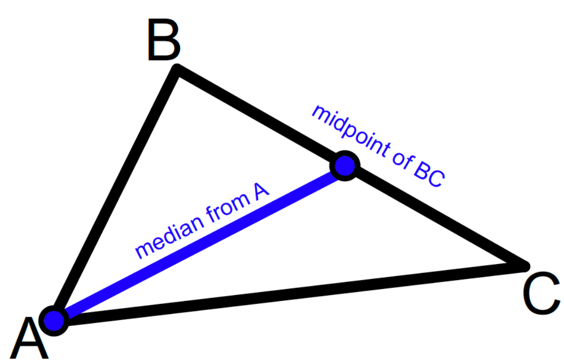
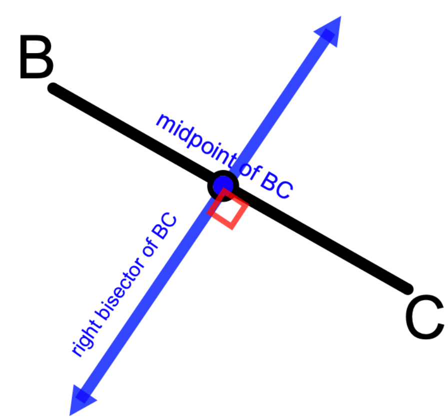
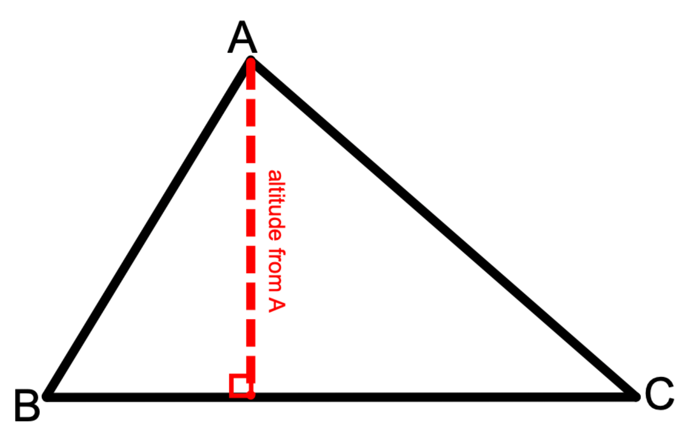
Unit 2- Analytic Geometry

Lesson Book

MPM2D







Unit 2 Outline

Unit Goal: By the end of this unit, you will be able to solve problems using analytic geometry involving properties of lines and line segments.

Section	Subject	Learning Goals	Curriculum Expectations
L1	Midpoint & Length	- calculate the midpoint of a line segment - calculate the length of a line segment	B2.1,2.2
L2	Medians, Right Bisectors, Altitudes	- determine the equation of the median of a triangle from a vertex - determine the equation of the altitude of a triangle from a vertex - determine the equation of a right bisector of a line segment	B2.4
L3	Circles	- determine the equation of a circle - determine the radius of a circle from its equation - know if a point lies inside, on, or outside a circle	B2.3
L4	Applications	- verify geometric properties of shapes using slope, midpoint, and length formulas	B3.2,3.3

Assessments	F/A/O	Ministry Code	P/O/C	KTAC
Note Completion	A		P	
Practice Worksheet Completion	F/A		P	
Quiz – medians, right bisectors, altitudes	F		P	
PreTest Review	F/A		P	
Test – Analytic Geometry	O	B2.1,2.2,2.3,2.4,3.2,3.3	P	K(30%), T(30%), A(30%), C(10%)

L1 – Midpoint and Length of a Line Segment Unit 2
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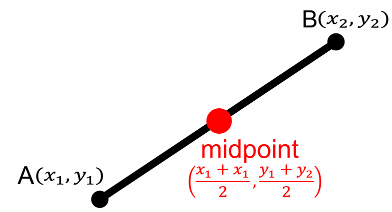
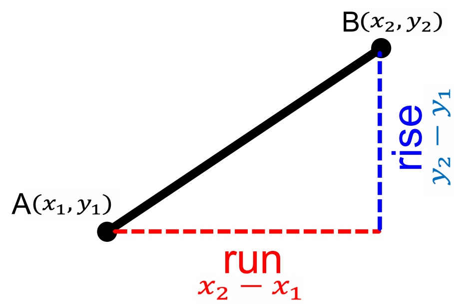
To find the ______________ of a line segment, you must find the middle (average) of both the x and y coordinates of the endpoints. If A has coordinates (x_1, y_1) and B has coordinates (x_2, y_2), then the coordinates of the midpoint of line segment AB are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$



