 7$  b)  $-12x - 73$  c)  $-20x - 9$   
4)a)  $x^2 + 6x + 5$  b)  $x^2 - 7x + 12$  c)  $c^2 - 6c - 16$   
d)  $2a^2 - 11a + 15$  e)  $4x^2 - 17x - 15$  f)  $9a^2 - 25$   
g)  $2x^2 + 16x + 30$  h)  $-8y^2 + 22y + 6$  i)  $3x^2 + 13xy + 4y^2$   
j)  $-6a^2 - ab + 12b^2$  k)  $2x^2 + 15x + 30$   
l)  $12x^2 + 3x + 20$  m)  $-20y^2 + 3y + 40$   
5)a)  $4x^2 + 12x + 9$  b)  $8x^3 + 36x^2 + 54x + 27$   
c)  $x^3 + 7x^2 + 10x - 8$  d)  $x^4 - x^3 - 6x^2 - 17x + 5$   
6)  $x^2 + x - 2$   
7)a)  $42 \text{ cm}^2$  b)  $A = (n + 2)(n + 3)$  c)  $930 \text{ cm}^2$

1) Factor each of the following expressions if possible.

a)  $15w + 25z$

$$= 5 \left( \frac{15w}{5} + \frac{25z}{5} \right)$$

$$= 5(3w + 5z)$$

b)  $17ca - 8cd$

$$= c \left( \frac{17ca}{c} - \frac{8cd}{c} \right)$$

$$= c(17a - 8d)$$

c)  $12b^4 + 18b^2$

$$= 2b^2 \left( \frac{12b^4}{2b^2} + \frac{18b^2}{2b^2} \right)$$

$$= 2b^2(6b^2 + 9)$$

d)  $7h + 3m - 5k$

not factorable

2) Factor each of the following expressions if possible.

a)  $14x^2y + 16xy^3$

$$= 2xy \left( \frac{14x^2y}{2xy} + \frac{16xy^3}{2xy} \right)$$

$$= 2xy(7x + 8y^2)$$

b)  $8s^2y + 11t^3$

not factorable

c)  $7gh + 2mn - 13pq$

not factorable

d)  $27r^2s^2 - 18r^3s^2 - 36rs^3$

$$= 9rs^2 \left( \frac{27r^2s^2}{9rs^2} - \frac{18r^3s^2}{9rs^2} - \frac{36rs^3}{9rs^2} \right)$$

$$= 9rs^2(3r - 2r^2 - 4s)$$

3) Factor each of the following expressions if possible.

a)  $3x(x + 8) + 5(x + 8)$

$$= (x+8) \left[ \frac{3x(x+8)}{x+8} + \frac{5(x+8)}{x+8} \right]$$

$$= (x+8)(3x+5)$$

b)  $a(b + 1) + 9c(b + 1)$

$$= (b+1) \left[ \frac{a(b+1)}{b+1} + \frac{9c(b+1)}{b+1} \right]$$

$$= (b+1)(a+9c)$$

c)  $2y(x - 5) + 4(x + 5)$

not factorable

4) Factor each of the following expressions if possible.

a)  $mx + my + 2x + 2y$

$$= m(x+y) + 2(x+y)$$
$$= (x+y)(m+2)$$

b)  $x^2 + 3x + 2x + 6$

$$= x(x+3) + 2(x+3)$$
$$= (x+3)(x+2)$$

c)  $ay^2 + 3ay + 4y + 12$

$$= ay(y+3) + 4(y+3)$$
$$= (y+3)(ay+4)$$

5) The formula for the surface area of a cylinder is  $SA = 2\pi r^2 + 2\pi rh$

a) Write the formula in factored form

$$SA = 2\pi r(r+h)$$

b) If  $r = 3$  and  $h = 8$ , find the surface area using both the original and factored form equations.

$$SA = 2\pi(3)^2 + 2\pi(3)(8)$$
$$= 18\pi + 48\pi$$
$$= 66\pi \text{ units}^2$$

$$SA = 2\pi(3)(3+8)$$
$$= 6\pi(11)$$
$$= 66\pi \text{ units}^2$$

6) Factor, if possible.

a)  $9a^3 + 27b^2$

$$= 9\left(\frac{9a^3}{9} + \frac{27b^2}{9}\right)$$
$$= 9(a^3 + 3b^2)$$

b)  $24xy^2 - 12xy + 36x^2y$

$$= 12xy\left(\frac{24xy^2}{12xy} - \frac{12xy}{12xy} + \frac{36x^2y}{12xy}\right)$$
$$= 12xy(2y - 1 + 3x)$$

c)  $xy + 12 + 4x + 3y$

$$= xy + 3y + 4x + 12$$
$$= y(x+3) + 4(x+3)$$
$$= (x+3)(y+4)$$

### Answers

1)a)  $5(3w + 5z)$  b)  $c(17a - 8d)$  c)  $6b^2(2b^2 + 3)$  d) not factorable

2)a)  $2xy(7x + 8y^2)$  b) not factorable c) not factorable d)  $9rs^2(3r - 2r^2 - 4s)$

