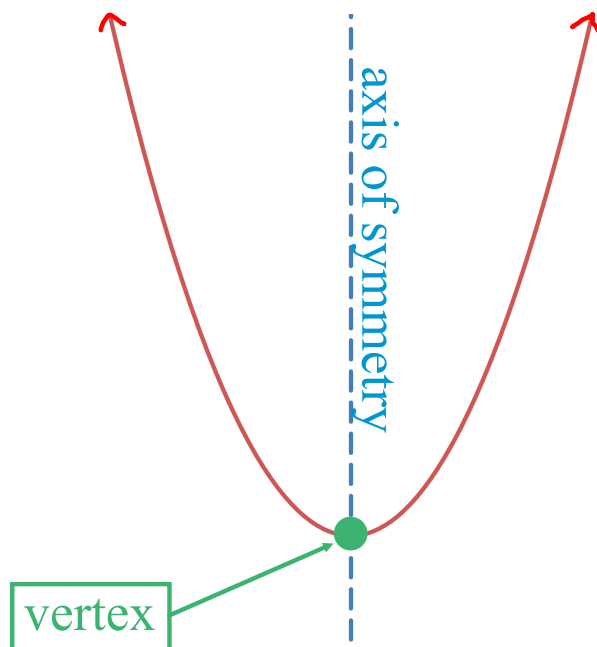


Unit 4 - Quadratics

Workbook

MPM2D



1) Use finite differences to determine if each relation is linear, quadratic, or neither.

a)

x	y
0	4
1	5
2	6
3	7
4	8
0	4

b)

x	y
0	3
1	4
2	7
3	12
4	19
0	3

c)

x	y
1	0
3	1
5	8
7	27
9	64
1	0

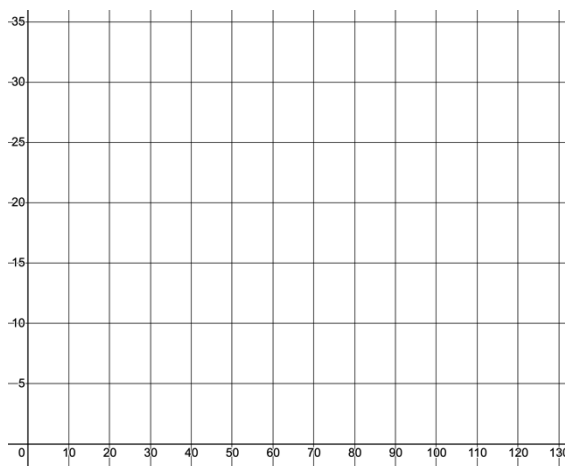
d)

x	y
-2	6
1	0
4	12
7	42
10	90
-2	6

2) The parabolic shape of the Humber River Pedestrian Bridge in Toronto can be approximated by the equation $h = -\frac{1}{144}x^2 + \frac{5}{6}x$, where x is the horizontal distance, in meters, from one end and h is the height, in meters, above the water.

a) Graph the quadratic relation using a table of values

x	y
0	
20	
40	
60	
80	
100	
120	



b) What is the height of the bridge 12 meters horizontally from one end?

c) How wide is the bridge at its base?

d) What is the maximum height of the bridge and at what horizontal distance does it reach that height?

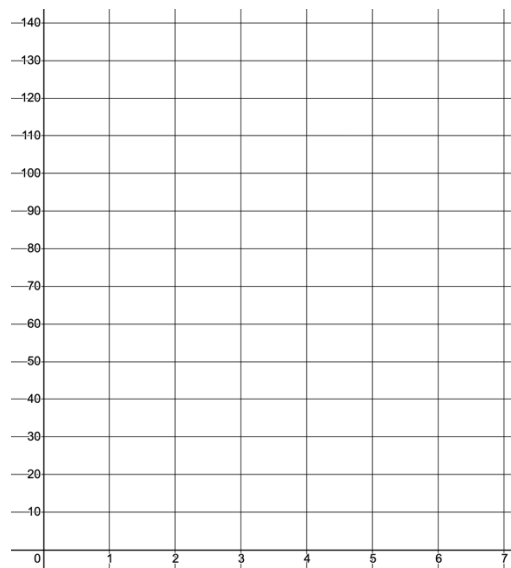
e) Identify the axis of symmetry of the bridge.

