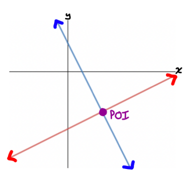
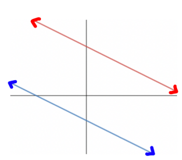
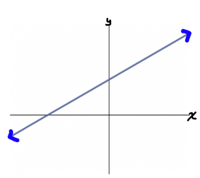
Unit 1- Linear Systems

Lessons

MPM2D

Graph	Slopes of Lines	Intercepts	Number of Solutions
Intersecting 	DIFFERENT	Usually different unless the lines intersect on an axis	1
Parallel & Distinct 	Same	Different	0
Parallel & Coincident 	Same	Same	Infinitely Many

Unit 1 Outline

Unit Goal: By the end of this unit, you will be able to model and solve problems involving the intersection of two straight lines.

Section	Subject	Learning Goals	Curriculum Expectations
L1	Solving by Graphing	- solve a system of 2 linear equations graphically - understand that the solution represents the point of intersection	B1.1
L2	Solving by Substitution	- solve a system of 2 linear equations using the method of substitution	B1.1
L3	Solving by Elimination	- solve a system of 2 linear equations using the method of elimination	B1.1
L4	Applications of Linear Systems	- solve problems that arise from realistic situations described in words or represented by linear systems of two equations involving two variables, by choosing an appropriate algebraic or graphical method	B1.2

Assessments	F/A/O	Ministry Code	P/O/C	KTAC
Note Completion	A		P	
Practice Worksheet Completion	F/A		P	
Quiz – Solving linear systems	F		P	
PreTest Review	F/A		P	
Test – Linear Systems	O	B1.1, B1.2	P	K(30%), T(30%), A(30%), C(10%)

L1 – Solving Linear Systems by GRAPHING Unit 1
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Linear System: Two or more linear equations that are considered at the same time.

Point of Intersection: The point where 2 or more lines cross.

To **solve** a linear system means to find the values of the variables that satisfy ALL of the equations in the system. Graphically speaking, this means you will find the ordered pair (x, y) where the lines intersect.

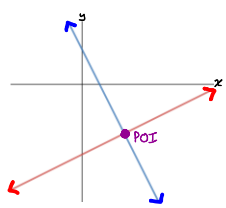
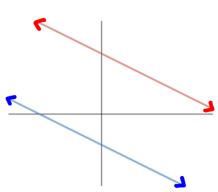
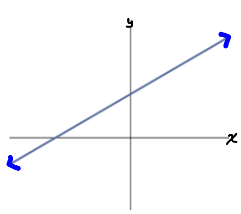
There are 3 main methods for solving a linear system:

1) Graphing
2) Substitution
3) Elimination

When solving by graphing, you can graph the lines by:

1) Using the slope and y-intercept (rearrange in to $y = mx + b$ form)
2) Use the x and y intercepts of each line
3) Create a table of values for each equation

A linear system could have 1, 0, or infinitely many solutions:

Graph	Slopes of Lines	Intercepts	Number of Solutions
Intersecting 	DIFFERENT	Usually different unless the lines intersect on an axis	1
Parallel & Distinct 	Same	Different	0
Parallel & Coincident 	Same	Same	Infinitely Many

Steps for Solving a Linear System by GRAPHING

1) Rearrange the equations in to slope y-intercept form ($y = mx + b$)

2) Graph equations and find the point of intersection

3) Verify that the point of intersection satisfies the equation of both lines

4) Clearly communicate your solution

Example 1: Find the point of intersection of the graphs of the following systems of equations.