3)a)  $(x + 8)(3x + 5)$  b)  $(b + 1)(a + 9c)$  c) not factorable

4)a)  $(x + y)(m + 2)$  b)  $(x + 3)(x + 2)$  c)  $(y + 3)(ay + 4)$

5)a)  $SA = 2\pi r(r + h)$  b)  $SA = 66\pi \text{ cm}^2$ , or  $207.3 \text{ cm}^2$

6)a)  $9(a^3 + 3b^2)$  b)  $12xy(2y - 1 + 3x)$  c)  $(x + 3)(y + 4)$



**W3 – Factor  $x^2 + bx + c$** 

Unit 3

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Jensen

1) Factor, if possible.

a)  $x^2 + 7x + 10$

$$\begin{array}{r} \underline{2} \times \underline{5} = 10 \\ \underline{2} + \underline{5} = 7 \end{array}$$

$$= (x+2)(x+5)$$

b)  $j^2 + 12j + 27$

$$\begin{array}{r} \underline{9} \times \underline{3} = 27 \\ \underline{9} + \underline{3} = 12 \end{array}$$

$$= (j+9)(j+3)$$

c)  $k^2 + 5k + 4$

$$\begin{array}{r} \underline{4} \times \underline{1} = 4 \\ \underline{4} + \underline{1} = 5 \end{array}$$

$$= (k+4)(k+1)$$

d)  $p^2 + 9p + 12$

$$\begin{array}{r} \underline{\quad} \times \underline{\quad} = 12 \\ \underline{\quad} + \underline{\quad} = 9 \end{array}$$

not factorable

e)  $w^2 + 11w + 25$

$$\begin{array}{r} \underline{\quad} \times \underline{\quad} = 25 \\ \underline{\quad} + \underline{\quad} = 11 \end{array}$$

not factorable

f)  $d^2 + 10d + 24$

$$\begin{array}{r} \underline{6} \times \underline{4} = 24 \\ \underline{6} + \underline{4} = 10 \end{array}$$

$$= (d+6)(d+4)$$

2) Factor, if possible.

a)  $m^2 - 7m + 10$

$$\begin{array}{r} \underline{-2} \times \underline{-5} = 10 \\ \underline{-2} + \underline{-5} = -7 \end{array}$$

$$= (m-2)(m-5)$$

b)  $x^2 - 5x + 7$

$$\begin{array}{r} \underline{\quad} \times \underline{\quad} = 7 \\ \underline{\quad} + \underline{\quad} = -5 \end{array}$$

not factorable

c)  $y^2 - 5y + 4$

$$\begin{array}{r} \underline{-4} \times \underline{-1} = 4 \\ \underline{-4} + \underline{-1} = -5 \end{array}$$

$$= (y-4)(y-1)$$

d)  $r^2 - 16r + 64$

$$\begin{array}{r} \underline{-8} \times \underline{-8} = 64 \\ \underline{-8} + \underline{-8} = -16 \end{array}$$

$$= (r-8)(r-8)$$

$$= (r-8)^2$$

e)  $w^2 - 9w + 24$

$$\begin{array}{r} \underline{\quad} \times \underline{\quad} = 24 \\ \underline{\quad} + \underline{\quad} = -9 \end{array}$$

NOT factorable

f)  $q^2 - 10q + 9$

$$\begin{array}{r} \underline{-9} \times \underline{-1} = 9 \\ \underline{-9} + \underline{-1} = -10 \end{array}$$

$$= (q-9)(q-1)$$

3) Factor, if possible.

a)  $a^2 - 3a - 10$

$$\begin{array}{r} \underline{-5} \times \underline{2} = -10 \\ \underline{-5} + \underline{2} = -3 \end{array}$$

$$= (a-5)(a+2)$$

b)  $s^2 + 3s - 10$

$$\begin{array}{r} \underline{5} \times \underline{-2} = -10 \\ \underline{5} + \underline{-2} = 3 \end{array}$$

$$= (s+5)(s-2)$$

c)  $d^2 - 8d - 9$

$$\begin{array}{r} \underline{-9} \times \underline{1} = -9 \\ \underline{-9} + \underline{1} = -8 \end{array}$$

$$= (d-9)(d+1)$$

$$\text{d) } f^2 + 7f - 6 \quad \begin{array}{|l} \underline{\quad} \times \underline{\quad} = -6 \\ \underline{\quad} + \underline{\quad} = 7 \end{array}$$

