

1) Expand and simplify each of the following

a)  $(x - 2)(x + 3)$

$$= x^2 + 3x - 2x - 6$$

$$= x^2 + x - 6$$

b)  $(y + 6)(y + 2)$

$$= y^2 + 2y + 6y + 12$$

$$= y^2 + 8y + 12$$

c)  $(x + 4)(x - 5)$

$$= x^2 - 5x + 4x - 20$$

$$= x^2 - x - 20$$

d)  $(x - 8)(x - 6)$

$$= x^2 - 6x - 8x + 48$$

$$= x^2 - 14x + 48$$

e)  $(x - 2y)(x + 2y)$

$$= x^2 + 2xy - 2xy - 4y^2$$

$$= x^2 - 4y^2$$

f)  $(2x + 1)(x - 3)$

$$= 2x^2 - 6x + x - 3$$

$$= 2x^2 - 5x - 3$$

g)  $(2x - 7y)(2x - 5y)$

$$= 4x^2 - 10xy - 14xy + 35y^2$$

$$= 4x^2 - 24xy + 35y^2$$

h)  $(3 - 2s)(2 - 3s)$

$$= 6 - 9s - 4s + 6s^2$$

$$= 6s^2 - 13s + 6$$

i)  $2(x - 7)(2x + 1)$

$$= (2x - 14)(2x + 1)$$

$$= 4x^2 + 2x - 28x - 14$$

$$= 4x^2 - 26x - 14$$

j)  $(x + 3)(x + 6) - 2(x + 1)$

$$= x^2 + 6x + 3x + 18 - 2x - 2$$

$$= x^2 + 7x + 16$$

$$\mathbf{k)} -(m+7)(m-1) + 4(2m+1)(3m-4)$$

$$\begin{aligned} &= -(m^2 - m + 7m - 7) + 4(6m^2 - 8m + 3m - 4) \\ &= -(m^2 + 6m - 7) + 4(6m^2 - 5m - 4) \\ &= -m^2 - 6m + 7 + 24m^2 - 20m - 16 \\ &= 23m^2 - 26m - 9 \end{aligned}$$

$$\mathbf{l)} -6(2x+1)(6x+1) + 3(4x-3)^2$$

$$\begin{aligned} &= -6(12x^2 + 2x + 6x + 1) + 3(16x^2 - 12x - 12x + 9) \\ &= -6(12x^2 + 8x + 1) + 3(16x^2 - 24x + 9) \\ &= -72x^2 - 48x - 6 + 48x^2 - 72x + 27 \\ &= -24x^2 - 120x + 21 \end{aligned}$$

2) Factor each of the following, if possible

$$\mathbf{a)} x^2 + 3x$$

$$= x(x+3)$$

$$\mathbf{b)} 2x^2 + 10x$$

$$= 2x(x+5)$$

$$\mathbf{c)} 3x^2 + 6x$$

$$= 3x(x+2)$$

$$\mathbf{d)} 3x + 6y$$

$$= 3(x+2y)$$

$$\mathbf{e)} 17ac - 34ad$$

$$= 17a(c-2d)$$

$$\mathbf{f)} 16x^2y^2 - 24xy$$

$$= 8xy(2xy-3)$$

$$\mathbf{g)} 27x^3y^3 + 18x^2y^2 + 9xy$$

$$= 9xy(3x^2y^2 + 2xy + 1)$$

$$\mathbf{h)} 2x(x+7) + 3(x+7)$$

$$= (x+7)(2x+3)$$

$$\mathbf{i)} a(b - 7) + 2(b - 7)$$

$$= (b-7)(a+2)$$

$$\mathbf{j)} 4s(r + u) - 3(r + u)$$

$$= (r+u)(4s-3)$$

$$\mathbf{k)} y(x + s) + z(x + s)$$

$$= (x+s)(y+z)$$

$$\mathbf{l)} ax + ay + 3x + 3y$$

$$= a(x+y) + 3(x+y)$$

$$= (x+y)(a+3)$$

$$\mathbf{m)} 4x^2 + 6xy + 12y + 8x$$

$$= 4x^2 + 8x + 6xy + 12y$$

$$= 4x(x+2) + 6y(x+2)$$

$$= (x+2)(4x+6y)$$

$$= 2(x+2)(2x+3y)$$

$$\mathbf{n)} 9x^2 - 6x + 6x - 4$$

$$= 3x(3x-2) + 2(3x-2)$$

$$= (3x-2)(3x+2)$$

$$\mathbf{o)} 16x^2 - 12xy - 12xy + 9y^2$$

$$= 4x(4x-3y) - 3y(4x-3y)$$

$$= (4x-3y)(4x-3y)$$

$$= (4x-3y)^2$$

$$\mathbf{p)} 3x^2 + 6x - 2x + 10$$

Not possible

3) Factor each of the following, if possible

a)  $x^2 + 5x + 6$

$$\begin{array}{l} \underline{2} \times \underline{3} = 6 \\ \underline{2} + \underline{3} = 5 \end{array}$$