3) A flying bird drops a seed that it had picked up off the ground. The height, h , in meters, of the seed above the ground can be modelled by the relation $h = -5t^2 + 125$, where t is seconds since the seed was dropped.

a) At what height was the seed dropped from?

b) Graph the quadratic relation using a table of values.

x	y
0	
1	
2	
3	
4	
5	



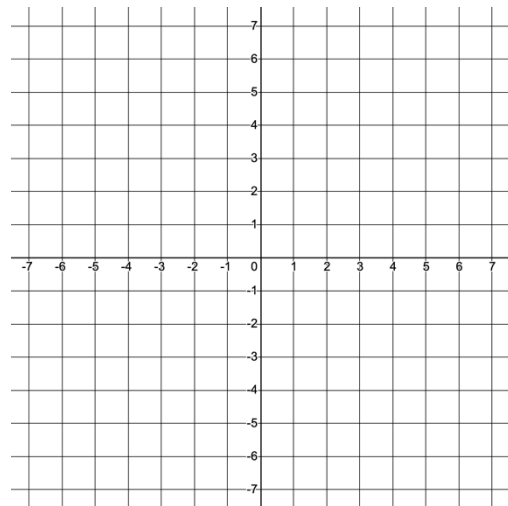
c) How long does it take from when the bird drops the seed until it hits the ground?

4) How can you tell if the vertex of a parabola is a maximum or minimum without graphing?

5) State the direction of opening and y -intercept of the given quadratic, then make a table of values and sketch the graph to verify.

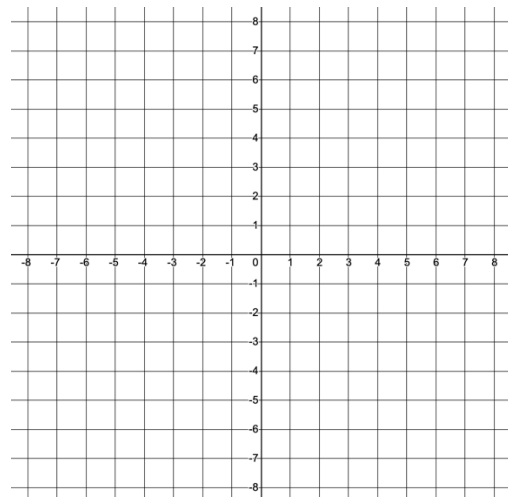
a) $y = x^2 + 2x - 3$

x	y
-4	
-3	
-2	
-1	
0	
1	
2	



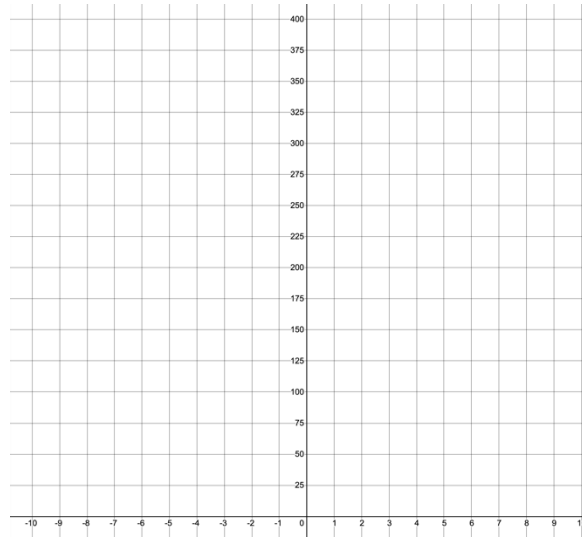
b) $y = -x^2 - 8x - 12$

x	y
-7	
-6	
-5	
-4	
-3	
-2	
-1	



6) An object dropped from the top of the Empire State Building has a height in meters from the ground, y , at any time in seconds, x , according to the formula: $y = -4.84x^2 + 381$

a) Using graphing technology to help you make a rough sketch of the graph of the function.



b) What is the vertex? Interpret the meaning of the vertex in this context.

c) Find the x -intercepts and interpret their meaning in this context.

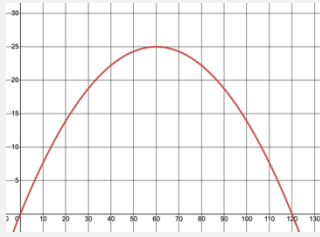
d) How far from the ground is the object after 3 seconds.

Answers

1) a) linear b) quadratic c) neither d) quadratic

2) a)

x	y
0	0
20	13.9
40	22.2
60	25
80	22.2
100	13.9
120	0



b) 9 m c) 120 m d) max height of 25 m at a horizontal distance of 60 m
e) $x = 60$

3) a) 125 m b)

x	y
0	125
1	120
2	105
3	80
4	45
5	0

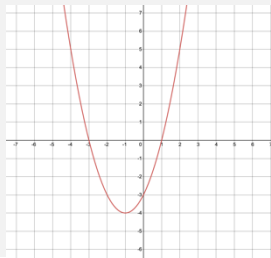


c) 5 seconds

4) By looking at the leading coefficient (the coefficient of the x^2 term). If the leading coefficient is positive, the parabola opens up and has a minimum point. If the leading coefficient is negative, the parabola opens down and has a maximum point.