not factorable

$$\text{e) } g^2 - 5g - 14 \quad \begin{array}{|l} \underline{-7} \times \underline{2} = -14 \\ \underline{-7} + \underline{2} = -5 \end{array}$$

$$= (g-7)(g+2)$$

$$\text{f) } r^2 + 2r - 6 \quad \begin{array}{|l} \underline{\quad} \times \underline{\quad} = -6 \\ \underline{\quad} + \underline{\quad} = 2 \end{array}$$

not factorable

$$\text{g) } x^2 + x - 42 \quad \begin{array}{|l} \underline{7} \times \underline{-6} = -42 \\ \underline{7} + \underline{-6} = 1 \end{array}$$

$$= (x+7)(x-6)$$

$$\text{h) } b^2 - 2b - 4 \quad \begin{array}{|l} \underline{\quad} \times \underline{\quad} = -4 \\ \underline{\quad} + \underline{\quad} = -2 \end{array}$$

not factorable

$$\text{i) } x^2 + xy - 42y^2 \quad \begin{array}{|l} \underline{7} \times \underline{-6} = -42 \\ \underline{7} + \underline{-6} = 1 \end{array}$$

$$= (x+7y)(x-6y)$$

$$\text{j) } x^2 - 8xy - 48y^2 \quad \begin{array}{|l} \underline{-12} \times \underline{4} = -48 \\ \underline{-12} + \underline{4} = -8 \end{array}$$

$$= (x-12y)(x+12y)$$

$$\text{k) } x^4 + 11x^2 + 24$$

Let  $k = x^2$

$$= k^2 + 11k + 24$$

$$= (k+8)(k+3)$$

$$= (x^2+8)(x^2+3)$$

$$\begin{array}{|l} \underline{8} \times \underline{3} = 24 \\ \underline{8} + \underline{3} = 11 \end{array}$$

$$\text{l) } x^2 - 9 \quad \begin{array}{|l} \underline{3} \times \underline{-3} = -9 \\ \underline{3} + \underline{-3} = 0 \end{array}$$

$$= (x+3)(x-3)$$

4) Factor, if possible.

$$\text{a) } 3x^2 + 12x + 9$$

$$= 3(x^2 + 4x + 3)$$

$$= 3(x+3)(x+1)$$

$$\begin{array}{|l} \underline{3} \times \underline{1} = 3 \\ \underline{3} + \underline{1} = 4 \end{array}$$

$$\text{b) } 2d^2 - 22d + 56$$

$$= 2(d^2 - 11d + 28)$$

$$= 2(d-7)(d-4)$$

$$\begin{array}{|l} \underline{-7} \times \underline{-4} = 28 \\ \underline{-7} + \underline{-4} = -11 \end{array}$$

$$\text{c) } 5z^2 + 40z + 60$$

$$= 5(z^2 + 8z + 12)$$

$$= 5(z+2)(z+6)$$

$$\begin{array}{|l} \underline{2} \times \underline{6} = 12 \\ \underline{2} + \underline{6} = 8 \end{array}$$

$$\text{d) } 4s^2 - 8s - 32$$

$$= 4(s^2 + 2s - 8)$$

$$= 4(s+4)(s-2)$$

$$\begin{array}{|l} \underline{4} \times \underline{-2} = -8 \\ \underline{4} + \underline{-2} = 2 \end{array}$$

5) Factor, if possible.

a)  $x^4 + 10x^2y + 9y^2$

Let  $k = x^2$

$$\begin{aligned} &= k^2 + 10ky + 9y^2 \\ &= (k + 9y)(k + y) \\ &= (x^2 + 9y)(x^2 + y) \end{aligned}$$

$$\begin{array}{l} \frac{9}{9} \times \frac{1}{1} = 9 \\ \frac{9}{9} + \frac{1}{1} = 10 \end{array}$$

b)  $(x + a)^2 + 3(x + a) + 2$

Let  $k = x + a$

$$\begin{aligned} &= k^2 + 3k + 2 \\ &= (k + 2)(k + 1) \\ &= (x + a + 2)(x + a + 1) \end{aligned}$$

$$\begin{array}{l} \frac{2}{2} \times \frac{1}{1} = 2 \\ \frac{2}{2} + \frac{1}{1} = 3 \end{array}$$

6) Determine binomials that represent the length and width of the rectangle. Then, determine the dimensions of the rectangle if  $x$  represents 15 cm.