a) ℓ_1: $y = x + 4$
ℓ_2: $y = -x + 2$

line 1

$y = 1x + 4$ (m, b)

slope $= m = \frac{1}{1}$ (rise, run)

y-int $= b = 4$

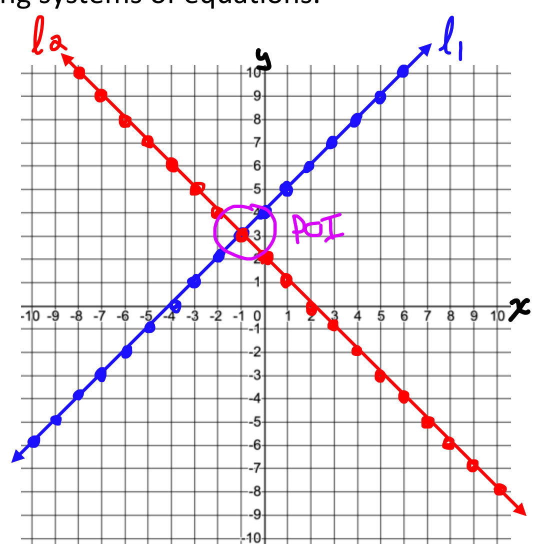
line 2

$y = -1x + 2$ (m, b)

slope $= m = -1 = \frac{-1}{1}$ (rise, run)

Check solution

ℓ_1		ℓ_2	
LS	RS	LS	RS
$= y$	$= x+4$	$= y$	$= -x+2$
$= 3$	$= -1+4$	$= 3$	$= -(-1)+2$
✓	$= 3$	✓	$= 3$



The Point of Intersection is $(-1, 3)$
The solution is $x = -1$, $y = 3$

b) ℓ_1: $2x + y = 5$
ℓ_2: $x - 2y = 10$

line 1

$2x + y = 5$

$y = -2x + 5$ (m, b)

slope $= m = -2 = \frac{-2}{1}$ (rise, run)

y-int $= b = 5$

line 2

$x - 2y = 10$

$x - 10 = 2y$

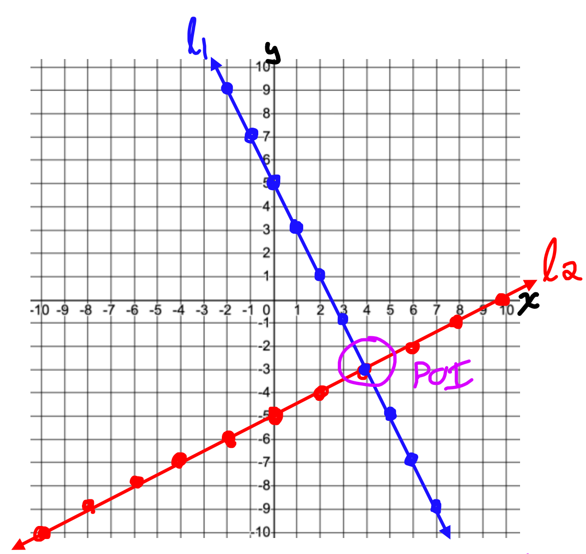
$\frac{1}{2}x - 5 = y$ (m, b)

slope $= m = \frac{1}{2}$ (rise, run)

y-int $= b = -5$

Check solution $x = 4$, $y = -3$

ℓ_1		ℓ_2	
LS	RS	LS	RS
$= 2x + y$	$= 5$	$= x - 2y$	$= 10$
$= 2(4) + (-3)$		$= 4 - 2(-3)$	
$= 5$ ✓		$= 10$ ✓	



The point of intersection is $(4, -3)$
The solution is $x = 4$, $y = -3$

c) ℓ_1: $2x + 5y = -20$
ℓ_2: $5x - 3y = -15$

line 1

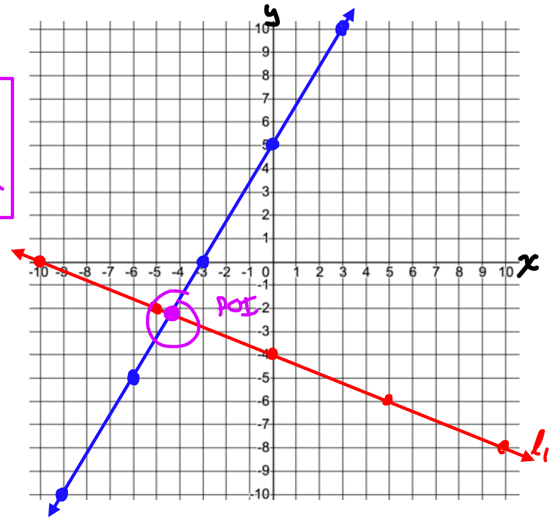
$2x + 5y = -20$
$5y = -2x - 20$
$y = \overset{m}{-\frac{2}{5}}x \overset{b}{-4}$
slope $= m = -\frac{2}{5}$ $\frac{\text{rise}}{\text{run}}$
y-int $= b = -4$