$$= (x+2)(x+3)$$

b)  $x^2 + 12x + 27$

$$\begin{array}{l} \underline{9} \times \underline{3} = 27 \\ \underline{9} + \underline{3} = 12 \end{array}$$

$$= (x+9)(x+3)$$

c)  $x^2 - 3x - 18$

$$\begin{array}{l} \underline{-6} \times \underline{3} = -18 \\ \underline{-6} + \underline{3} = -3 \end{array}$$

$$= (x-6)(x+3)$$

d)  $d^2 - 12d + 35$

$$\begin{array}{l} \underline{-7} \times \underline{-5} = 35 \\ \underline{-7} + \underline{-5} = -12 \end{array}$$

$$= (d-7)(d-5)$$

e)  $c^2 - 11c + 30$

$$\begin{array}{l} \underline{-6} \times \underline{-5} = 30 \\ \underline{-6} + \underline{-5} = -11 \end{array}$$

$$= (c-6)(c-5)$$

f)  $x^2 - x - 72$

$$\begin{array}{l} \underline{-9} \times \underline{8} = -72 \\ \underline{-9} + \underline{8} = -1 \end{array}$$

$$= (x-9)(x+8)$$

g)  $3x^2 - 12x - 36$

$$= 3(x^2 - 4x - 12)$$

$$= 3(x-6)(x+2)$$

$$\begin{array}{l} \underline{-6} \times \underline{2} = -12 \\ \underline{-6} + \underline{2} = -4 \end{array}$$

h)  $-2x^2 + 2x + 4$

$$= -2(x^2 - x - 2)$$

$$= -2(x-2)(x+1)$$

$$\begin{array}{l} \underline{-2} \times \underline{1} = -2 \\ \underline{-2} + \underline{1} = -1 \end{array}$$

i)  $6x^2 - 42x + 72$

$$= 6(x^2 - 7x + 12)$$

$$= 6(x-4)(x-3)$$

$$\begin{array}{l} \underline{-4} \times \underline{-3} = 12 \\ \underline{-4} + \underline{-3} = -7 \end{array}$$

j)  $2x^2 + 7x + 3$

$$= 2x^2 + 6x + 1x + 3$$

$$= 2x(x+3) + 1(x+3)$$

$$= (x+3)(2x+1)$$

$$\begin{array}{l} \underline{6} \times \underline{1} = 6 \\ \underline{6} + \underline{1} = 7 \end{array}$$

$$\text{k) } 6x^2 + 11x + 4$$

$$\begin{array}{r} \underline{8} \times \underline{3} = 24 \\ \underline{8} + \underline{3} = 11 \end{array}$$

$$\begin{aligned} &= 6x^2 + 8x + 3x + 4 \\ &= 2x(3x + 4) + 1(3x + 4) \\ &= (3x + 4)(2x + 1) \end{aligned}$$

$$\text{l) } 6x^2 + 10x - 4$$

$$\begin{array}{r} \underline{6} \times \underline{-1} = -6 \\ \underline{6} + \underline{-1} = 5 \end{array}$$

$$\begin{aligned} &= 2(3x^2 + 5x - 2) \\ &= 2(3x^2 + 6x - 1x - 2) \\ &= 2[3x(x + 2) - 1(x + 2)] \\ &= 2(x + 2)(3x - 1) \end{aligned}$$

$$\text{m) } 2d^2 - 11d - 6$$

$$\begin{array}{r} \underline{-12} \times \underline{1} = -12 \\ \underline{-12} + \underline{1} = -11 \end{array}$$

$$\begin{aligned} &= 2d^2 - 12d + 1d - 6 \\ &= 2d(d - 6) + 1(d - 6) \\ &= (d - 6)(2d + 1) \end{aligned}$$

$$\text{n) } 6x^2 - 29x + 35$$

$$\begin{array}{r} \underline{-14} \times \underline{-15} = 210 \\ \underline{-14} + \underline{-15} = -29 \end{array}$$