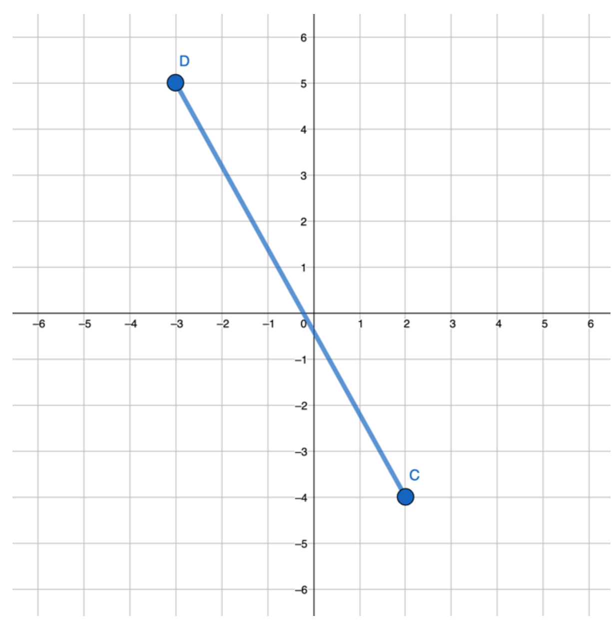
To find the ______________ of a line segment, we first construct a right-angle triangle using the rise and run of the line segment. The run is the difference in the x-coordinates of the endpoints, and the rise is the difference in the y-coordinates of the endpoints. You can then use Pythagorean Theorem, $a^2 + b^2 = c^2$ to calculate the length of the line segment.



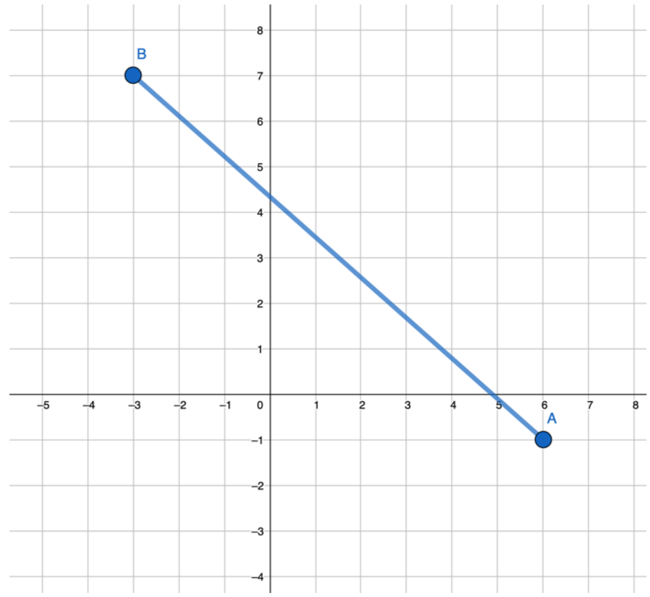
$$(length\ of\ AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$length\ of\ AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 1: Calculate the length and midpoint of the line segment joining the endpoints $C(2, -4)$ and $D(-3,5)$.



Example 2: Calculate the length and midpoint of the line segment joining the endpoints $A(6,-1)$ and $B(-3,7)$.

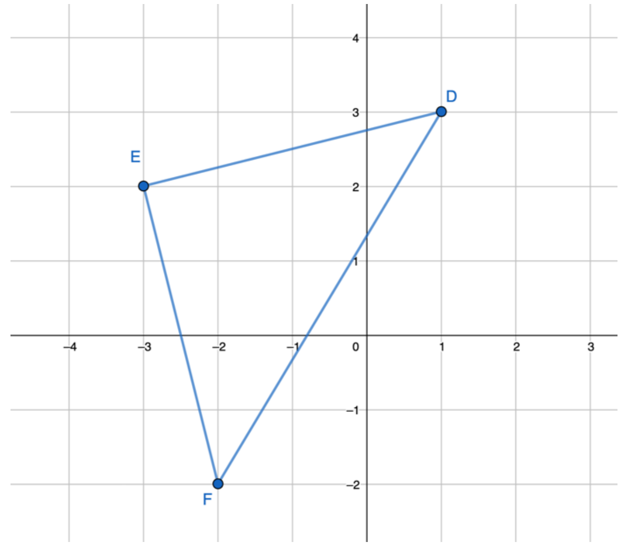


Example 3: Calculate the length and midpoint of the line segment joining the endpoints $E\left(-\frac{5}{8},\frac{1}{8}\right)$ and $F(4,\frac{3}{4})$.