line 2

$5x - 3y = -15$
$5x + 15 = 3y$
$\overset{m}{\frac{5}{3}}x + \overset{b}{5} = y$
slope $= m = \frac{5}{3}$ $\frac{\text{rise}}{\text{run}}$
y-int $= b = 5$

POI: $(-4.3, -2.2)$
Solution: $x = -4.3$, $y = -2.2$

Note: *Our solution to this system is an estimate. The solution will not exactly verify the original equations but should be close!*



d) ℓ_1: $y = 2x + 3$
ℓ_2: $y = 2x - 4$

line 1

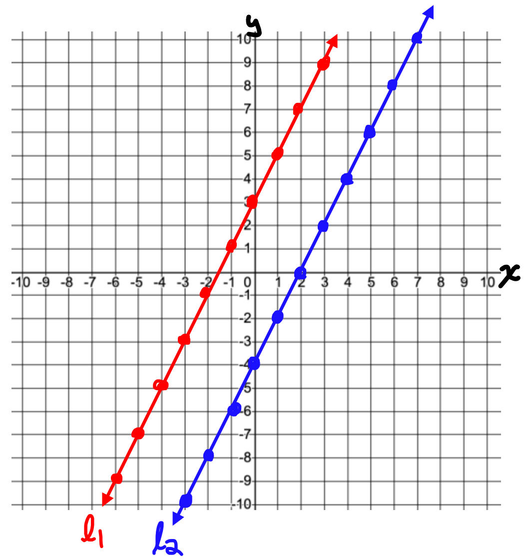
$y = \overset{m}{2}x + \overset{b}{3}$
slope $= m = 2 = \frac{2}{1}$ $\frac{\text{rise}}{\text{run}}$
y-int $= b = 3$

line 2

$y = \overset{m}{2}x \overset{b}{-4}$
slope $= m = 2 = \frac{2}{1}$ $\frac{\text{rise}}{\text{run}}$
y-int $= b = -4$

Notice the functions have the same slope but different y-intercepts. They will be parallel but distinct.

The lines are parallel and distinct.
∴ there are NO solutions.



e) ℓ_1: $x + y = 3$
ℓ_2: $2x + 2y = 6$

line 1

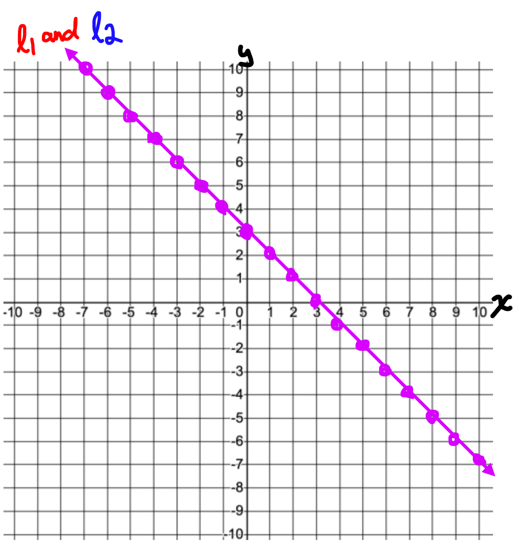
$x + y = 3$
$y = \overset{m}{-}x + \overset{b}{3}$
slope $= m = -1 = \frac{-1}{1}$ $\frac{\text{rise}}{\text{run}}$
y-int $= b = 3$

line 2

$2x + 2y = 6$
$2y = -2x + 6$
$y = \overset{m}{-}x + \overset{b}{3}$
slope $= m = -1 = \frac{-1}{1}$ $\frac{\text{rise}}{\text{run}}$
y-int $= b = 3$

Notice the lines have the same slope and same y-int. They will be parallel and coincident

The lines are parallel and coincident.
∴ there are INFINITE solutions.