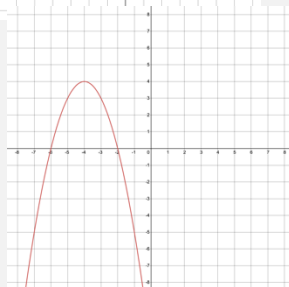
5) a) opens up; y-int at $(-3, 0)$

x	y
-4	5
-3	0
-2	-3
-1	-4
0	-3
1	0
2	5

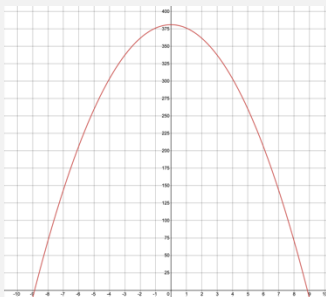


b) opens down; y-int at $(0, -12)$

x	y
-7	-5
-6	0
-5	3
-4	4
-3	3
-2	0
-1	-5



6) a)



b) $(0, 381)$, the object is dropped from a height of 381 meters

c) The x -intercepts are at ± 8.872 . The negative -8.872 x -intercept can be ignored since it does not fit the domain of this scenario. The x -intercept of 8.872 means that it took 8.872 seconds for the object to hit the ground.

d) 337.44 meters

1) Complete the table of properties for each quadratic

a) $y = (x - 4)^2$

b) $y = -3(x + 2)^2 - 5$

Vertex	
Axis of Symmetry	
Direction of Opening	
Values x may take (domain)	
Values y may take (range)	

Vertex	
Axis of Symmetry	
Direction of Opening	
Values x may take (domain)	
Values y may take (range)	

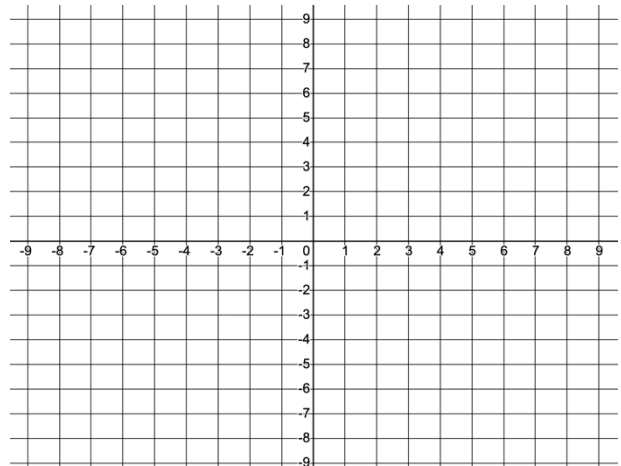
2) Write an equation for the parabola with vertex at $(2,3)$, opening upward, and with no vertical stretch.

3) Write an equation for the parabola with vertex at $(-3,0)$, opening downward, and with a vertical stretch by a factor of 2.

4) Write an equation for the parabola with vertex at $(4, -1)$, opening upward, and with a vertical compression by a factor of $\frac{1}{3}$.

5) The graph of $y = x^2$ is stretched vertically by a factor of 3 and then translated 2 units to the left and 1 unit down. Write the equation of the parabola and then sketch it using a table of values.

x	y



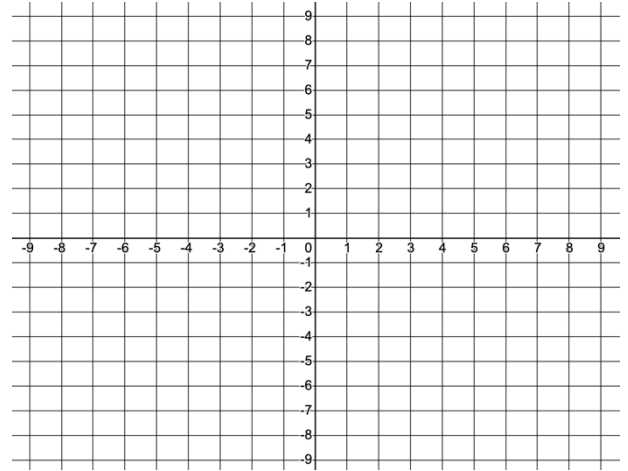
6) For each of the following functions, i) describe the transformations compared to $y = x^2$, ii) complete the table of properties, iii) graph the function by making a table of values

a) $y = -\frac{1}{4}(x - 3)^2 + 6$

Transformations:

Vertex	
Axis of Symmetry	
Direction of Opening	
Values x may take (domain)	
Values y may take (range)	

x	y
1	5
2	5.75
3	6
4	5.75
5	5

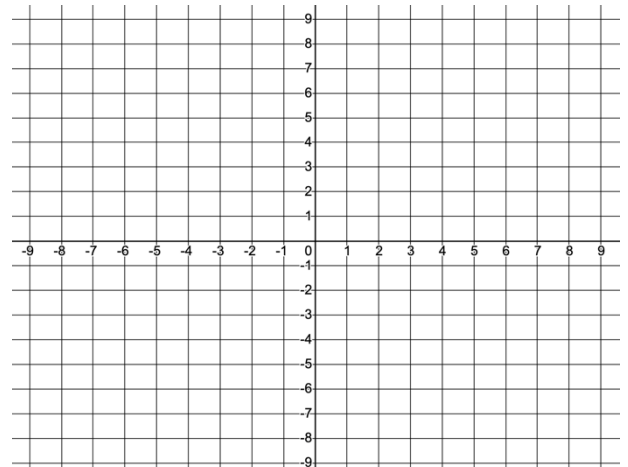


b) $y = 2(x + 1)^2 - 5$

Transformations:

Vertex	$(-1, -5)$
Axis of Symmetry	$x = -1$
Direction of Opening	UP
Values x may take (domain)	$\{X \in \mathbb{R}\}$
Values y may take (range)	$\{Y \in \mathbb{R} y \geq -5\}$

x	y
-3	3
-2	-3
-1	-5
0	-3
1	3

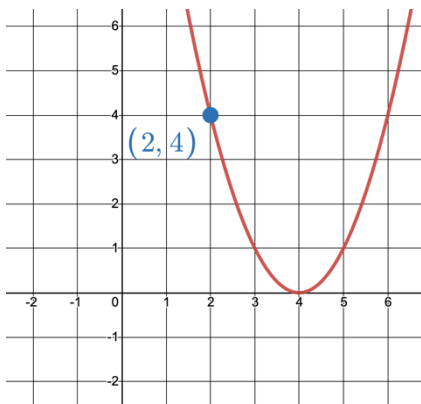


7) The graph of $y = x^2$ is compressed vertically by a factor of $\frac{1}{2}$, reflected in the x -axis, and then translated 2 units up. Write the equation of the parabola.

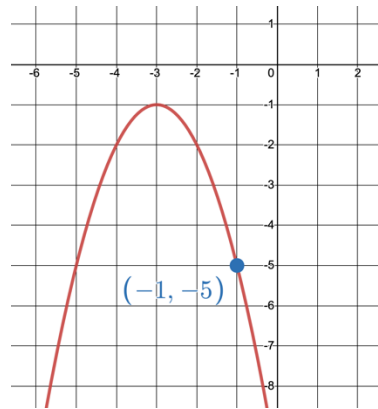
8) Describe the transformations from $y = x^2$ to $y = -5(x + 4)^2 + 7$

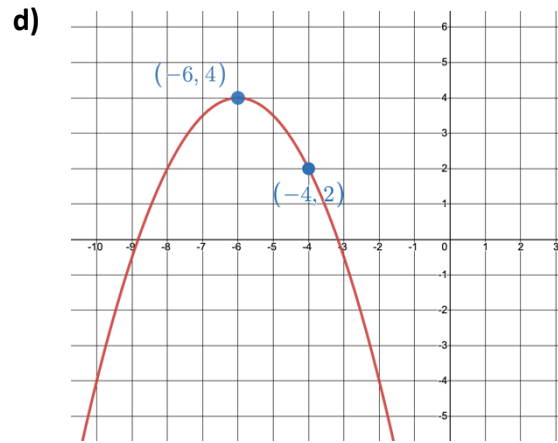
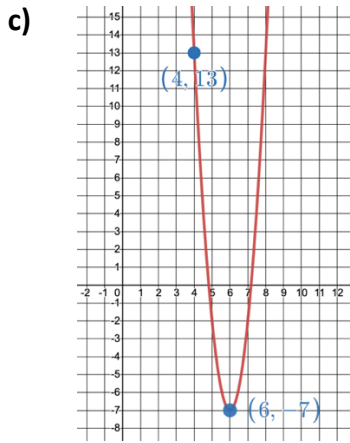
9) Write an equation, in vertex form, for each parabola.

a)



b)





10) A baseball is batted at a height of 1 meter above the ground and reaches a maximum height of 33 meters at a horizontal distance of 4 meters.

a) Determine an equation to model the path of the baseball in vertex form.

b) What is the height of the baseball once it has travelled a horizontal distance of 6 meters?

c) At what other horizontal distance is the baseball at the same height as in part b) ?

11) The flight path of a firework is modeled by the relation $h = -5(t - 5)^2 + 127$, where h is the height, in meters, of the firework above the ground and t is the time, in seconds, since the firework was fired.

a) What is the maximum height reached by the firework? How many seconds after it was fired does the firework reach this height?

b) How high was the firework above the ground when it was fired?

