Unit 2 Pretest Review Unit 2
MPM2D
Jensen

1) Find the coordinates of the midpoint of each line segment.

a) $P(2,-7)$ and $Q(-3,5)$

$mid_{PQ} = \left(\frac{2+(-3)}{2}, \frac{-7+5}{2}\right)$

$= \left(\frac{-1}{2}, -1\right)$

b) $S(6,-2)$ and $T(2,2)$

$mid_{ST} = \left(\frac{6+2}{2}, \frac{-2+2}{2}\right)$

$= (4,0)$

c) $M(2,-5)$ and $N(5,-1)$

$mid_{MN} = \left(\frac{2+5}{2}, \frac{-5+(-1)}{2}\right)$

$= \left(\frac{7}{2}, -3\right)$

d) $A\left(\frac{7}{2},\frac{1}{2}\right)$ and $B\left(-\frac{5}{3},\frac{3}{2}\right)$

$mid_{AB} = \left(\frac{\frac{7}{2}+\left(\frac{-5}{3}\right)}{2}, \frac{\frac{1}{2}+\frac{3}{2}}{2}\right)$

$= \left(\frac{\frac{21}{6}+\frac{-10}{6}}{2}, \frac{\frac{4}{2}}{2}\right)$

$= \left(\frac{11}{12}, 1\right)$

2) For a line segment KL, one endpoint is $K(5,1)$ and the midpoint is $M(1,4)$. Find the coordinates of endpoint L.

$(1,4) = \left(\frac{5+x}{2}, \frac{1+y}{2}\right)$

$1 = \frac{5+x}{2}$ $\quad$ $4 = \frac{1+y}{2}$

$2 = 5+x$ $\quad$ $8 = 1+y$

$x = -3$ $\quad$ $y = 7$

The other endpoint is $(-3,7)$

3) Find the exact length of the line segment joining each pair of points.

a) $A(7,9)$ and $B(1,1)$

$length_{AB} = \sqrt{(1-7)^2+(1-9)^2}$

$= \sqrt{100}$

$= 10$

b) $W(4,5)$ and $X(-2,3)$

$length_{WX} = \sqrt{(-2-4)^2+(3-5)^2}$

$= \sqrt{40}$

$= 2\sqrt{10}$

c) $E(-2,8)$ and $F(-5,5)$

$length_{EF} = \sqrt{[-5-(-2)]^2+(5-8)^2}$

$= \sqrt{18}$

$= 3\sqrt{2}$

d) $R(-10,5)$ and $T(4,-1)$

$length_{RT} = \sqrt{[4-(-10)]^2+(-1-5)^2}$

$= \sqrt{232}$

$= 2\sqrt{58}$

4) ΔABC has vertices $A(4,5)$, $B(-1,2)$, and $C(5,1)$.

a) Classify the triangle by side length.

$$\text{length}_{AB} = \sqrt{(-1-4)^2 + (2-5)^2} = \sqrt{34}$$

$$\text{length}_{BC} = \sqrt{[5-(-1)]^2 + (1-2)^2} = \sqrt{37}$$

$$\text{length}_{AC} = \sqrt{(5-4)^2 + (1-5)^2} = \sqrt{17}$$

All 3 sides are different lengths; it is a SCALENE triangle.

b) Determine the perimeter of the triangle, to the nearest tenth.

$$P = \sqrt{34} + \sqrt{37} + \sqrt{17}$$

$$\simeq 16.0 \text{ units}$$

5) ΔABC has vertices $A(10,2)$, $B(4,-2)$, and $C(-4,6)$. Draw and determine the equation of…

a) the median from vertex C

$$\text{mid}_{AB} = \left(\frac{10+4}{2}, \frac{2+(-2)}{2}\right) = (7,0)$$

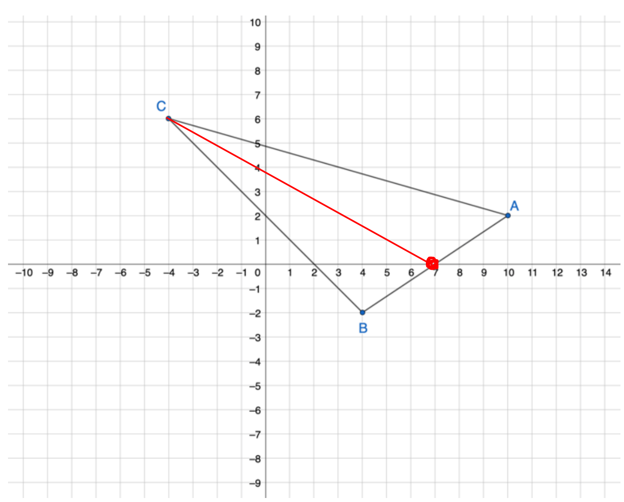
$$\text{slope} = \frac{0-6}{7-(-4)} = \frac{-6}{11}$$