L2 – Solving Linear Systems by SUBSTITUTION Unit 1
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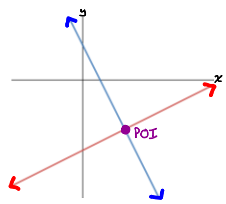
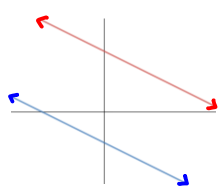
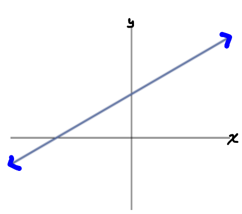
Remember that **solving** a linear system means to find the values of the variables that satisfy ALL of the equations in the system. Graphically speaking, this means you will find the ordered pair (x, y) where the lines intersect.

There are 3 main methods for solving a linear system:

1) Graphing
2) Substitution
3) Elimination

A linear system could have 1, 0, or infinitely many solutions:

Graph	Slopes of Lines	Intercepts	Number of Solutions	What happens algebraically
Intersecting 	DIFFERENT	Usually different unless the lines intersect on an axis	1	You will get a single solution for each variable that will satisfy both equations
Parallel & Distinct 	Same	Different	0	You will get an equation that is not true for any value of the variable Ex: $0x = 5$
Parallel & Coincident 	Same	Same	Infinitely Many	You will get an equation that is true for ALL values of the variable Ex: $0x = 0$

Steps for Solving by Substitution:

1) Rearrange either equation to isolate a variable (x or y)

2) Substitute what the isolated variable is equal to into the OTHER equation

3) Solve the new equation for the variable

4) Plug your answer back in to EITHER original equation to solve for the OTHER variable.

5) Check your answer in BOTH equations

Example 1: Solve the following systems using the method of substitution

a) $\ell_1:\ x + 4y = 6$
$\ell_2:\ 2x - 3y = 1$

① Isolate x in ℓ_1:
$x = -4y + 6$

② Sub in for x in ℓ_2
$2x - 3y = 1$
$2(-4y+6) - 3y = 1$

③ Now solve for y
$-8y + 12 - 3y = 1$
$-11y = 1 - 12$
$-11y = -11$
$y = 1$

④ Sub $y=1$ in to ℓ_1
$x = -4(1) + 6$
$x = 2$

⑤ Check solution $x=2$, $y=1$

ℓ_1
LS $= x + 4y = 2 + 4(1) = 6$ ✓
RS $= 6$

ℓ_2
LS $= 2x - 3y = 2(2) - 3(1) = 1$ ✓
RS $= 1$

The solution is $x=2$, $y=1$
The POI is $(2,1)$

b) $\ell_1:\ 5x - 3y - 2 = 0$
$\ell_2:\ 7x + y = 0$

① Isolate y in ℓ_2
$7x + y = 0$
$y = -7x$

② sub in for y in ℓ_1
$5x - 3y - 2 = 0$
$5x - 3(-7x) - 2 = 0$

③ Solve for x
$5x + 21x - 2 = 0$
$26x = 2$
$x = \frac{2}{26}$
$x = \frac{1}{13}$

④ sub $x = \frac{1}{13}$ into ℓ_2
$y = -7x$
$y = -7\left(\frac{1}{13}\right)$
$y = \frac{-7}{13}$

⑤ Check solution $x = \frac{1}{13}$, $y = \frac{-7}{13}$

ℓ_1
LS $= 5x - 3y - 2 = 5\left(\frac{1}{13}\right) - 3\left(\frac{-7}{13}\right) - 2 = \frac{5}{13} + \frac{21}{13} - 2 = \frac{26}{13} - 2 = 2 - 2 = 0$ ✓
RS $= 0$

ℓ_2
LS $= 7x + y = 7\left(\frac{1}{13}\right) + \left(\frac{-7}{13}\right) = \frac{7}{13} - \frac{7}{13} = 0$ ✓
RS $= 0$