$$A = x^2 + 18x + 80$$

$$A = \underbrace{(x + 10)}_{\text{length}} \underbrace{(x + 8)}_{\text{width}}$$

$$\begin{array}{l} \frac{10}{10} \times \frac{8}{8} = 80 \\ \frac{10}{10} + \frac{8}{8} = 18 \end{array}$$

$$\text{Area is } x^2 + 18x + 80$$

$$A = (15 + 10)(15 + 8)$$

$$A = (25)(23)$$

$$A = 575 \text{ cm}^2$$

The length is 25 cm  
The width is 23 cm

### Answers

- 1) a)  $(x + 5)(x + 2)$  b)  $(j + 9)(j + 3)$  c)  $(k + 4)(k + 1)$  d) not possible e) not possible f)  $(d + 6)(d + 4)$   
2) a)  $(m - 2)(m - 5)$  b) not possible c)  $(y - 4)(y - 1)$  d)  $(r - 8)^2$  e) not possible f)  $(q - 9)(q - 1)$   
3) a)  $(a - 5)(a + 2)$  b)  $(s + 5)(s - 2)$  c)  $(d - 9)(d + 1)$  d) not possible e)  $(g - 7)(g + 2)$  f) not possible  
g)  $(x + 7)(x - 6)$  h) not possible i)  $(x + 7y)(x - 6y)$  j)  $(x - 12y)(x + 4y)$  k)  $(x^2 + 8)(x^2 + 3)$  l)  $(x - 3)(x + 3)$   
4) a)  $3(x + 3)(x + 1)$  b)  $2(d - 7)(d - 4)$  c)  $5(z + 6)(z + 2)$  d)  $4(s - 4)(s + 2)$   
5) a)  $(x^2 + 9y)(x^2 + y)$  b)  $(x + a + 2)(x + a + 1)$   
6)  $A = (x + 10)(x + 8)$ ; the rectangle is 25 cm by 23 cm.

**W4 – Factor  $ax^2 + bx + c$  where  $a \neq 1$** 

Unit 3

MPM2D

Jensen

1) Factor, if possible.

$$\begin{aligned} \text{a) } 2x^2 + 7x + 5 & \quad \boxed{\begin{array}{l} \underline{2} \times \underline{5} = 10 \\ \underline{2} + \underline{5} = 7 \end{array}} \\ = 2x^2 + 2x + 5x + 5 & \\ = 2x(x+1) + 5(x+1) & \\ = (x+1)(2x+5) & \end{aligned}$$

$$\begin{aligned} \text{b) } 6y^2 + 19y + 8 & \quad \boxed{\begin{array}{l} \underline{16} \times \underline{3} = 48 \\ \underline{16} + \underline{3} = 19 \end{array}} \\ = 6y^2 + 16y + 3y + 8 & \\ = 2y(3y+8) + 1(3y+8) & \\ = (3y+8)(2y+1) & \end{aligned}$$

$$\begin{aligned} \text{c) } 4k^2 + 15k + 9 & \quad \boxed{\begin{array}{l} \underline{12} \times \underline{3} = 36 \\ \underline{12} + \underline{3} = 15 \end{array}} \\ = 4k^2 + 12k + 3k + 9 & \\ = 4k(k+3) + 3(k+3) & \\ = (k+3)(4k+3) & \end{aligned}$$

$$\begin{aligned} \text{d) } 3m^2 + 10m + 8 & \quad \boxed{\begin{array}{l} \underline{6} \times \underline{4} = 24 \\ \underline{6} + \underline{4} = 10 \end{array}} \\ = 3m^2 + 6m + 4m + 8 & \\ = 3m(m+2) + 4(m+2) & \\ = (m+2)(3m+4) & \end{aligned}$$

$$\begin{aligned} \text{e) } 10w^2 + 15w + 3 & \quad \boxed{\begin{array}{l} \underline{\quad} \times \underline{\quad} = 30 \\ \underline{\quad} + \underline{\quad} = 15 \end{array}} \\ \text{not factorable} & \end{aligned}$$

$$\begin{aligned} \text{f) } 12q^2 + 17q + 6 & \quad \boxed{\begin{array}{l} \underline{9} \times \underline{8} = 72 \\ \underline{9} + \underline{8} = 17 \end{array}} \\ = 12q^2 + 9q + 8q + 6 & \\ = 3q(4q+3) + 2(4q+3) & \\ = (4q+3)(3q+2) & \end{aligned}$$

$$\begin{aligned} \text{g) } 4x^2 - 11x + 6 & \quad \boxed{\begin{array}{l} \underline{-8} \times \underline{-3} = 24 \\ \underline{-8} + \underline{-3} = -11 \end{array}} \\ = 4x^2 - 8x - 3x + 6 & \\ = 4x(x-2) - 3(x-2) & \\ = (x-2)(4x-3) & \end{aligned}$$

$$\begin{aligned} \text{h) } 5n^2 - 11n + 6 & \quad \boxed{\begin{array}{l} \underline{-6} \times \underline{-5} = 30 \\ \underline{-6} + \underline{-5} = -11 \end{array}} \\ = 5n^2 - 6n - 5n + 6 & \\ = n(5n-6) - 1(5n-6) & \\ = (5n-6)(n-1) & \end{aligned}$$

