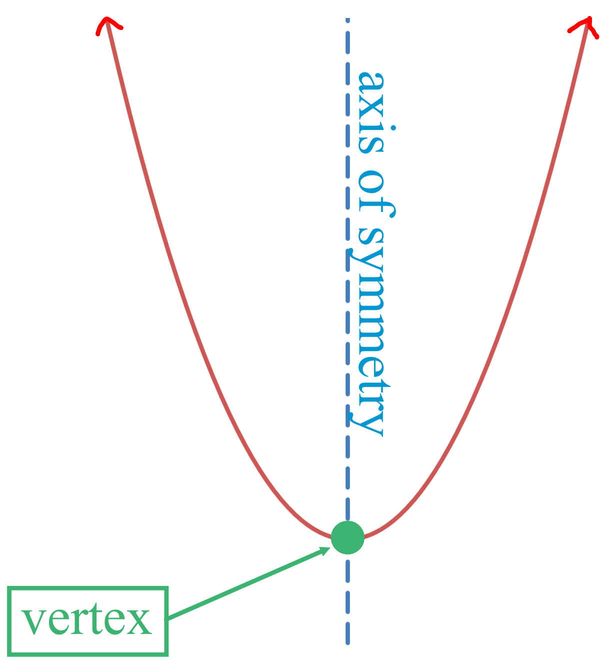
Unit 4 - Quadratics

Lessons

MPM2D



Unit 4 Outline

Unit Goal: By the end of this unit, you will understand the basic properties of a quadratic in standard, vertex, and factored form.

Section	Subject	Learning Goals	Curriculum Expectations
L1	Intro to Quadratics	- understand the shape and key properties of a quadratic function	A1.2,1.3
L2	Vertex Form	- Learn how the parameters $a, h,$ and k transform a quadratic	A2.1,2.2,2.3,2.4
L3	Completing the Square	- Convert from standard form to vertex form	A3.5,3.6
L4	Factored Form	- Understand how the factors of a quadratic relate to the x-intercepts of a quadratic	A3.4

Assessments	F/A/O	Ministry Code	P/O/C	KTAC
Note Completion	A		P	
Practice Worksheet Completion	F/A		P	
Quiz – Completing the Square	F		P	
PreTest Review	F/A		P	
Test – Quadratics	O	A1.2,1.3,2.1-2.4,3.4-3.6	P	K(30%), T(30%), A(30%), C(10%)

L1 – Intro to Quadratics
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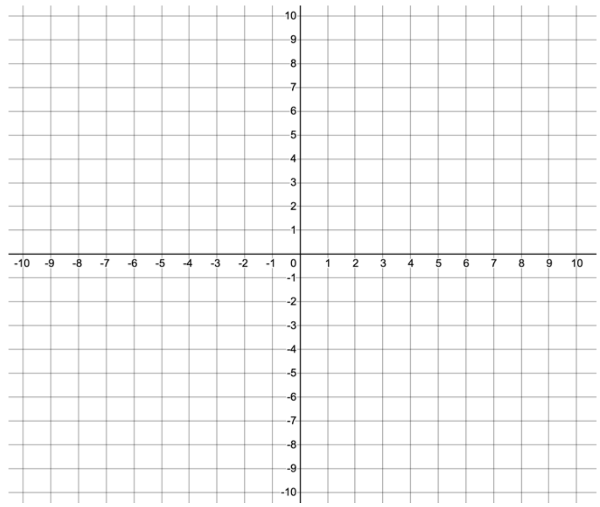
Jensen

Unit 4

Section 1: Properties of Quadratics

The simplest form a ___________ relationship is $y = x$

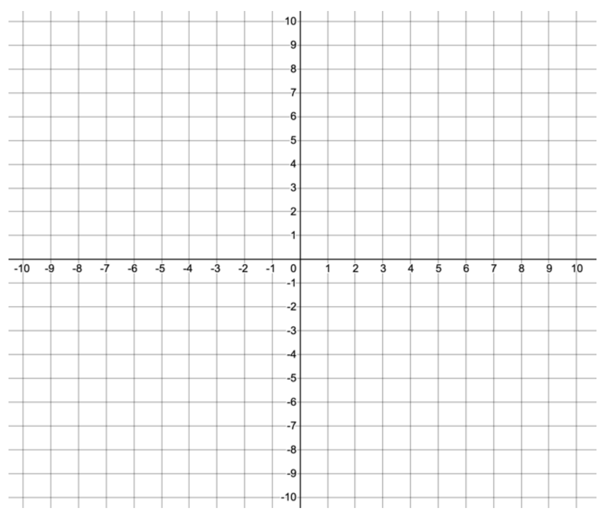
x	y	1st Differences
-3		
-2		
-1		
0		
1		
2		
3		



Notice that the column of 1st finite differences is ___________ for linear relationships.