Example 4: If line segment AB has point $A(5,7)$ and a midpoint at $(4,8)$, what are the coordinates of point B?

Example 5: Triangle DEF has vertices $D(1,3)$, $E(-3,2)$, and $F(-2,-2)$.

a) Classify the triangle by side length



b) Determine the perimeter of the triangle rounded to the nearest tenth.

c) Is it a right-angle triangle? Give proof.

L2 – Median, Right Bisector, Altitude Unit 2
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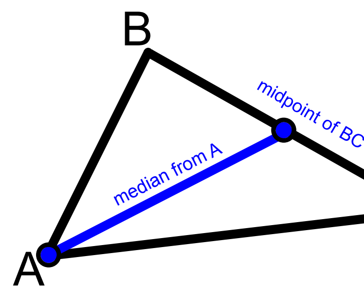
Median of a Triangle:

A median of a triangle is the line segment that joins a vertex to the midpoint of the opposite side.

To find the equation of the median from a vertex:

1) Find the midpoint of the opposite side
2) Find the slope of the line connecting the vertex to the midpoint of the opposite side
3) Calculate the y-intercept of the line
4) Write the equation of the line.

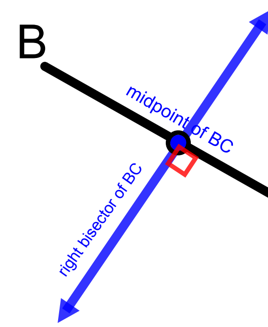


Right Bisector

The line that passes through the midpoint of a line segment and intersects it at a $90°$ *angle.*

To find the equation of the right bisector of line BC:

1) Find the midpoint of BC
2) Find the slope of BC.
3) Find the slope of a line perpendicular to BC
4) Use the slope perpendicular to BC and the midpoint of BC to calculate the y-intercept of the right bisector
5) Write the equation of the right bisector

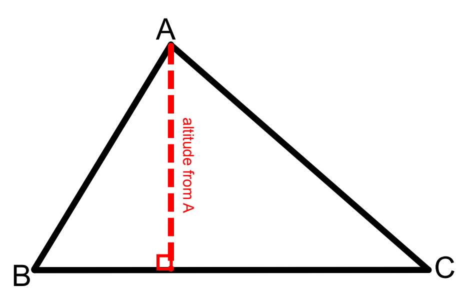


Altitude

An altitude of a triangle is a line segment from a vertex of a triangle to the opposite side, that is perpendicular to that side.

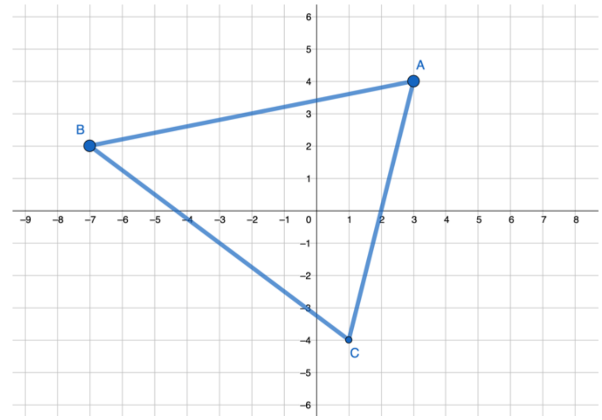
To find the equation of an altitude from a vertex:

1) Find the slope of the side opposite from the vertex
2) Find the slope of the altitude which is perpendicular to the slope of the side opposite from the vertex
3) Use the altitude's slope and the point from the vertex to calculate the y-intercept of the altitude
4) Write the equation of the altitude