$$\begin{aligned} \text{i) } 9b^2 - 24b + 7 & \quad \boxed{\begin{array}{l} \underline{-21} \times \underline{-3} = 63 \\ \underline{-21} + \underline{-3} = -24 \end{array}} \\ = 9b^2 - 21b - 3b + 7 & \\ = 3b(3b-7) - 1(3b-7) & \\ = (3b-7)(3b-1) & \end{aligned}$$

$$\begin{aligned} \text{j) } 3y^2 + 4y - 7 & \quad \boxed{\begin{array}{l} 7 \times -3 = -21 \\ 7 + -3 = 4 \end{array}} \\ & = 3y^2 + 7y - 3y - 7 \\ & = y(3y+7) - 1(3y+7) \\ & = (3y+7)(y-1) \end{aligned}$$

$$\begin{aligned} \text{k) } 8k^2 - 6k - 5 & \quad \boxed{\begin{array}{l} -10 \times 4 = -40 \\ -10 + 4 = -6 \end{array}} \\ & = 8k^2 - 10k + 4k - 5 \\ & = 2k(4k-5) + 1(4k-5) \\ & = (4k-5)(2k+1) \end{aligned}$$

$$\begin{aligned} \text{l) } 5h^2 - 14h - 3 & \quad \boxed{\begin{array}{l} -15 \times 1 = -15 \\ -15 + 1 = -14 \end{array}} \\ & = 5h^2 - 15h + h - 3 \\ & = 5h(h-3) + 1(h-3) \\ & = (h-3)(5h+1) \end{aligned}$$

$$\begin{aligned} \text{m) } 3x^2 + 7xy + 2y^2 & \quad \boxed{\begin{array}{l} 6 \times 1 = 6 \\ 6 + 1 = 7 \end{array}} \\ & = 3x^2 + 6xy + 1xy + 2y^2 \\ & = 3x(x+2y) + y(x+2y) \\ & = (x+2y)(3x+y) \end{aligned}$$

$$\begin{aligned} \text{n) } 2p^2 - 11pq + 5q^2 & \quad \boxed{\begin{array}{l} -10 \times -1 = 10 \\ -10 + -1 = -11 \end{array}} \\ & = 2p^2 - 10pq - 1pq + 5q^2 \\ & = 2p(p-5q) - q(p-5q) \\ & = (p-5q)(2p-q) \end{aligned}$$

$$\begin{aligned} \text{o) } 9x^2 - 9xy - 4y^2 & \quad \boxed{\begin{array}{l} -12 \times 3 = -36 \\ -12 + 3 = -9 \end{array}} \\ & = 9x^2 - 12xy + 3xy - 4y^2 \\ & = 3x(3x-4y) + y(3x-4y) \\ & = (3x-4y)(3x+y) \end{aligned}$$

$$\begin{aligned} \text{p) } 8k^2 - 16k + 6 & \quad \boxed{\begin{array}{l} -6 \times -2 = 12 \\ -6 + -2 = -8 \end{array}} \\ & = 2(4k^2 - 8k + 3) \\ & = 2(4k^2 - 6k - 2k + 3) \\ & = 2[2k(2k-3) - 1(2k-3)] \\ & = 2(2k-3)(2k-1) \end{aligned}$$

$$\begin{aligned} \text{q) } 6m^2 - 14m - 12 & \quad \boxed{\begin{array}{l} -9 \times 2 = -18 \\ -9 + 2 = -7 \end{array}} \\ & = 2(3m^2 - 7m - 6) \\ & = 2(3m^2 - 9m + 2m - 6) \\ & = 2[3m(m-3) + 2(m-3)] \\ & = 2(m-3)(3m+2) \end{aligned}$$

$$\begin{aligned} \text{r) } 10r^2 - 22r + 4 & \quad \boxed{\begin{array}{l} -10 \times -1 = 10 \\ -10 + -1 = -11 \end{array}} \\ & = 2(5r^2 - 11r + 2) \\ & = 2(5r^2 - 10r - 1r + 2) \\ & = 2[5r(r-2) - 1(r-2)] \\ & = 2(r-2)(5r-1) \end{aligned}$$

$$\text{s) } 2x^3 + 9x^2 + 4x$$

$$= x(2x^2 + 9x + 4)$$

$$= x(2x^2 + 8x + 1x + 4)$$

$$= x[2x(x+4) + 1(x+4)]$$

$$= x(x+4)(2x+1)$$

$$\begin{array}{l} \underline{8} \times \underline{1} = 8 \\ \underline{8} + \underline{1} = 9 \end{array}$$

$$\text{t) } 5x^2y - 7xy + 2y$$

$$= y(5x^2 - 7x + 2)$$

$$= y(5x^2 - 2x - 5x + 2)$$

$$= y[x(5x-2) - 1(5x-2)]$$

$$= y(5x-2)(x-1)$$

$$\begin{array}{l} \underline{-2} \times \underline{-5} = 10 \\ \underline{-2} + \underline{-5} = -7 \end{array}$$