Answers

1)a)

Vertex	(4,0)
Axis of Symmetry	$x = 4$
Direction of Opening	UP
Values x may take (domain)	$\{X \in \mathbb{R}\}$
Values y may take (range)	$\{Y \in \mathbb{R} y \geq 0\}$

b)

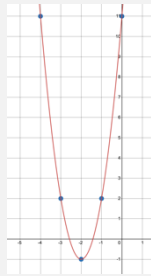
Vertex	(-2,-5)
Axis of Symmetry	$x = -2$
Direction of Opening	Down
Values x may take (domain)	$\{X \in \mathbb{R}\}$
Values y may take (range)	$\{Y \in \mathbb{R} y \leq -5\}$

2) $y = (x - 2)^2 + 3$

3) $y = -2(x + 3)^2$

4) $y = \frac{1}{3}(x - 4)^2 - 1$

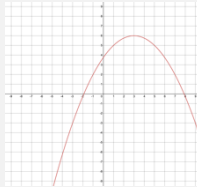
5) $y = 3(x + 2)^2 - 1$;



6)a) Vertical reflection; vertical compression by a factor of $\frac{1}{4}$; shift right 3; shift up 6

Vertex	(3,6)
Axis of Symmetry	$x = 3$
Direction of Opening	Down
Values x may take (domain)	$\{X \in \mathbb{R}\}$
Values y may take (range)	$\{Y \in \mathbb{R} y \leq 6\}$

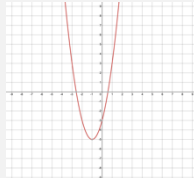
x	y
1	5
2	5.75
3	6
4	5.75
5	5



b) Vertical stretch by a factor of 2; shift left 1 unit; shift down 5 units

Vertex	(-1,-5)
Axis of Symmetry	$x = -1$
Direction of Opening	UP
Values x may take (domain)	$\{X \in \mathbb{R}\}$
Values y may take (range)	$\{Y \in \mathbb{R} y \geq -5\}$

x	y
-3	3
-2	-3
-1	-5
0	-3
1	3



7) $y = -\frac{1}{2}x^2 + 2$

8) Vertical reflection; vertical stretch by a factor of 5; shift left 4 units; shift up 7 units

9)a) $y = (x - 4)^2$ **b)** $y = -(x + 3)^2 - 1$ **c)** $y = -5(x - 4)^2 + 13$ **d)** $y = -\frac{1}{2}(x + 6)^2 + 4$

10)a) $h = -2(d - 4)^2 + 33$ **b)** 25 m **c)** 2 m

11)a) 127 m; 5 seconds **b)** 2 meters

W3 –Completing the Square

Unit 4

MPM2D

Jensen

1) For each quadratic that is in standard form, determine the value of 'c' that makes each expression a perfect square trinomial (remember, the 'c' value is half of the 'b' value squared)

a) $x^2 + 6x + c$

b) $x^2 - 12x + c$

c) $x^2 + 2x + c$

2) Rewrite each relation in the form $y = a(x - h)^2 + k$ by completing the square

a) $y = x^2 + 6x - 1$

b) $y = x^2 + 10x + 20$

c) $y = x^2 - 6x - 4$

d) $y = x^2 - 12x + 8$

e) $y = -x^2 + 80x - 100$

f) $y = 3x^2 + 90x + 50$

g) $y = -7x^2 + 14x - 3$

h) $y = 4x^2 + 64x + 156$

3) Find the maximum or minimum point of each parabola by completing the square.

a) $y = -x^2 - 10x - 9$

b) $y = 2x^2 + 120x + 75$

c) $y = -5x^2 - 200x - 120$

4) The path of a ball is modeled by the equation $y = -x^2 + 4x + 1$, where x is the horizontal distance, in meters, travelled and y is the height, in meters, of the ball above the ground. What is the maximum height of the ball, and at what horizontal distance does it occur?

5) The path of a rocket is given by the equation, $h = -3t^2 + 30t + 73$, where ' h ' is the height in meters and ' t ' is the time in seconds.

a) What is the max height of the rocket

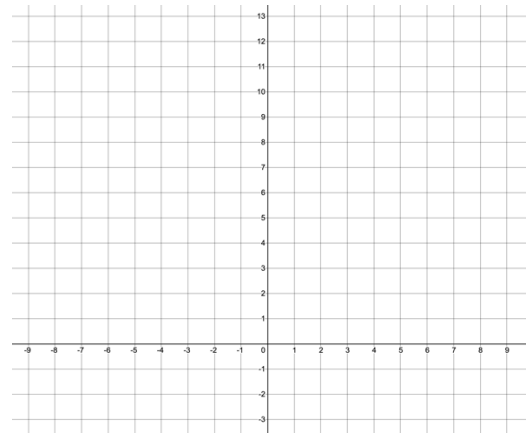
b) At what time does the rocket reach its maximum height

6) For each of the following functions, **i)** convert to vertex form by completing the square, **ii)** complete the table of properties, **iii)** graph the function by making a table of values

a) $y = 2x^2 - 12x + 22$

Vertex	
Axis of Symmetry	
Direction of Opening	
Values x may take (domain)	
Values y may take (range)	