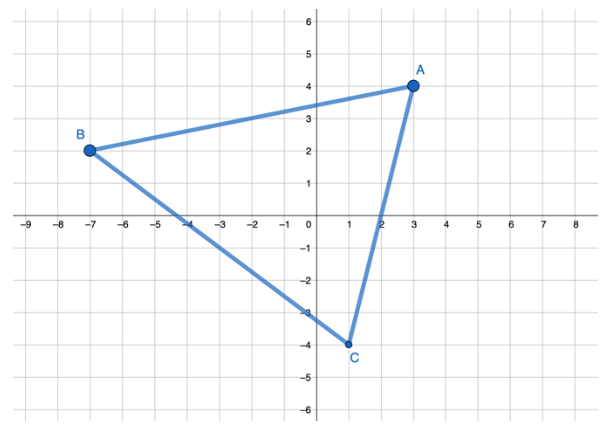


Example 1: ΔABC has vertices $A(3,4)$, $B(-7,2)$, and $C(1,-4)$. Determine…

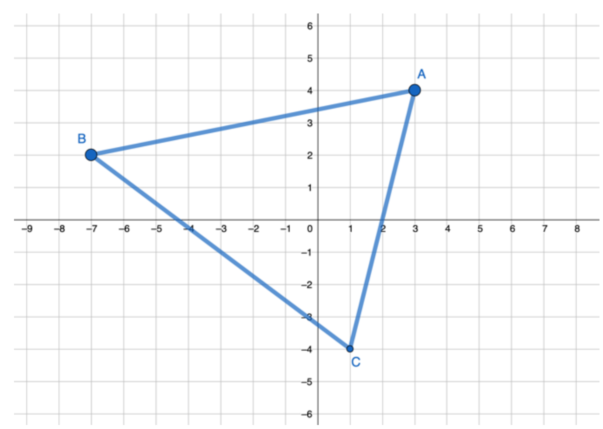
a) an equation for the median from vertex C