The simplest form a ___________ relationship is $y = x^2$

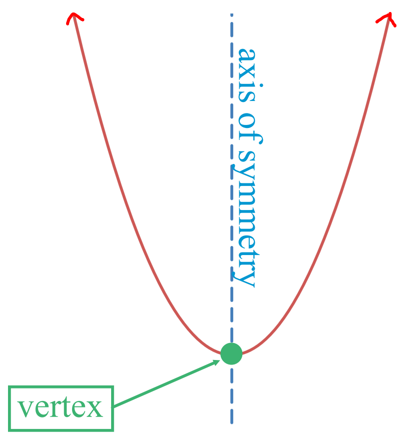
x	y	1st Differences	2nd Differences
-3			
-2			
-1			
0			
1			
2			
3			

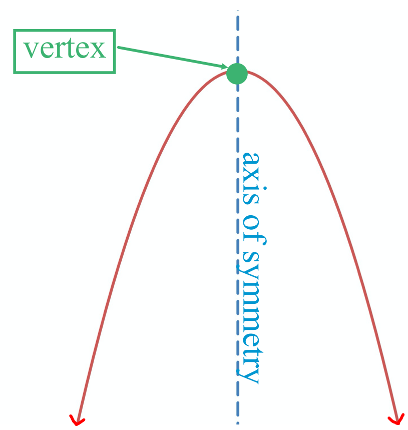


Notice that the column of 2nd column of finite differences is ___________ for quadratic relationships.

Properties of Quadratics

- The shape of the graph of a quadratic relation is called a ____________
- A parabola has a maximum or minimum point called a ____________
- If the parabola opens up, the vertex is a ____________ point
- If the parabola opens down, the vertex is a ____________ point
- Parabolas are symmetrical
- The vertical line that passes through the vertex is the ________________________



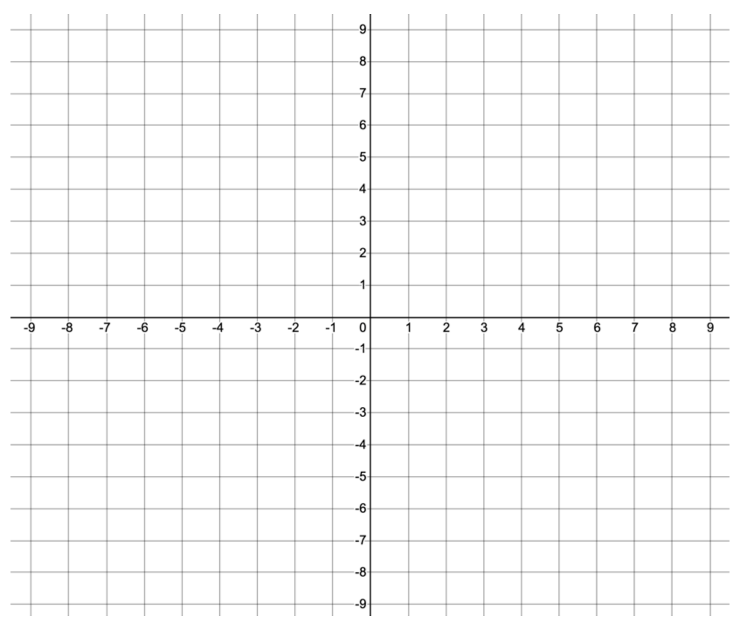


Section 2: Quadratics in Standard Form

The standard form of a quadratic equation is

Example 1: For the function $y = x^2 + 2x + 1$, sketch a graph by completing the given table of values, then state the vertex and axis of symmetry.