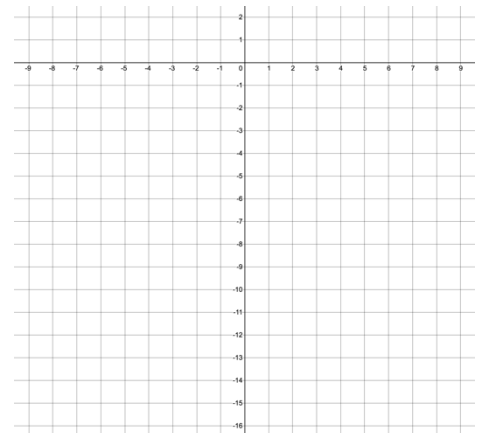
x	y



b) $y = \frac{1}{2}x^2 - 4x - 7$

Vertex	
Axis of Symmetry	
Direction of Opening	
Values x may take (domain)	
Values y may take (range)	

x	y



Answers

1)a) 9 b) 36 c) 1

2)a) $y = (x + 3)^2 - 10$ b) $y = (x + 5)^2 - 5$ c) $y = (x - 3)^2 - 13$ d) $y = (x - 6)^2$

e) $y = -(x - 40)^2 + 1500$ f) $y = 3(x + 15)^2 - 625$ g) $y = -7(x - 1)^2 + 4$ h) $y = 4(x + 8)^2 - 100$

3)a) max at $(-5, 15)$ b) min at $(-30, -1725)$ c) max at $(-20, 1880)$

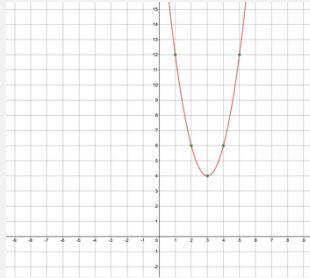
4) max height of 5m occurs at a horizontal distance of 2m

5)a) 148m b) 5 seconds

6)a)

Vertex	$(3, 4)$
Axis of Symmetry	$x = 3$
Direction of Opening	Up
Values x may take (domain)	$\{x \in \mathbb{R}\}$
Values y may take (range)	$\{y \in \mathbb{R} \mid y \geq 4\}$

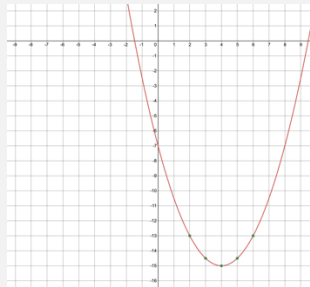
x	y
1	12
2	6
3	4
4	6
5	12



b)

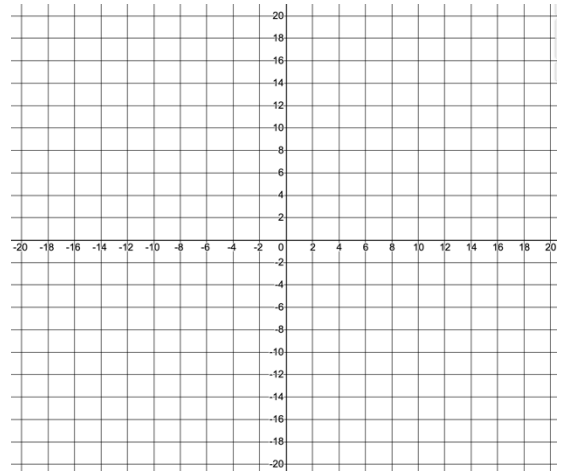
Vertex	$(4, -15)$
Axis of Symmetry	$x = 4$
Direction of Opening	Up
Values x may take (domain)	$\{x \in \mathbb{R}\}$
Values y may take (range)	$\{y \in \mathbb{R} \mid y \geq -15\}$

x	y
2	-13
3	-14.5
4	-15
5	-14.5
6	-13

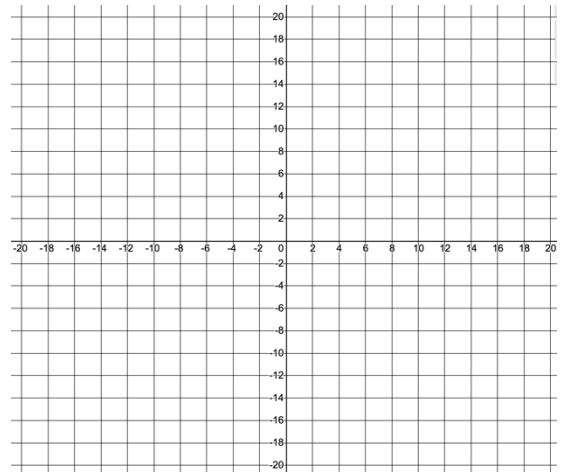


1) Given the following quadratic equations, determine the **i)** x -intercepts using the zero product rule, **ii)** the axis of symmetry, **iii)** the vertex **iv)** graph the quadratic