2) A rectangle has area defined by  $6x^2 + 13x - 8$ .

a) Factor to find algebraic expressions for the length and width

$$A = 6x^2 + 13x - 8$$

$$= 2x(3x+8) - 1(3x+8)$$

$$= (3x+8)(2x-1)$$

$$\text{Length} = 3x+8$$

$$\text{Width} = 2x-1$$

$$\begin{array}{l} \underline{16} \times \underline{-3} = -48 \\ \underline{16} + \underline{-3} = 13 \end{array}$$

Width?

Area is  
 $6x^2 + 13x - 8$

Length?

b) If  $x = 10$  cm, what is the perimeter and area of the rectangle?

$$P = 2l + 2w$$

$$P = 2(3x+8) + 2(2x-1)$$

$$P = 2[3(10)+8] + 2[2(10)-1]$$

$$P = 2(38) + 2(19)$$

$$P = 114 \text{ cm}$$

$$A = lw$$

$$A = (3x+8)(2x-1)$$

$$A = [3(10)+8][2(10)-1]$$

$$A = (38)(19)$$

$$A = 722 \text{ cm}^2$$

### Answers

1) a)  $(2x+5)(x+1)$  b)  $(3y+8)(2y+1)$  c)  $(4k+3)(k+3)$  d)  $(3m+4)(m+2)$  e) not possible f)  $(3q+2)(4q+3)$

g)  $(x-2)(4x-3)$  h)  $5n-6)(n-1)$  i)  $(3b-1)(3b-7)$  j)  $(3y+7)(y-1)$  k)  $(2k+1)(4k-5)$  l)  $(5h+1)(h-3)$

m)  $(3x+y)(x+2y)$  n)  $(2p-q)(p-5q)$  o)  $(3x+y)(3x-4y)$  p)  $2(2k-1)(2k-3)$  q)  $2(3m+2)(m-3)$

r)  $2(5r-1)(r-2)$  s)  $x(2x+1)(x+4)$  t)  $y(5x-2)(x-1)$

2) length is  $3x+8$ ; width is  $2x-1$  b)  $P = 114 \text{ cm}$ ;  $A = 722 \text{ cm}^2$

## 1) Expand

a)  $(x - 4)(x + 4)$

$$= x^2 - (4)^2$$

$$= x^2 - 16$$

b)  $(3y - 2)(3y + 2)$

$$= (3y)^2 - (2)^2$$


$$= 9y^2 - 4$$

c)  $(5x - 1)(5x + 1)$

$$= (5x)^2 - (1)^2$$

$$= 25x^2 - 1$$


d)  $(x + 4)^2$

$$= (x+4)(x+4)$$


$$= x^2 + 4x + 4x + 16$$

$$= x^2 + 8x + 16$$

e)  $(3x + 2)^2$

$$= (3x+2)(3x+2)$$


$$= 9x^2 + 6x + 6x + 4$$

$$= 9x^2 + 12x + 4$$

f)  $(3x + 7y)^2$

$$= (3x+7y)(3x+7y)$$

$$= 9x^2 + 21xy + 21xy + 49y^2$$

$$= 9x^2 + 42xy + 49y^2$$

## 2) Factor

a)  $x^2 - 25$

$$= (x)^2 - (5)^2$$

$$= (x-5)(x+5)$$

b)  $y^2 - 49$

$$= (y)^2 - (7)^2$$

$$= (y-7)(y+7)$$

c)  $9k^2 - 1$

$$= (3k)^2 - (1)^2$$

$$= (3k-1)(3k+1)$$

d)  $16k^2 - 49$

$$= (4k)^2 - (7)^2$$

$$= (4k-7)(4k+7)$$

e)  $25w^2 - 36$

$$= (5w)^2 - (6)^2$$

$$= (5w-6)(5w+6)$$

f)  $4 - 9w^2$

$$= (2)^2 - (3w)^2$$

$$= (2-3w)(2+3w)$$

### 3) Factor

a)  $x^2 - y^2$

$$= (x-y)(x+y)$$

b)  $36x^2 - y^2$

$$= (6x)^2 - (y)^2 \\ = (6x-y)(6x+y)$$

c)  $25r^2 - 36s^2$

$$= (5r)^2 - (6s)^2 \\ = (5r-6s)(5r+6s)$$

d)  $144r^2 - 49s^2$

$$= (12r)^2 - (7s)^2 \\ = (12r-7s)(12r+7s)$$

e)  $121x^2 - 9y^2$

$$= (11x)^2 - (3y)^2 \\ = (11x-3y)(11x+3y)$$

f)  $100r^2 - 81s^2$

$$= (10r)^2 - (9s)^2 \\ = (10r-9s)(10r+9s)$$

### 4) Factor

a)  $x^2 + 14x + 49$

$$\begin{array}{l} \underline{7} \times \underline{7} = 49 \\ \underline{7} + \underline{7} = 14 \\ = (x+7)(x+7) \\ = (x+7)^2 \end{array}$$