$$\begin{aligned} &= 6x^2 - 14x - 15x + 35 \\ &= 2x(3x - 7) - 5(3x - 7) \\ &= (3x - 7)(2x - 5) \end{aligned}$$

$$\text{o) } 4r^2 - 20r + 25$$

$$\begin{array}{r} \underline{-10} \times \underline{-10} = 100 \\ \underline{-10} + \underline{-10} = -20 \end{array}$$

$$\begin{aligned} &= 4r^2 - 10r - 10r + 25 \\ &= 2r(2r - 5) - 5(2r - 5) \\ &= (2r - 5)(2r - 5) \\ &= (2r - 5)^2 \end{aligned}$$

$$\text{p) } -6x^2 - 34x + 12$$

$$\begin{array}{r} \underline{18} \times \underline{-1} = -18 \\ \underline{18} + \underline{-1} = 17 \end{array}$$

$$\begin{aligned} &= -2(3x^2 + 17x - 6) \\ &= -2(3x^2 + 18x - 1x - 6) \\ &= -2[3x(x + 6) - 1(x + 6)] \\ &= -2(x + 6)(3x - 1) \end{aligned}$$

$$\text{q) } 6x^2 - 5xy - 4y^2$$

$$\begin{array}{r} \underline{-8} \times \underline{3} = -24 \\ \underline{-8} + \underline{3} = -5 \end{array}$$

$$\begin{aligned} &= 6x^2 - 8xy + 3xy - 4y^2 \\ &= 2x(3x - 4y) + y(3x - 4y) \\ &= (3x - 4y)(2x + y) \end{aligned}$$

$$\text{r) } x^4 - 7x^2 - 18$$

$$\begin{array}{r} \underline{-9} \times \underline{2} = -18 \\ \underline{-9} + \underline{2} = -7 \end{array}$$

$$\begin{aligned} &= (x^2)^2 - 7(x^2) - 18 \\ &\text{Let } k = x^2 \\ &= k^2 - 7k - 18 \\ &= (k - 9)(k + 2) \\ &= (x^2 - 9)(x^2 + 2) \\ &= (x - 3)(x + 3)(x^2 + 2) \end{aligned}$$

4) Expand each of the following

a)  $(x + 3)^2$

$$\begin{aligned} &= (x+3)(x+3) \\ &= x^2 + 3x + 3x + 9 \\ &= x^2 + 6x + 9 \end{aligned}$$

b)  $(x - 6)^2$

$$\begin{aligned} &= (x-6)(x-6) \\ &= x^2 - 6x - 6x + 36 \\ &= x^2 - 12x + 36 \end{aligned}$$

c)  $(3x + 2y)^2$

$$\begin{aligned} &= (3x+2y)(3x+2y) \\ &= 9x^2 + 6xy + 6xy + 4y^2 \\ &= 9x^2 + 12xy + 4y^2 \end{aligned}$$

d)  $(3x + y)(3x - y)$

$$\begin{aligned} &= 9x^2 - 3xy + 3xy - y^2 \\ &= 9x^2 - y^2 \end{aligned}$$

e)  $(v - 2)(v + 2)$

$$\begin{aligned} &= v^2 + 2v - 2v - 4 \\ &= v^2 - 4 \end{aligned}$$

f)  $(x + 6)(x - 6)$

$$\begin{aligned} &= x^2 - 6x + 6x - 36 \\ &= x^2 - 36 \end{aligned}$$

5) Factor each of the following

a)  $x^2 - 25$

$$\begin{aligned} &= (x)^2 - (5)^2 \\ &= (x-5)(x+5) \end{aligned}$$

b)  $y^2 - 49$

$$\begin{aligned} &= (y)^2 - (7)^2 \\ &= (y-7)(y+7) \end{aligned}$$

c)  $36x^2 - y^2$

$$\begin{aligned} &= (6x)^2 - (y)^2 \\ &= (6x-y)(6x+y) \end{aligned}$$

d)  $x^2 + 14x + 49$

$\underline{7} \times \underline{7} = 49$
$\underline{7} + \underline{7} = 14$

$$\begin{aligned} &= (x+7)(x+7) \\ &= (x+7)^2 \end{aligned}$$

e)  $x^2 - 6x + 9$

$\underline{-3} \times \underline{-3} = 9$
$\underline{-3} + \underline{-3} = -6$

$$\begin{aligned} &= (x-3)(x-3) \\ &= (x-3)^2 \end{aligned}$$

f)  $4x^2 - 12xy + 9y^2$

$\underline{-6} \times \underline{-6} = 36$
$\underline{-6} + \underline{-6} = -12$