b) an equation for the right bisector of AB

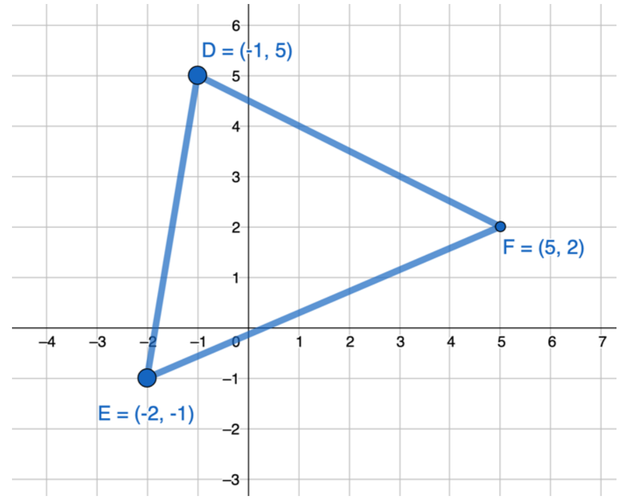


c) an equation for the altitude from vertex C

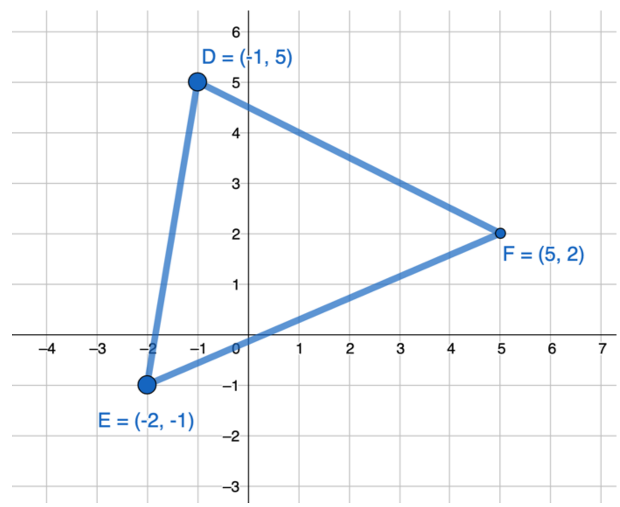


Example 2: ΔDEF has vertices $D(-1,5)$, $E(-2,-1)$, and $F(5,2)$. Determine…

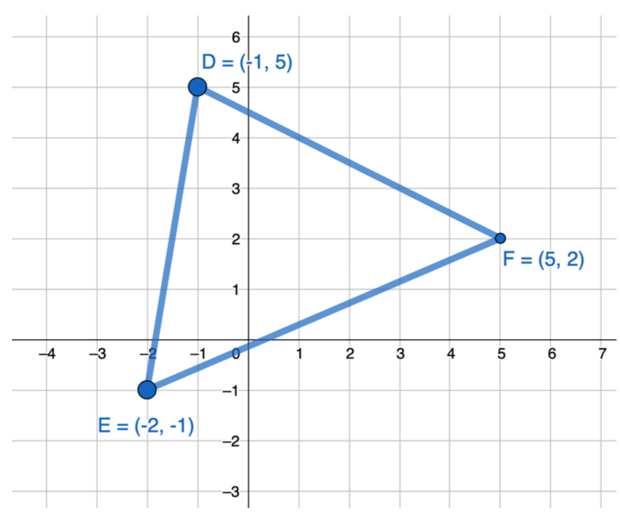
a) an equation for the median from vertex E



b) an equation for the right bisector of DF



c) an equation for the altitude from vertex E



L3 – Equation of a Circle Unit 2
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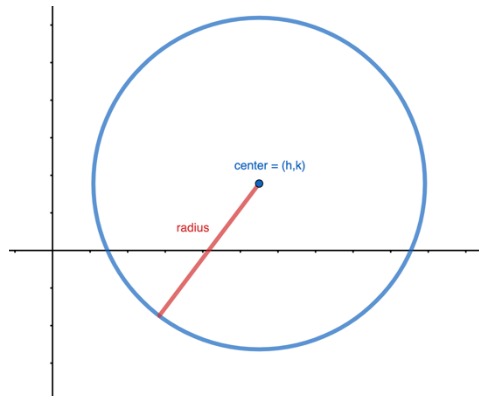
A ________________ is the set of all points that are the same distance from a fixed point, the center.

The ________________ is the distance from the center of the circle to any point on the circle.

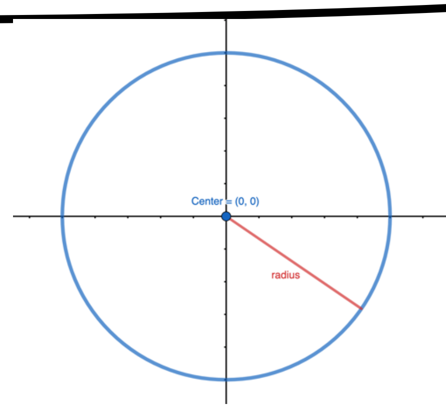
Equation of any circle:

The equation of a circle is defined based on the location of its center (h, k) and length of its radius, r.



Equation of a circle with center at ORIGIN

If the radius of the circle is at the origin $(0,0)$, the equation simplifies.



Example 1: Write the equation of a circle with center $(0,0)$ and a radius of

a) 3

b) $\frac{1}{2}$

Example 2: What is the radius of a circle defined by the equation $x^2 + y^2 = 36$

Example 3: A circle has a center at the origin and passes through the point $(5,3)$. Determine the equation of the circle.

Example 4: Is the point $(-5,9)$ inside, outside, or on the circle $x^2 + y^2 = 100$

Tip:

If point (x, y) is **ON** the circle $\rightarrow x^2 + y^2 = r^2$

If point (x, y) is **OUTSIDE** the circle $\rightarrow x^2 + y^2 > r^2$

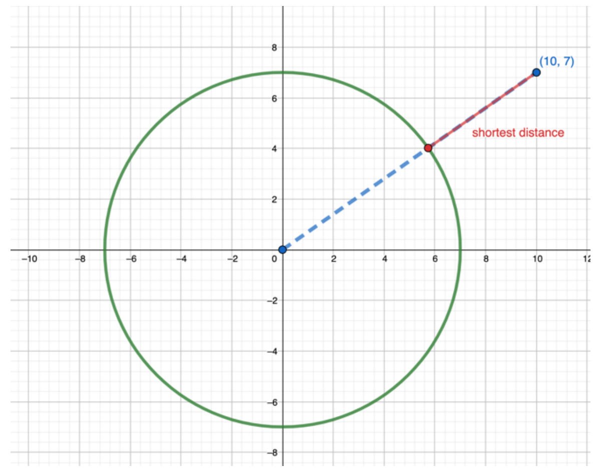
If point (x, y) is **INSIDE** the circle $\rightarrow x^2 + y^2 < r^2$

Example 5: Determine the equation of a circle with center at $(3,4)$ and a radius of 8.