b)  $x^2 - 6x + 9$

$$\begin{array}{l} \underline{-3} \times \underline{-3} = 9 \\ \underline{-3} + \underline{-3} = -6 \\ = (x-3)(x-3) \\ = (x-3)^2 \end{array}$$

c)  $x^2 - 8x + 16$

$$\begin{array}{l} \underline{-4} \times \underline{-4} = 16 \\ \underline{-4} + \underline{-4} = -8 \\ = (x-4)(x-4) \\ = (x-4)^2 \end{array}$$

d)  $100 - 20x + x^2$

$$\begin{array}{l} = x^2 - 20x + 100 \\ \underline{-10} \times \underline{-10} = 100 \\ \underline{-10} + \underline{-10} = -20 \\ = (x-10)(x-10) \\ = (x-10)^2 \end{array}$$

e)  $4x^2 - 12xy + 9y^2$

$$\begin{array}{l} \underline{-6} \times \underline{-6} = 36 \\ \underline{-6} + \underline{-6} = -12 \\ = 4x^2 - 6xy - 6xy + 9y^2 \\ = 2x(2x-3y) - 3y(2x-3y) \\ = (2x-3y)(2x-3y) \\ = (2x-3y)^2 \end{array}$$

f)  $49x^2 + 56xy + 16y^2$

$$\begin{array}{l} \underline{7} \times \underline{7} = 49 \\ \underline{7} + \underline{7} = 14 \\ = 49x^2 + 28xy + 28xy + 16y^2 \\ = 7x(7x+4y) + 4y(7x+4y) \\ = (7x+4y)(7x+4y) \\ = (7x+4y)^2 \end{array}$$

### 5) Factor if possible

a)  $2a^2 + 12a + 18$

$$\begin{array}{l} \underline{3} \times \underline{3} = 9 \\ \underline{3} + \underline{3} = 6 \\ = 2(a^2 + 6a + 9) \\ = 2(a+3)(a+3) \\ = 2(a+3)^2 \end{array}$$

b)  $25x^2 - 16y$

NOT factorable

c)  $75x^2 + 210xy + 147y^2$

$$\begin{array}{l} = 3(25x^2 + 70xy + 49y^2) \\ \underline{35} \times \underline{35} = 1225 \\ \underline{35} + \underline{35} = 70 \\ = 3(25x^2 + 35xy + 35xy + 49y^2) \\ = 3[5x(5x+7y) + 7y(5x+7y)] \\ = 3(5x+7y)(5x+7y) \\ = 3(5x+7y)^2 \end{array}$$



$$d) 9x^3y - 16xy^3$$

$$= xy(9x^2 - 16y^2)$$

$$= xy[(3x)^2 - (4y)^2]$$

$$= xy(3x-4y)(3x+4y)$$

$$e) 36m^2 - 96mn + 64n^2$$

$$= 4(9m^2 - 24mn + 16n^2)$$

$$= 4(9m^2 - 12mn - 12mn + 16n^2)$$

$$= 4[3m(3m-4n) - 4n(3m-4n)]$$

$$= 4(3m-4n)(3m-4n)$$

$$= 4(3m-4n)^2$$

$$f) 20x^2 + 20xy + 5y^2$$

$$= 5(4x^2 + 4xy + y^2)$$

$$= 5(4x^2 + 2xy + 2xy + y^2)$$

$$= 5[2x(2x+y) + y(2x+y)]$$

$$= 5(2x+y)(2x+y)$$

$$= 5(2x+y)^2$$

6) Determine the value(s) of  $k$  such that each trinomial is a perfect square.