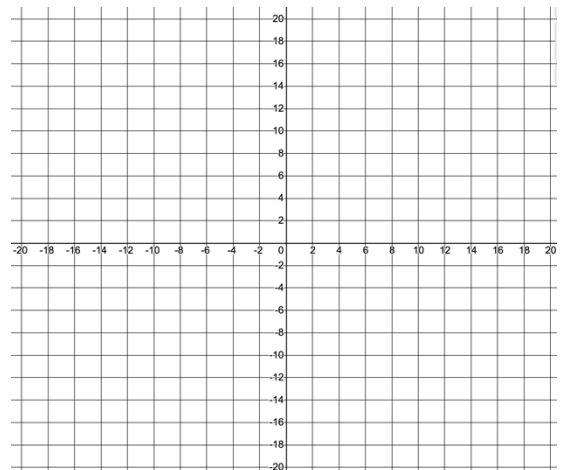
a) $y = (x + 3)(x - 1)$



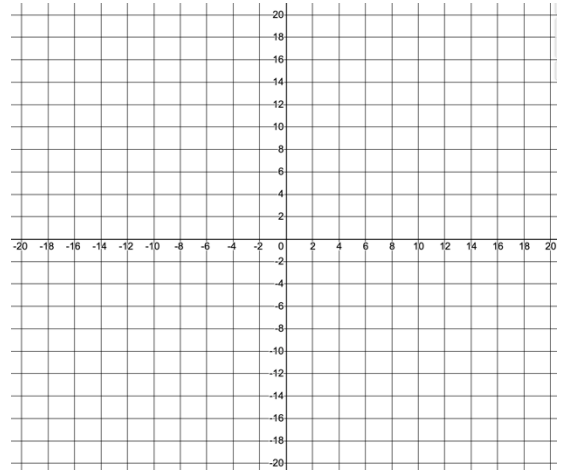
b) $y = 2(x + 4)(x - 2)$



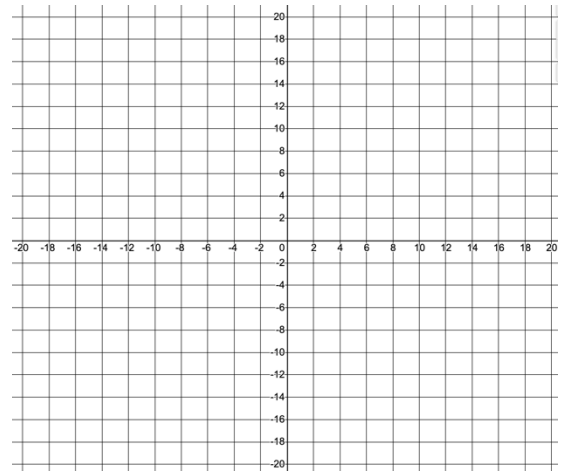
c) $y = -\frac{1}{2}(x + 2)(x - 6)$



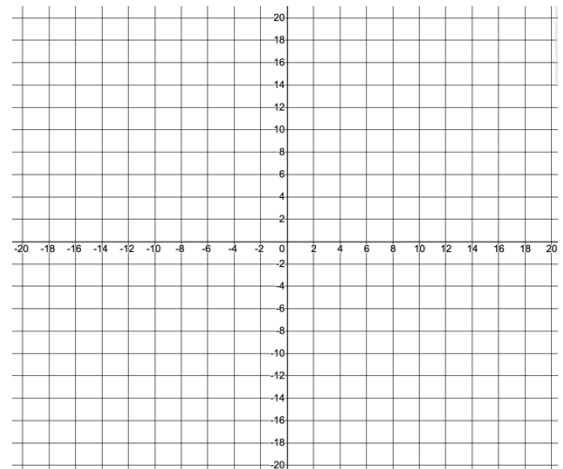
d) $y = 3x(x - 4)$



e) $y = 2x^2 + x - 10$

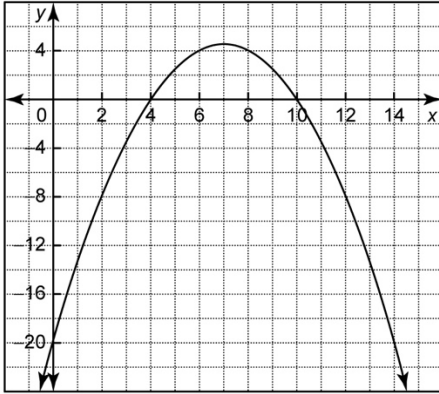


f) $y = \frac{1}{4}(4x - 3)(x - 6)$

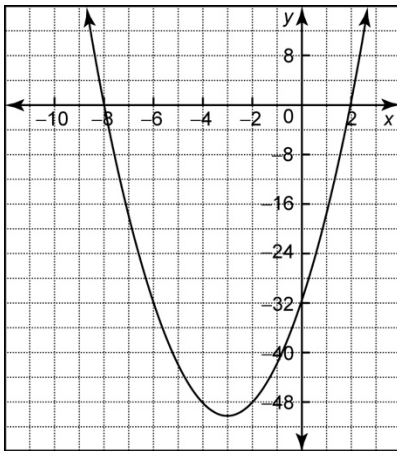


2) Determine an equation in factored form to represent each parabola shown on the graph.