x	y
-4	
-3	
-2	
-1	
0	
1	
2	



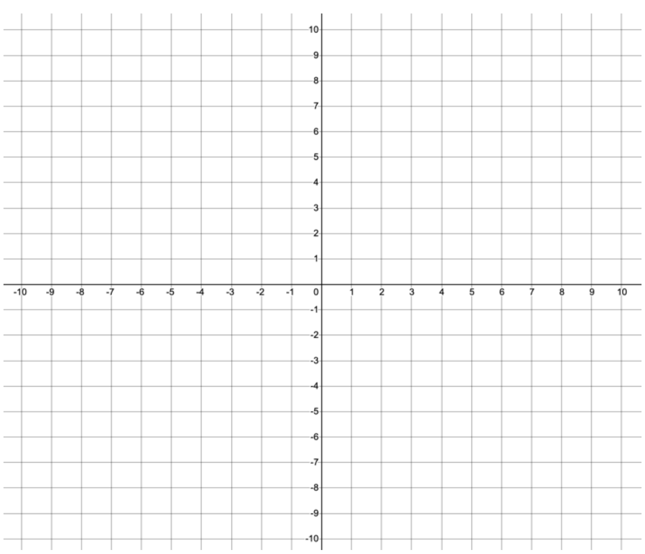
Properties of Quadratics from the Standard Form Equation → $y = ax^2 + bx + c$

- If $a > 0$, the parabola opens ___________
- If $a < 0$, the parabola opens ___________
- The ______________________ is at $(0, c)$

Example 2: State the direction of opening and y-intercept of the given quadratic, then make a table of values and sketch the graph to verify.

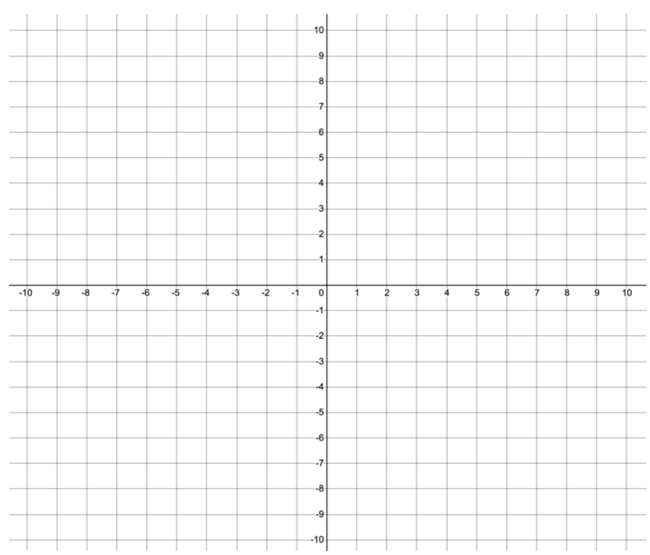
a) $y = -3x^2 + 2$

x	y



b) $y = 2x^2 - 8x + 3$

x	y



L2 –Quadratics in Vertex Form Unit 4
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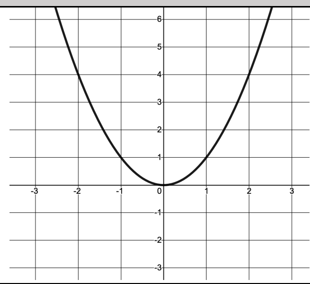
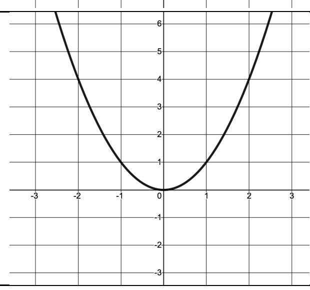
Standard Form: $y = ax^2 + bx + c$

Vertex Form: $y = a(x-h)^2 + k$

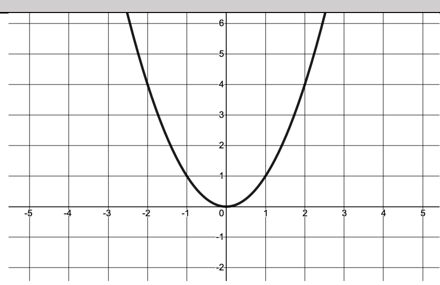
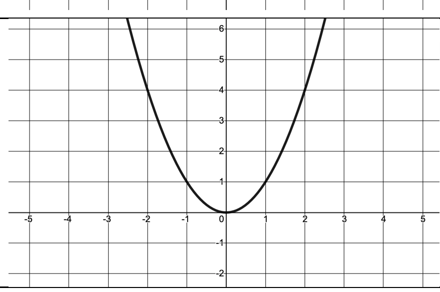
Factored Form: $y = a(x-r)(x-s)$