The solution is $x = \frac{1}{13}$, $y = \frac{-7}{13}$
The POI is $\left(\frac{1}{13}, \frac{-7}{13}\right)$

c) ℓ_1: $2x + 2y = 7$
ℓ_2: $x + y = 6$

① Isolate x in ℓ_2

$x + y = 6$
$x = 6 - y$

② sub in for x into ℓ_1

$2x + 2y = 7$
$2(6-y) + 2y = 7$

③ solve for y

$12 - 2y + 2y = 7$
$-2y + 2y = 7 - 12$
$0y = -5$

There are NO solutions to this equation

The system has NO solutions.
The lines are parallel and distinct.

d) ℓ_1: $3x + 4y = 2$
ℓ_2: $9x + 12y = 6$

① Isolate for y in ℓ_1

$3x + 4y = 2$
$4y = -3x + 2$
$y = -\frac{3}{4}x + \frac{1}{2}$

② sub in for y in ℓ_2

$9x + 12y = 6$
$9x + 12\left(-\frac{3}{4}x + \frac{1}{2}\right) = 6$

③ solve for x

$9x - \frac{36}{4}x + \frac{12}{2} = 6$
$9x - 9x + 6 = 6$
$9x - 9x = 6 - 6$
$0x = 0$

There infinite solutions to this equation

The system has infinitely many solutions.
The lines are parallel and coincident.

Remember that **solving** a linear system means to find the values of the variables that satisfy ALL of the equations in the system. Graphically speaking, this means you will find the ordered pair (x, y) where the lines intersect.

There are 3 main methods for solving a linear system:

1) Graphing
2) Substitution
3) Elimination

Steps for Solving by ELIMINATION:

1) Get rid of decimals or fractions if necessary

2) Rewrite the equations with like terms in the same column $(x + y = \#)$

3) Multiply one or both equations by a number so that you have two equations in which the coefficients of one variable are the same or opposite

4) Add or subtract the equations to eliminate a variable and solve the resulting equation for the remaining variable

5) Substitute your solution for one of the variables in to either of the original equations to solve for the other variable

6) Check that the solutions satisfy BOTH of the original equations

Example 1: Solve each of the following linear systems using the method of ELIMINATION

a) ℓ_1: $3x + 2y = 19$
ℓ_2: $5x - 2y = 5$

$\ell_1 \rightarrow 3x + 2y = 19$
$\ell_2 \rightarrow 5x - 2y = 5$ +

$8x + 0y = 24$
$8x = 24$
$x = 3$

Sub $x = 3$ into ℓ_1

$3x + 2y = 19$
$3(3) + 2y = 19$
$9 + 2y = 19$
$2y = 10$
$y = 5$

Check solution $x = 3, y = 5$

ℓ_1

LS	RS
$= 3x + 2y$	$= 19$
$= 3(3) + 2(5)$	
$= 19$ ✓	

ℓ_2

LS	RS
$= 5x - 2y$	$= 5$
$= 5(3) - 2(5)$	
$= 5$ ✓	

The solution is $x = 3, y = 5$
The POI is $(3, 5)$

b) $\ell_1:\ x + 4y = 6$
$\ell_2:\ 2x - 3y = 1$

$$2 \times \ell_1 \rightarrow \quad 2x + 8y = 12$$
$$\ell_2 \rightarrow \quad 2x - 3y = 1 \quad -$$
$$0x + 11y = 11$$
$$11y = 11$$
$$y = 1$$

sub $y=1$ into ℓ_1

$$x + 4y = 6$$
$$x + 4(1) = 6$$
$$x = 2$$

Check solution $x=2, y=1$

$\boxed{\ell_1}$

LS $= x + 4y = 2 + 4(1) = 6$ ✓

RS $= 6$

$\boxed{\ell_2}$

LS $= 2x - 3y = 2(2) - 3(1) = 1$ ✓

RS $= 1$

The solution is $x=2, y=1$
The POI is $(2,1)$

c) $\ell_1:\ 3x + 2y = 2$
$\ell_2:\ 4x + 5y = 12$

$$4 \times \ell_1 \rightarrow \quad 12x + 8y = 8$$
$$3 \times \ell_2 \rightarrow \quad 12x + 15y = 36 \quad -$$
$$0x - 7y = -28$$
$$-7y = -28$$
$$y = 4$$