Example 6: Determine the shortest distance from the point $(10,7)$ to the edge of the circle $x^2 + y^2 = 49$

Tip:

The shortest distance is going be in the direction of a line that goes through the center of the circle.



geogebra link

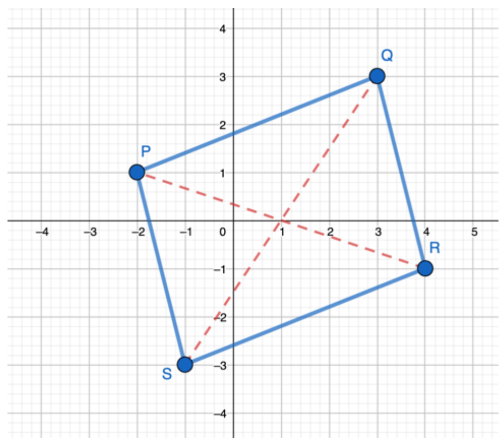
Formulas we will need:

Midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Length $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Equation of a circle: $x^2 + y^2 = r^2$

Example 1: Verify that the diagonals of the parallelogram with vertices $P(-2,1), Q(3,3), R(4,-1)$, and $S(-1,-3)$ bisect each other.



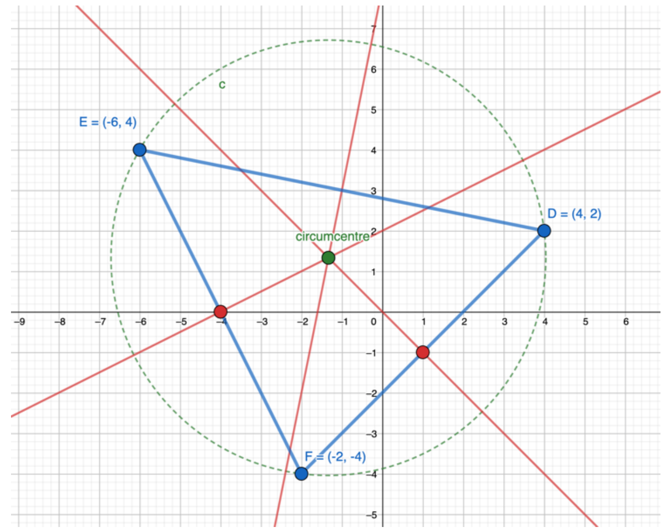
Example 2: The vertices of a triangle are $A(-3,6)$, $B(1,-6)$ and $C(5,2)$. If M is the midpoint of AB and N is the midpoint of AC, verify that

a) MN is parallel to BC

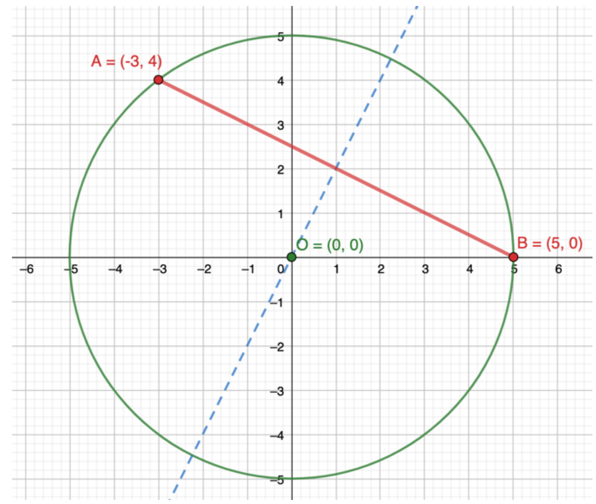
b) MN is half the length of BC

Example 3: ΔDEF has vertices $D(4,2)$, $E(-6,4)$, and $F(-2,-4)$. Determine the coordinates of the circumcentre of ΔDEF. The circumcentre is the point of intersection of the right bisectors of the sides of a triangle.

https://www.geogebra.org/calculator/brabwjsq



Example 4: The equation of a circle with centre $O(0,0)$ is $x^2 + y^2 = 25$. The points $A(-3,4)$ and $B(5,0)$ are the endpoints of chord AB. Verify that the centre of the circle lies on the right bisector of chord AB.



Example 5: Find the distance from the point $P(-1,3)$ to the line $x + y - 5 = 0$, to the nearest tenth of a unit.

Steps to find shortest distance from a point to a line:

1) Write an equation for the line that is perpendicular to the given line and intersects the point given

2) Find the point of intersection of the perpendicular line with the given line

3) Find the distance between the POI and the given point.