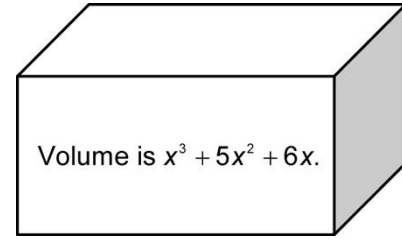
$$\begin{aligned} &= 4x^2 - 6xy - 6xy + 9y^2 \\ &= 2x(2x-3y) - 3y(2x-3y) \\ &= (2x-3y)(2x-3y) \\ &= (2x-3y)^2 \end{aligned}$$

6) Determine expressions to represent the dimensions of this rectangular prism.

$$V = x(x^2 + 5x + 6)$$

$$V = x(x+2)(x+3)$$

$$\begin{array}{l} \underline{2} \times \underline{3} = 6 \\ \underline{2} + \underline{3} = 5 \end{array}$$



$$\text{Length} = x$$

$$\text{Width} = x+2$$

$$\text{Height} = x+3$$

7) A square has side length  $4a$ . One dimension is increased by 6 and the other is decreased by 6.

a) Write an algebraic expression to represent the area of the resulting rectangle.

$$A = (4a+6)(4a-6)$$

b) Expand this expression and simplify.

$$A = 16a^2 - 24a + 24a - 36$$

$$A = 16a^2 - 36$$

**ANSWERS:**

**1)a)**  $x^2+x-6$  **b)**  $y^2+8y+12$  **c)**  $x^2-x-20$  **d)**  $x^2-14x+48$  **e)**  $x^2-4y^2$  **f)**  $2x^2-5x-3$  **g)**  $4x^2-24xy+35y^2$   
**h)**  $6s^2-13s+6$  **i)**  $4x^2-26x-14$  **j)**  $x^2+7x+16$  **k)**  $23m^2-26m-9$  **l)**  $-24x^2-120x+21$

**2)a)**  $x(x+3)$  **b)**  $2x(x+5)$  **c)**  $3x(x+2)$  **d)**  $3(x+2y)$  **e)**  $17a(c-2d)$  **f)**  $8xy(2xy-3)$  **g)**  $9xy(3x^2y^2+2xy+1)$  **h)**  $(x+7)(2x+3)$   
**i)**  $(b-7)(a+2)$  **j)**  $(r+u)(4s-3)$  **k)**  $(x+s)(y+z)$  **l)**  $(a+3)(x+y)$  **m)**  $2(x+2)(2x+3y)$  **n)**  $(3x+2)(3x-2)$  **o)**  $(4x-3y)^2$  **p)** not factorable

**3)a)**  $(x+3)(x+2)$  **b)**  $(x+3)(x+9)$  **c)**  $(x-6)(x+3)$  **d)**  $(d-5)(d-7)$  **e)**  $(c-5)(c-6)$  **f)**  $(x-9)(x+8)$  **g)**  $3(x-6)(x+2)$   
**h)**  $-2(x-2)(x+1)$  **i)**  $6(x-4)(x-3)$  **j)**  $(x+3)(2x+1)$  **k)**  $(2x+1)(3x+4)$  **l)**  $2(x+2)(3x-1)$  **m)**  $(d-6)(2d+1)$   
**n)**  $(2x-5)(3x-7)$  **o)**  $(2r-5)(2r-5)$  **p)**  $-2(3x-1)(x+6)$  **q)**  $(3x-4y)(2x+y)$  **r)**  $(x-3)(x+3)(x^2+2)$

**4)a)**  $x^2+6x+9$  **b)**  $x^2-12x+36$  **c)**  $9x^2+12xy+4y^2$  **d)**  $9x^2-y^2$  **e)**  $v^2-4$  **f)**  $x^2-36$

**5)a)**  $(x-5)(x+5)$  **b)**  $(y-7)(y+7)$  **c)**  $(6x-y)(6x+y)$  **d)**  $(x+7)^2$  **e)**  $(x-3)^2$  **f)**  $(2x-3y)^2$

**6)**  $x(x+3)(x+2)$

**7)a)**  $(4a+6)(4a-6)$  **b)**  $16a^2-36$