sub $y=4$ into ℓ_1

$$3x + 2y = 2$$
$$3x + 2(4) = 2$$
$$3x + 8 = 2$$
$$3x = -6$$
$$x = -2$$

Check solution $x=-2, y=4$

$\boxed{\ell_1}$

LS $= 3x + 2y = 3(-2) + 2(4) = 2$ ✓

RS $= 2$

$\boxed{\ell_2}$

LS $= 4x + 5y = 4(-2) + 5(4) = 12$ ✓

RS $= 12$

The solution is $x=-2, y=4$
The POI is $(-2,4)$

d) ℓ_1: $0.6x - 0.3y = 2.4$
ℓ_2: $-0.4y + 0.7x - 2.9 = 0 \rightarrow 0.7x - 0.4y = 2.9$

$10 \times \ell_1 \rightarrow 6x - 3y = 24 \xrightarrow{\times 4} 24x - 12y = 96$
$10 \times \ell_2 \rightarrow 7x - 4y = 29 \xrightarrow{\times 3} 21x - 12y = 87$ −

$3x + 0y = 9$
$3x = 9$
$x = 3$

Sub $x = 3$ into ℓ_1
$6x - 3y = 24$
$6(3) - 3y = 24$
$18 - 3y = 24$
$-3y = 6$
$y = -2$

$\boxed{\text{The solution is } x = 3, y = -2 \quad \text{The POI is } (3, -2)}$

Check solution $x=3, y=-2$

ℓ_1

LS
$= 0.6x - 0.3y$
$= 0.6(3) - 0.3(-2)$
$= 1.8 + 0.6$
$= 2.4$ ✓

RS
$= 2.4$

ℓ_2

LS
$= -0.4y + 0.7x - 2.9$
$= -0.4(-2) + 0.7(3) - 2.9$
$= 0.8 + 2.1 - 2.9$
$= 0$ ✓

RS
$= 0$

e) ℓ_1: $\frac{x}{2} + \frac{y}{8} = 4$
ℓ_2: $\frac{x}{3} - \frac{y}{2} = -2$

$8 \times \ell_1 \rightarrow 4x + y = 32 \xrightarrow{\times 3} 12x + 3y = 96$
$6 \times \ell_2 \rightarrow 2x - 3y = -12 \longrightarrow 2x - 3y = -12$ +

$14x + 0y = 84$
$14x = 84$
$x = 6$

Sub $x = 6$ into ℓ_1
$4x + y = 32$
$4(6) + y = 32$
$24 + y = 32$
$y = 8$

$\boxed{\text{The solution is } x = 6, y = 8 \quad \text{the POI is } (6, 8)}$

Check solution

$\boxed{\ell_1}$

LS
$= \frac{x}{2} + \frac{y}{8}$
$= \frac{6}{2} + \frac{8}{8}$
$= 3 + 1$
$= 4$ ✓

RS
$= 4$

$\boxed{\ell_2}$

LS
$= \frac{x}{3} - \frac{y}{2}$
$= \frac{6}{3} - \frac{8}{2}$
$= 2 - 4$
$= -2$ ✓

RS
$= -2$

f) ℓ_1: $5x + 2y = 2$
ℓ_2: $10x + 4y = -4$

$$2 \times \ell_1 \rightarrow \quad 10x + 4y = 4$$
$$\ell_2 \rightarrow \quad \underline{10x + 4y = -4} \quad -$$
$$0x + 0y = 8$$
$$0 = 8$$

There are NO SOLUTIONS to this equation.

There are NO SOLUTIONS to the linear system.
The lines are parallel and distinct.