Eq^n: $y = mx + b$

$$0 = \frac{-6}{11}(7) + b$$

$$b = \frac{42}{11}$$

$$\boxed{y = \frac{-6}{11}x + \frac{42}{11}}$$



b) the right bisector of AB

$mid_{AB} = (7,0)$

$slope_{AB} = \frac{-2-2}{4-10} = \frac{-4}{-6} = \frac{2}{3}$

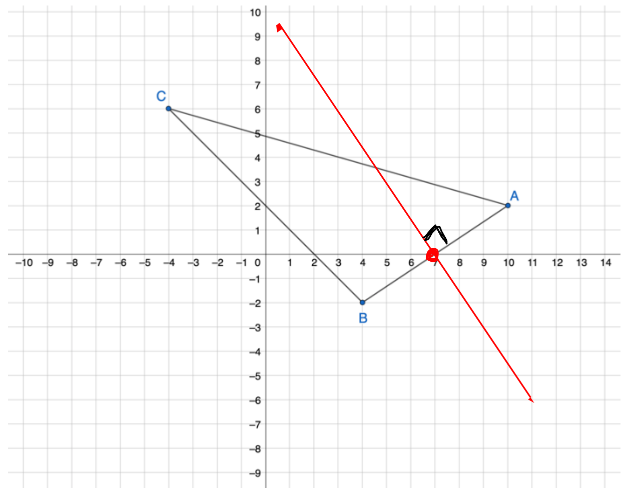
slope of RB $= -\frac{3}{2}$

Eq^n: $y = mx+b$

$0 = -\frac{3}{2}(7) + b$

$b = \frac{21}{2}$

$$\boxed{y = \frac{-3}{2}x + \frac{21}{2}}$$



c) The altitude from vertex B

$slope_{AC} = \frac{2-6}{10-(-4)} = \frac{-4}{14} = \frac{-2}{7}$

slope of altitude $= \frac{7}{2}$

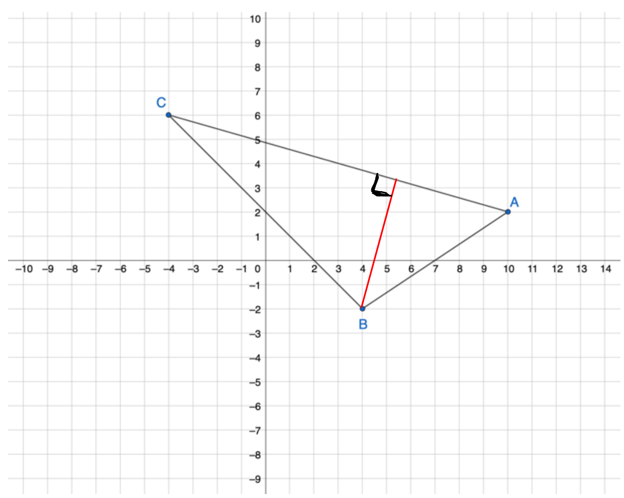
Eq^n: $y = mx+b$

$-2 = \left(\frac{7}{2}\right)(4) + b$

$-2 = 14 + b$

$b = -16$

$$\boxed{y = \frac{7}{2}x - 16}$$



6) A triangle has vertices $A(1,4)$, $B(-3,-2)$, and $C(3,0)$. Determine the exact length of the median from vertex B.