$$a) x^2 + kx + 16$$

$$= x^2 + kx + (4)^2$$

$$kx = 2(x)(4)$$

$$kx = 8x$$

$$k = \pm 8$$

$$b) 9x^2 + kx + 49$$

$$= (3x)^2 + kx + (7)^2$$

$$kx = 2(3x)(7)$$

$$kx = 42x$$

$$k = \pm 42$$

$$c) x^2 + 4x + k$$

$$4x = 2(\sqrt{x^2})(\sqrt{k})$$

$$4x = 2x\sqrt{k}$$

$$2 = \sqrt{k}$$

$$k = 4$$

$$d) 4x^2 - 12x + k$$

$$= (2x)^2 - 12x + (\sqrt{k})^2$$

$$-12x = 2(2x)(\sqrt{k})$$

$$-12x = 4x\sqrt{k}$$

$$-3 = \sqrt{k}$$

$$k = 9$$

$$e) kx^2 + 40x + 16$$

$$= (x\sqrt{k})^2 + 40x + (4)^2$$

$$2(x\sqrt{k})(4) = 40x$$

$$8\sqrt{k} = 40$$

$$\sqrt{k} = 5$$

$$k = 25$$

$$f) kx^2 - 24xy + 9y^2$$

$$= (x\sqrt{k})^2 - 24xy + (3y)^2$$

$$2(x\sqrt{k})(3y) = -24xy$$

$$6\sqrt{k} = -24$$

$$\sqrt{k} = -4$$

$$k = 16$$

7) Find an algebraic expression for the area of the shaded region in factored form

$$A = (3x+4)^2 - (x-2)^2$$

$$= 9x^2 + 24x + 16 - (x^2 - 4x + 4)$$

$$= 9x^2 + 24x + 16 - x^2 + 4x - 4$$

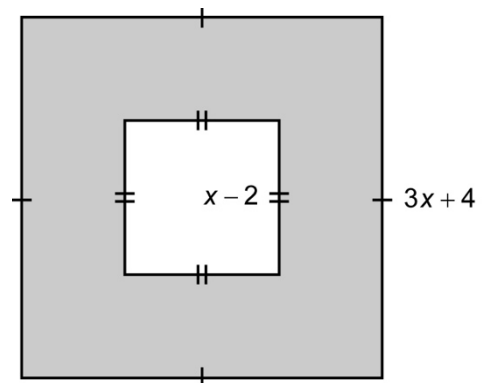
$$= 8x^2 + 28x + 12$$

$$= 4(2x^2 + 7x + 3)$$

$$= 4(2x^2 + 6x + x + 3)$$

$$= 4[2x(x+3) + 1(x+3)]$$

$$= 4(x+3)(2x+1)$$



8) The area of a volleyball court can be represented by the trinomial  $2x^2 - 4x + 2$ .

$$\begin{array}{l} \underline{-1} \times \underline{-1} = 1 \\ \underline{-1} + \underline{-1} = -2 \end{array}$$

a) Factor the trinomial completely

$$\begin{aligned} A &= 2(x^2 - 2x + 1) \\ &= 2(x^2 - x - x + 1) \\ &= 2[x(x-1) - 1(x-1)] \\ &= 2(x-1)^2 \end{aligned}$$

b) If the length of the court is twice the width, use the factors to write the expressions that represent the length and width.

$$\begin{aligned} A &= 2(x-1)(x-1) \\ \text{length} &= 2(x-1) \\ \text{width} &= x-1 \end{aligned}$$

c) If  $x = 10$  m, what are the length and width of the court.

$$\begin{aligned} \text{length} &= 2(10-1) \\ &= 2(9) \\ &= 18 \text{ m} \end{aligned} \qquad \begin{aligned} \text{width} &= 10-1 \\ &= 9 \text{ m} \end{aligned}$$

## Answers

- 1)a)  $x^2 - 16$  b)  $9y^2 - 4$  c)  $25x^2 - 1$  d)  $x^2 + 8x + 16$  e)  $9x^2 + 12x + 4$  f)  $9x^2 + 42xy + 49y^2$   
2)a)  $(x-5)(x+5)$  b)  $(y-7)(y+7)$  c)  $(3k-1)(3k+1)$  d)  $(4k-7)(4k+7)$  e)  $(5w-6)(5w+6)$  f)  $(2-3w)(2+3w)$   
3)a)  $(x-y)(x+y)$  b)  $(6x-y)(6x+y)$  c)  $(5r-6s)(5r+6s)$  d)  $(12r-7s)(12r+7s)$  e)  $(11x-3y)(11x+3y)$   
f)  $(10r-9s)(10r+9s)$   
4)a)  $(x+7)^2$  b)  $(x-3)^2$  c)  $(x-4)^2$  d)  $(10-x)^2$  e)  $(2x-3y)^2$  f)  $(7x+4y)^2$   
5)a)  $2(a+3)^2$  b) not possible c)  $3(5x+7y)^2$  d)  $xy(3x-4y)(3x+4y)$  e)  $4(3m-4n)^2$  f)  $5(2x+y)^2$   
6)a)  $\pm 8$  b)  $\pm 42$  c) 4 d) 9 e) 25 f) 16  
7)  $A = 4(x+3)(2x+1)$   
8)a)  $2(x-1)^2$  b)  $2(x-1), x-1$  c) 18m by 9m