Helpful tip:

When coefficients of a variable have opposite signs, ***ADDING*** *will eliminate them*
When coefficients of a variable have the same sign, ***SUBTRACTING*** *will eliminate them*

Many problems with 2 unknowns can be solved using a system of 2 linear equations. To solve these types of problems you should:

1) Assign variables to each of the unknowns
2) Write 2 equations showing the relationships between the variables. Each equation should include both variables.
3) Solve the system of equations using any method (graphing, substitution, elimination)
4) Check your solution
5) Clearly communicate your final answer

Example 1: Find the value of two numbers if their sum is 13 and their difference is 5.

x = first #
y = second #

Eq^n 1: $x + y = 13$
Eq^n 2: $x - y = 5$

solve system with elimination

① $x + y = 13$
② $x - y = 5$ +
$2x = 18$
$x = 9$

$x + y = 13$
$9 + y = 13$
$y = 4$

$\therefore$ the two numbers are 9 and 4.

Example 2: The Sports Shop sells Nike running shoes for $82 a pair and Air Jensen basketball shoes for $95 a pair. One day, the Sports Shop sells 75 pairs of Nike and Air Jensen shoes totaling $6241 in sales. How many pairs of each shoe were sold?

x = # of Nike sold
y = # of AJ sold.

① $x + y = 75$
② $82x + 95y = 6241$

solve system using substitution

① $x + y = 75$
$y = 75 - x$

② $82x + 95y = 6241$
$82x + 95(75 - x) = 6241$
$82x + 7125 - 95x = 6241$
$-13x = -884$
$x = 68$

$y = 75 - 68$
$y = 7$

$\therefore$ they sold 68 Nike's and 7 AJ's

Example 3: A blue spruce tree grows an average of 15 cm per year. An eastern hemlock grows an average of 10 cm per year. When they were planted, a blue spruce was 120 cm tall and an eastern hemlock was 180 cm tall. How many years after planting will the trees reach the same height? How tall will that be?

x = # of years
y = height

① $y = 120 + 15x$

② $y = 180 + 10x$

Solve system using substitution

① $y = 120 + 15x$ ② $y = 180 + 10x$

$120 + 15x = 180 + 10x$
$5x = 60$
$x = 12$

$y = 120 + 15(12)$
$y = 120 + 180$
$y = 300$

∴ After 12 years both trees will be 300 cm tall.

Example 4: Tia had \$10 000 to invest. She invested part of it in a term deposit paying 4% per annum and the remainder in bonds paying 5% per annum. If the total interest earned after one year was \$440, how much did she invest in each account?

x = amount in term deposit
y = amount in bonds

① $x + y = 10\,000$

② $0.04x + 0.05y = 440$

Solve using elimination:

$4 \times$ ① $4x + 4y = 40\,000$
$100 \times$ ② $4x + 5y = 44\,000$ −

$-y = -4000$
$y = 4000$

① $x + y = 10\,000$
$x + 4000 = 10\,000$
$x = 6000$

∴ She invested \$6000 in a term deposit and \$4000 in bonds

Example 5: A chemistry teacher needs to make 10L of 42% sulfuric acid solution. The acid solutions available are 30% sulfuric acid and 50% sulfuric acid, by volume. How many liters of each solution must be mixed to make the 42% solution?

x = amount of 30% acid
y = amount of 50% acid

① $x + y = 10$ (volume of solution)
② $0.3x + 0.5y = 0.42(10)$ (amount of pure acid)

solve using elimination:

$3 \times$ ① $3x + 3y = 30$
$10 \times$ ② $3x + 5y = 42$ −

$-2y = -12$
$y = 6$

① $x + y = 10$
$x + 6 = 10$
$x = 4$

$\therefore$ you need 4L of 30% acid and 6L of 50% acid.

Example 6: A riverboat took 2 hours to travel 24km, down a river with the current and 3 hours to make the return trip against the current. Find the speed of the boat in still water and the speed of the current.

Note:

Speed travelling with current = boat speed + current speed
Speed travelling against current = boat speed - current speed

Remember:

$distance = speed \times time$

x = speed of boat in still water
y = speed of current

① $2(x+y) = 24$ (with current)
② $3(x-y) = 24$ (against current)

solve using elimination

① $x + y = 12$
② $x - y = 8$ +

$2x = 20$
$x = 10$

① $x + y = 12$
$10 + y = 12$
$y = 2$

$\therefore$ the speed of the boat in still water is 10 km/h and the speed of the current is 2 km/h.