$mid_{AC} = \left(\frac{1+3}{2}, \frac{4+0}{2}\right) = (2,2)$

length from $B(-3,-2)$ to $(2,2)$ $= \sqrt{[2-(-3)]^2 + [2-(-2)]^2} = \sqrt{41}$ units

7) Write an equation for the circle with center $(0,0)$ and the given radius.

a) radius 12

$x^2+y^2=12^2$

$\boxed{x^2+y^2=144}$

b) radius 20

$x^2+y^2=20^2$

$\boxed{x^2+y^2=400}$

c) radius $\sqrt{87}$

$x^2+y^2=(\sqrt{87})^2$

$\boxed{x^2+y^2=87}$

8) The following equations model circles with center $(0,0)$. Determine the radius of each circle. Round to the nearest tenth, if necessary.

a) $x^2+y^2=121$

$r^2=121$

$r=\sqrt{121}$

$\boxed{r=11}$

b) $x^2+y^2=20$

$r^2=20$

$r=\sqrt{20}$

$r=2\sqrt{5}$

$\boxed{r\simeq 4.5}$

c) $x^2+y^2=0.49$

$r^2=0.49$

$r=\sqrt{0.49}$

$\boxed{r=0.7}$

9) Find the equation of the circle that is centered at the origin and passes through the point $(5,-1)$

$x^2+y^2=r^2$

$(5)^2+(-1)^2=r^2$

$26=r^2$

$\boxed{x^2+y^2=26}$

10) Do each of the following points line inside, outside, or on the circle defined by $x^2+y^2=58$

a) $(5,5)$

$5^2+5^2 \overset{?}{=} 58$

$50<58$

$\boxed{\text{inside}}$

b) $(-3,7)$

$(-3)^2+7^2 \overset{?}{=} 58$

$58=58$

$\boxed{\text{on}}$

c) $(8,1)$

$8^2+1^2 \overset{?}{=} 58$

$65>58$

$\boxed{\text{outside}}$

11) Write the equation of the circle that is centered at $(2,4)$ and has a radius of 6.

$$(x-2)^2+(y-4)^2=6^2$$

$$\boxed{(x-2)^2+(y-4)^2=36}$$

12) Write the equation of the circle that is centered at $(-3,2)$ and goes through the point $(-6,4)$

$$(x+3)^2+(y-2)^2=r^2$$

$$(-6+3)^2+(4-2)^2=r^2$$

$$13=r^2$$

$$\boxed{(x+3)^2+(y-2)^2=13}$$

13) Given ΔDEF with vertices $D(-4,-1)$, $E(4,3)$, and $F(0,-5)$, verify that
a) ΔDEF is isosceles
b) the line segment joining the midpoints of the equal sides is parallel to the third side and half the length of the third side.

a) $\text{length}_{DE}=\sqrt{[4-(-4)]^2+[3-(-1)]^2}=\sqrt{80}$

$\text{length}_{EF}=\sqrt{(0-4)^2+(-5-3)^2}=\sqrt{80}$

$\text{length}_{DF}=\sqrt{[0-(-4)]^2+[-5-(-1)]^2}=\sqrt{32}$

2 sides are equal; it's isosceles

b) $\text{mid}_{DE}=\left(\frac{-4+4}{2},\frac{-1+3}{2}\right)=(0,1)$

$\text{mid}_{EF}=\left(\frac{4+0}{2},\frac{3+(-5)}{2}\right)=(2,-1)$

slope of line connecting midpoints $=\frac{-1-1}{2-0}=\frac{-2}{2}=-1$

length of line connecting midpoints $=\sqrt{(2-0)^2+(-1-1)^2}=\sqrt{8}=2\sqrt{2}$

slope of DF $=\frac{-5-(-1)}{0-(-4)}=\frac{-4}{4}=-1$

length $=$ DF $=\sqrt{[0-(-4)]^2+[-5-(-1)]^2}=\sqrt{32}=4\sqrt{2}$

14) ΔABC has vertices $A(4,2)$, $B(0,4)$, and $C(-2,-2)$. Determine the coordinates of the circumcenter of ΔABC.