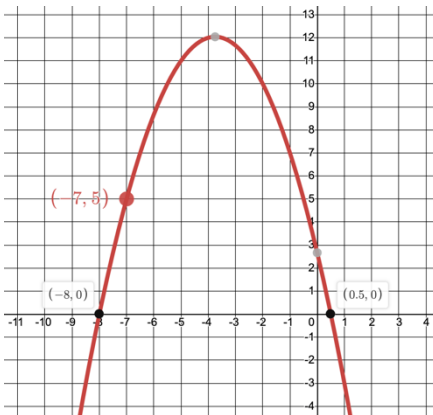
a)



b)



c)



3) A parabola has x -intercepts -2 and -8 , and has vertex $(-5, -18)$. Determine the equation of this parabola in the form $y = a(x - r)(x - s)$

4) A parabola has x -intercepts 3 and 7 , and has vertex $(5, 2)$. Determine the equation of this parabola in factored form.

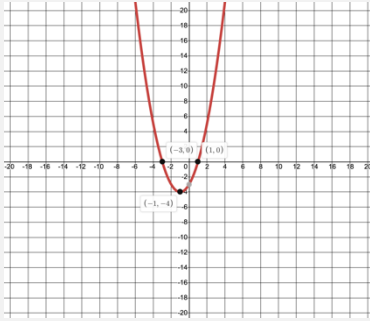
5) Determine the x -intercepts of each of the following functions.

a) $y = x^2 + 5x - 24$

b) $y = x^2 - 11x + 10$

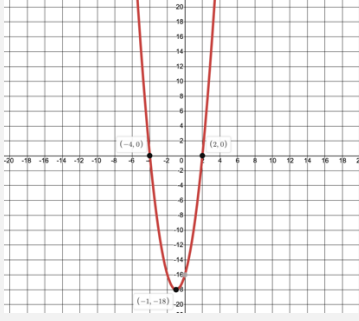
Answers

1)a)



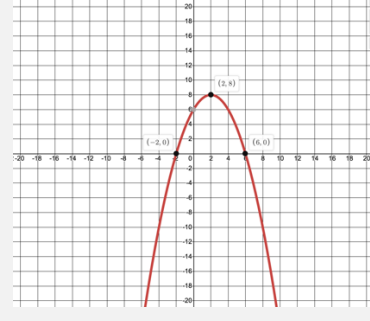
x-int: $(-3,0), (1,0)$
axis of symmetry: $x = -1$
vertex: $(-1, -4)$

b)



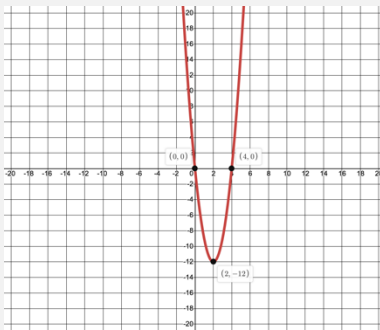
x-int: $(-4,0), (2,0)$
axis of symmetry: $x = -1$
vertex: $(-1, -18)$

c)



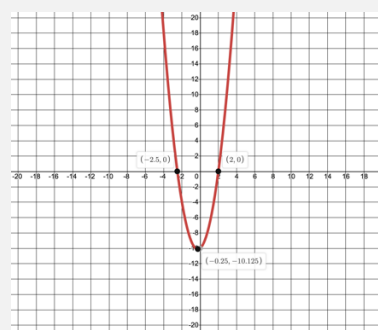
x-int: $(-2,0), (6,0)$
axis of symmetry: $x = 2$
vertex: $(2,8)$

d)



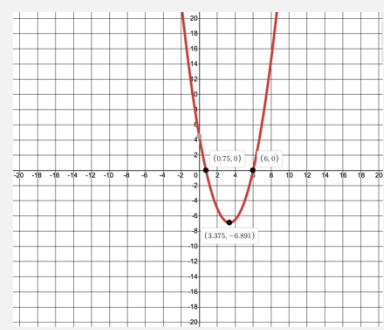
x-int: $(0,0), (4,0)$
axis of symmetry: $x = 2$
vertex: $(2, -12)$

e)



x-int: $(-2.5,0), (2,0)$
axis of symmetry: $x = -0.25$
vertex: $(-0.25, -10.125)$

f)



x-int: $(0.75,0), (6,0)$
axis of symmetry: $x = 3.375$
vertex: $(3.375, -6.891)$

2)a) $y = -\frac{1}{2}(x - 4)(x - 10)$ **b)** $y = 2(x + 8)(x - 2)$ **c)** $y = -\frac{1}{3}(2x - 1)(x + 8)$

3) $y = 2(x + 2)(x + 8)$

4) $y = -\frac{1}{2}(x - 3)(x - 7)$

5)a) $(-8,0)$ and $(3,0)$ **b)** $(10,0)$ and $(1,0)$