Part 1: Effects of a, h, and k on transforming the graph of $y = x^2$

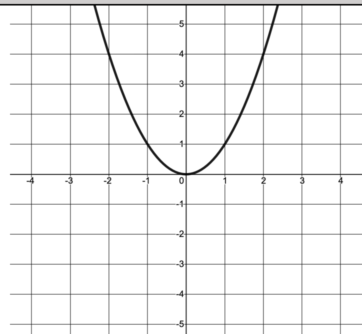
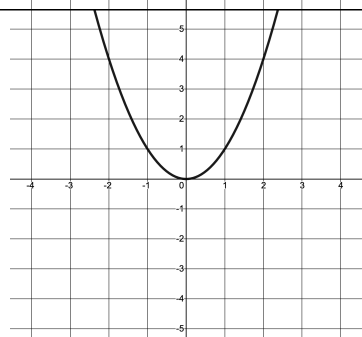
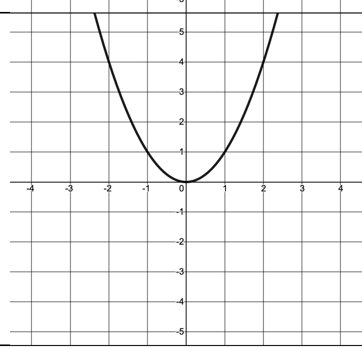
The effects of the k parameter on the graph of $y = x^2 + k$

Function	Graph	Vertex	Axis of Symmetry	Transformations
$y = x^2 + 3$				
$y = x^2 - 2$				

The effects of the h parameter on the graph of $y = (x-h)^2$

Function	Graph	Vertex	Axis of Symmetry	Transformations
$y = (x-2)^2$				
$y = (x+3)^2$				

The effects of the a parameter on the graph of $y = ax^2$

Function	Graph	Vertex	Axis of Symmetry	Transformations
$y = 2x^2$				
$y = \frac{1}{2}x^2$				
$y = -\frac{1}{2}x^2$				

Properties of $y = a(x - h)^2 + k$

$a > 0$ →
$a < 0$ →
$a > 1$ or $a < -1$ →
$-1 < a < 1$ →

$h > 0$ →
$h < 0$ →

$k > 0$ →
$k < 0$ →

Vertex is at

Axis of symmetry is at

The domain (values x may take) of all quadratic functions is $X \in \mathbb{R}$

The range (values y may take) depends on the location of the vertex

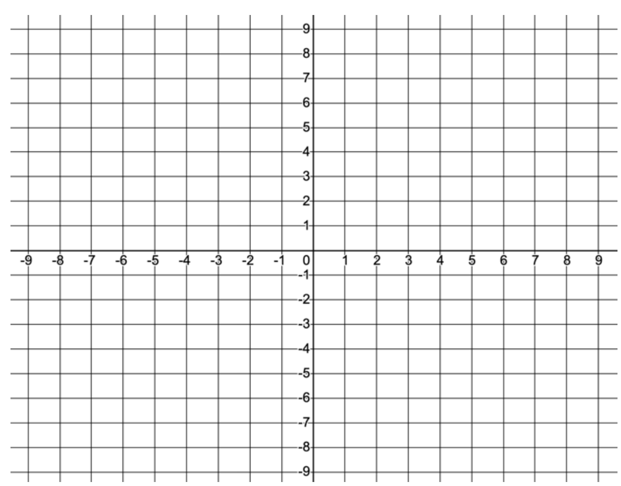
Example 1: For each of the following functions, **i)** describe the transformations compared to $y = x^2$, **ii)** complete the table of properties, **iii)** graph the function by making a table of values

a) $y = -3(x + 2)^2$

Transformations:

Vertex	
Axis of Symmetry	
Direction of Opening	
Values x may take (domain)	
Values y may take (range)	

x	y

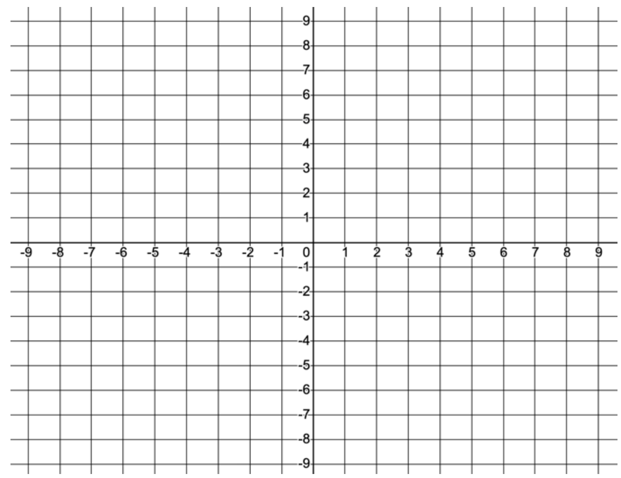


b) $y = 2x^2 - 5$

Transformations:

Vertex	
Axis of Symmetry	
Direction of Opening	
Values x may take (domain)	
Values y may take (range)	

x	y

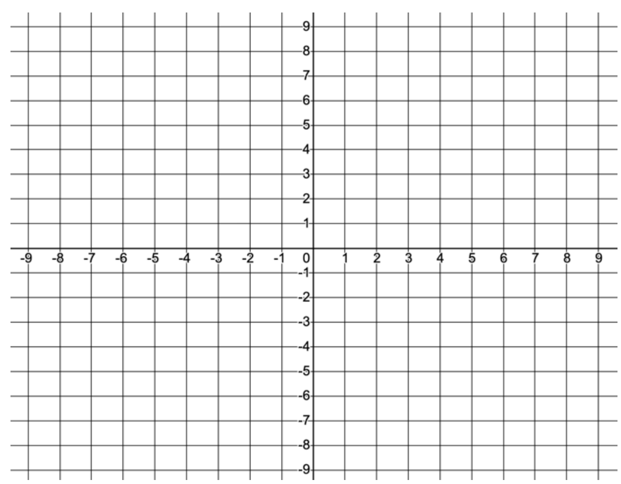


c) $y = 2(x - 3)^2 + 1$

Transformations:

Vertex	
Axis of Symmetry	
Direction of Opening	
Values x may take (domain)	
Values y may take (range)	

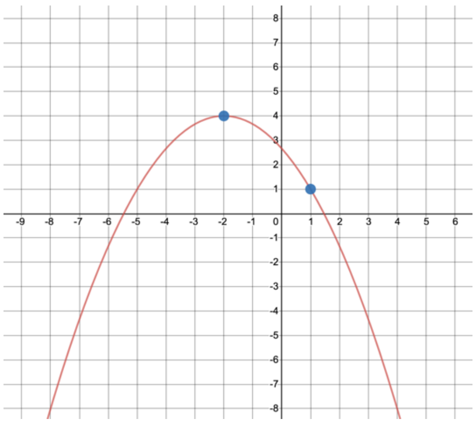
x	y



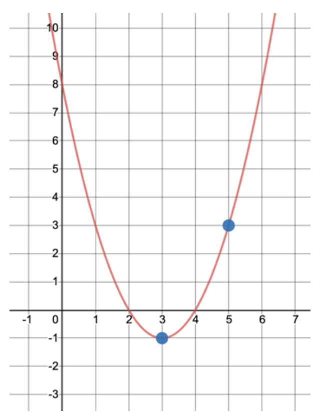
Example 2: Determine the vertex form equation of the parabola with its vertex at $(1,5)$ and passes through the point $(0,2)$

Example 3: Determine the vertex form equation of the following parabolas

a)



b)



Example 4: The graph of $y = x^2$ is reflected vertically in the x-axis, compressed vertically by a factor of $\frac{1}{4}$, shifted 1 unit to the left, and 2 units down. Write the vertex form equation of this parabola.

Example 5: At a fireworks display, a firework is launched from a height of 2 meters above the ground and reaches a max height of 40 meters at a horizontal distance of 10 meters. The firework continues to travel an additional 1 meter horizontally after it reaches its max height before it explodes. What is the height when it explodes?

Part 1: Perfect Square Trinomials

Completing the square is a process for changing a standard form quadratic equation into vertex form

$$y = ax^2 + bx + c \rightarrow y = a(x-h)^2 + k$$

Notice that vertex form contains a $(x-h)^2$. A binomial squared can be obtained when factoring a perfect square trinomial:

$$a^2 + 2ab + b^2 = (a+b)^2$$
$$a^2 + 2ab + b^2 = (a-b)^2$$

The process of completing the square involves creating this perfect square trinomial within the standard form equation so that it can be factored to create the vertex form equation.