Right Bisector of AB

$mid_{AB} = \left(\frac{4+0}{2}, \frac{2+4}{2}\right) = (2,3)$

$slope_{AB} = \frac{4-2}{0-4} = \frac{2}{-4} = -\frac{1}{2}$

slope of right bisector $= 2$

Eq^n: $y = mx+b$

$3 = 2(2)+b$

$b = -1$

① $y = 2x-1$

Right Bisector of BC

$mid_{BC} = \left(\frac{0+(-2)}{2}, \frac{4+(-2)}{2}\right) = (-1,1)$

$slope_{BC} = \frac{-2-4}{-2-0} = \frac{-6}{-2} = 3$

slope of right bisector $= \frac{-1}{3}$

Eq^n $=$ $1 = -\frac{1}{3}(-1)+b$

$1 = \frac{1}{3}+b$

$\frac{3}{3} - \frac{1}{3} = b$

$b = \frac{2}{3}$

② $y = -\frac{1}{3}x + \frac{2}{3}$

PoI of right Bisectors

① $y - 2x = -1$

$6\times$② $6y + 2x = 4$ +

$7y = 3$

$y = \frac{3}{7}$

sub $y = \frac{3}{7}$ into ①

$\frac{3}{7} = 2x-1$

$\frac{3}{7} + \frac{7}{7} = 2x$

$\frac{10}{7} = 2x$

$\frac{10}{14} = x$

$x = \frac{5}{7}$

$\boxed{\text{circumcenter is at } \left(\frac{5}{7}, \frac{3}{7}\right)}$

15) ΔPQR has vertices $P(1,3)$, $Q(-1,-1)$, and $R(5,1)$. Determine the coordinates of the centroid of ΔPQR.

Median from R

$mid_{PQ} = \left(\frac{1+(-1)}{2}, \frac{3+(-1)}{2}\right) = (0,1)$

slope of median $= \frac{1-1}{5-0} = \frac{0}{5} = 0$

Eq^n: $y = mx+b$

$1 = 0(0)+b$

$b = 1$

① $y = 1$

Median from P

$mid_{QR} = \left(\frac{-1+5}{2}, \frac{-1+1}{2}\right) = (2,0)$

slope of median $= \frac{0-3}{2-1} = \frac{-3}{1} = -3$

Eq^n: $y = mx+b$

$0 = -3(2)+b$

$b = 6$

② $y = -3x+6$

PoI of Medians

① $y = 1$ ② $y = -3x+6$

$1 = -3x+6$

$-5 = -3x$

$x = \frac{5}{3}$

$\boxed{\text{The centroid is @ } \left(\frac{5}{3}, 1\right)}$

16) The equation of a circle with center $O(0,0)$ is $x^2 + y^2 = 20$. The points $P(2,-4)$ and $Q(4,2)$ are endpoints of chord PQ. AB right bisects the chord PQ at C. Verify that the center of the circles lies on the right bisector of chord PQ.

$C = mid_{PQ} = \left(\frac{2+4}{2}, \frac{-4+2}{2}\right) = (3,-1)$

$slope_{PQ} = \frac{2-(-4)}{4-2} = 3$

$slope_{AB} = -\frac{1}{3}$

Eq^n of AB: $y = mx+b$

$-1 = \frac{-1}{3}(3) + b$

$-1 = -1 + b$

$b = 0$

$y = \frac{-1}{3}x$

Check if $(0,0)$ is on $y = \frac{-1}{3}x$

LS	RS
$= y$	$= \frac{-1}{3}x$
$= 0$	$= \frac{-1}{3}(0)$
	$= 0$

$LS = RS$

$\therefore$ the center of the circle lies on the right bisector.

Answers

1)a) $\left(-\frac{1}{2},-1\right)$ **b)** $(4,0)$ **c)** $\left(\frac{7}{2},-3\right)$ **d)** $\left(\frac{11}{12},1\right)$
2) $(-3,7)$
3)a) 10 **b)** $\sqrt{40} = 2\sqrt{10}$ **c)** $\sqrt{18} = 3\sqrt{2}$ **d)** $\sqrt{232} = 2\sqrt{58}$
4)a) scalene **b)** 16.0 units
5)a) $y = -\frac{6}{11}x + \frac{42}{11}$ **b)** $y = -\frac{3}{2}x + \frac{21}{2}$ **c)** $y = \frac{7}{2}x - 16$
6) $\sqrt{41}$ units
7)a) $x^2 + y^2 = 144$ **b)** $x^2 + y^2 = 400$ **c)** $x^2 + y^2 = 87$
8)a) 11 **b)** 4.5 **c)** 0.7
9) $x^2 + y^2 = 26$
10)a) inside **b)** on **c)** outside
11) $(x-2)^2 + (y-4)^2 = 36$
12) $(x+3)^2 + (y-2)^2 = 13$
13) see posted solution
14) $\left(\frac{5}{7},\frac{3}{7}\right)$
15) $\left(\frac{5}{3},1\right)$
16) see posted solutions