Let's start by analyzing the following perfect square trinomials. Specifically notice how the middle term is 2 times the product of the square roots of the first and last terms.

$x^2 + 10x + 25$ $\qquad\qquad$ $x^2 - 12x + 36$

Example 1: Determine the value of k that would make each quadratic a perfect square trinomial. Then factor the trinomial.

a) $x^2 + 14x + k$ $\qquad\qquad$ **b)** $x^2 - 24x + k$

Tip: You can calculate the constant term that makes the quadratic a PST by squaring half of the coefficient of the x term.

Note: *this only works when the coefficient of x^2 is 1.*

Part 2: Completing the Square

Completing the Square Steps

$$ax^2 + bx + c \rightarrow a(x-h)^2 + k$$

1) Put brackets around the first 2 terms
2) Factor out the constant in front of the x^2 term
3) Look at the last term in the brackets, divide it by 2 and then square it
4) Add AND subtract that term behind the last term in the brackets
5) Move the negative term outside the brackets by multiplying it by the $'a'$ value
6) Simplify the terms outside the brackets
7) Factor the perfect square trinomial

$$a^2 + 2ab + b^2 = (a+b)^2$$

Example 2: Rewrite each quadratic in vertex form by completing the square. Then state the vertex, whether it is a max or min point, and the axis of symmetry.

a) $y = x^2 + 2x + 7$

b) $y = 5x^2 - 30x + 41$

c) $y = -5x^2 + 20x + 2$

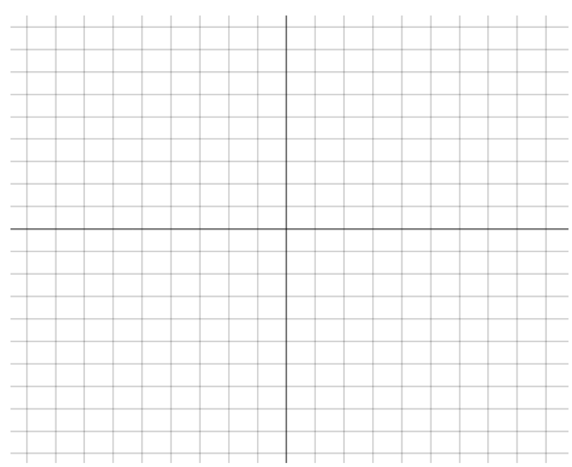
d) $y = 3x^2 + 8x - 5$

Example 3: For each of the following functions, **i)** convert to vertex form by completing the square, **ii)** complete the table of properties, **iii)** graph the function by making a table of values

a) $y = x^2 + 6x + 8$

Vertex	
Axis of Symmetry	
Direction of Opening	
Values x may take (domain)	
Values y may take (range)	

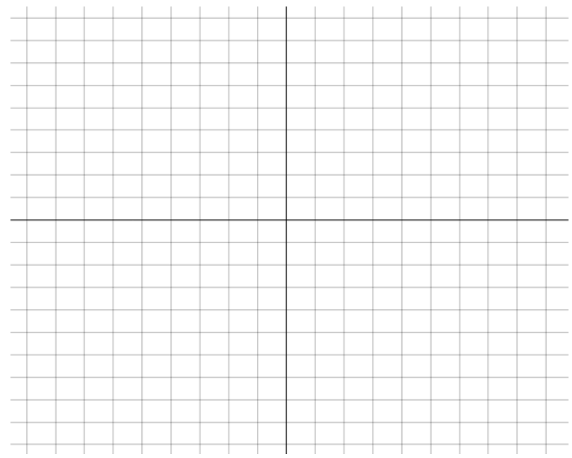
x	y



b) $y = 3x^2 - 24x + 11$

Vertex	
Axis of Symmetry	
Direction of Opening	
Values x may take (domain)	
Values y may take (range)	

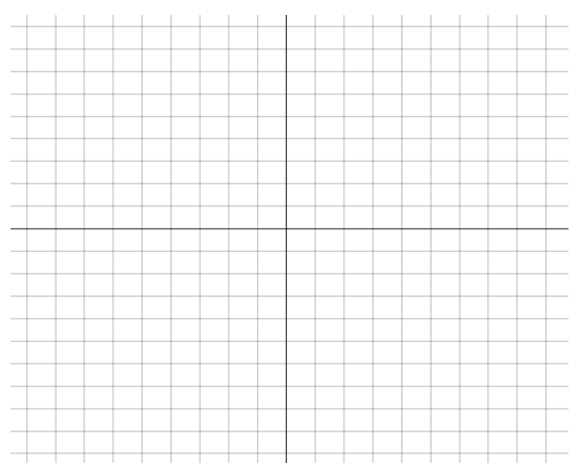
x	y



c) $y = 6x - 3x^2$

Vertex	
Axis of Symmetry	
Direction of Opening	
Values x may take (domain)	
Values y may take (range)	

x	y



L4 –Quadratics in Factored Form Unit 4
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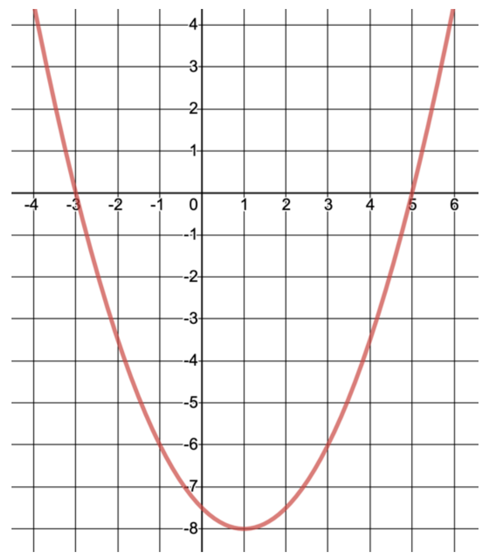
Standard Form: $y = ax^2 + bx + c$

Vertex Form: $y = a(x - h)^2 + k$

Factored Form: $y = a(x - r)(x - s)$

Part 1: Analysis of a Quadratic in Factored Form

Example 1: Given the graph of $y = 2(x + 3)(x - 5)$



a) What are the x-intercepts and how do they relate to the equation?

b) What is the vertex? How does the x-coordinate of the vertex relate to the x-intercepts?

c) What is the equation of the axis of symmetry?

d) What is direction of opening?

Properties of $y = a(x - r)(x - s)$

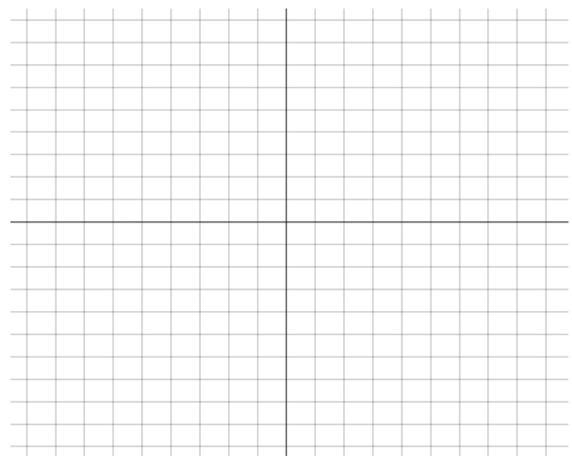
-
-
-
-
-

Example 1: Given the following quadratic equations, determine the **i)** x-intercepts using the zero product rule, **ii)** the axis of symmetry, **iii)** the vertex **iv)** graph the quadratic

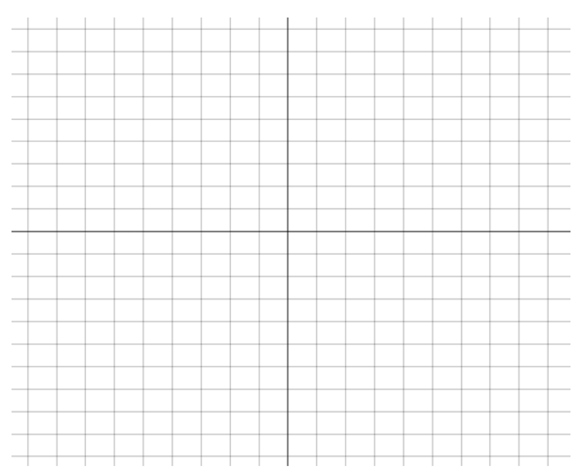
Zero product rule: The product of factors is zero if one or more of the factors are zero.

$ab = 0$ if $a = 0$ or $b = 0$ (or both)

a) $y = 2(x + 1)(x - 3)$

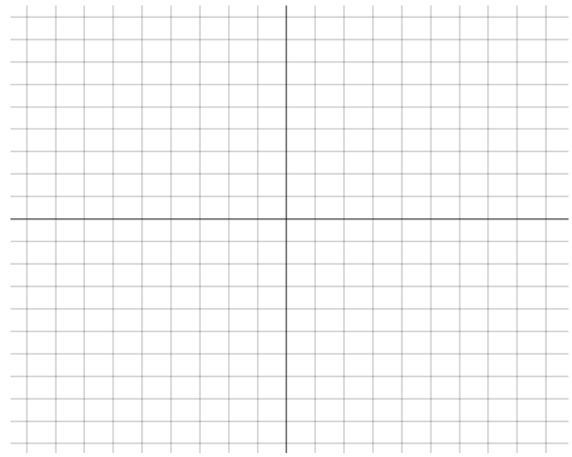


b) $y = \frac{1}{2}(x + 6)(x + 2)$

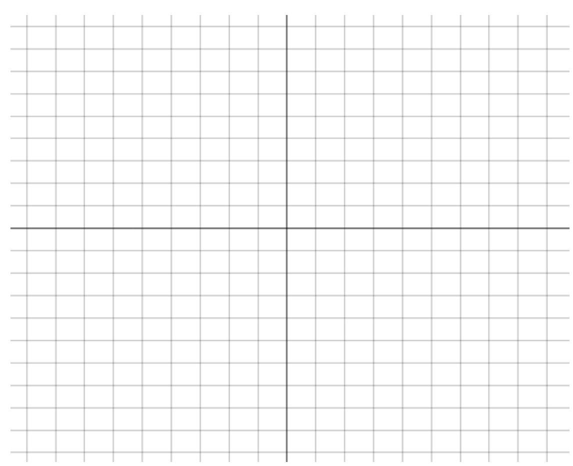


c) $y = x^2 + 2x - 8$

Note: Factor the standard form quadratic in to factored form so that you can more easily find the x-intercepts.



d) $y = x^2 - 9$

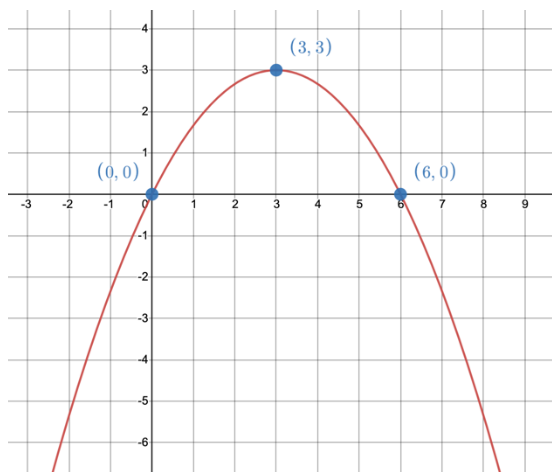


Algorithm for Determining Factored Form Equation from a Graph

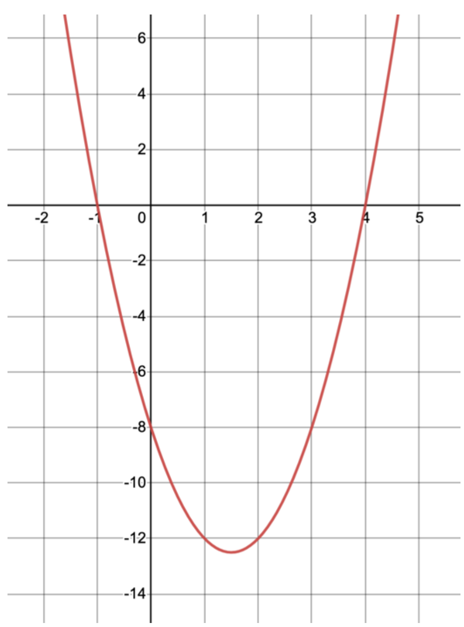
- Find the x-intercepts (r and s)
- Find another point on the graph (x, y)
- Plug the values of $r, s, x,$ and y in to $y = a(x-r)(x-s)$ and solve for a
- Write the final equation by plugging in $a, r,$ and s. NOT x and y.

Example 2: Determine the factored form equation of each of the following quadratic relations.

a)



b)



Example 3: Determine the factored form equation of the parabola with x-intercepts at -3 and -5 and passes through the point $(-4,1)$.