Transformations of Quadratic Functions

Transformations of Functions

Transformation:

A change made to a figure or a relation such that the figure or the graph of the relation is shifted or changed in shape.

Translations, stretches, and reflections are types of transformations.

The general function:

$f(x)$ parent function you are transforming

$$g(x) = af[k(x-d)] + c$$

a transformed function

takes f(x) and performs transformations to it

<u>Changes to the *y*-coordinates (vertical changes)</u>

c: vertical translation $g(x) = f(x) + c$

The graph of $g(x) = f(x) + c$ is a vertical translation of the graph of $f(x)$ by c units.

If c > 0, the graph shifts **up**
If c < 0, the graph shifts **down**

a: vertical stretch/compression $g(x) = af(x)$

The graph of $g(x) = af(x)$ is a vertical stretch or compression of the graph of $f(x)$ by a factor of a.

If $a > 1$ or $a < -1$, **vertical stretch** by a factor of a.
If $-1 < a < 1$, **vertical compression** by a factor of a.
If $a < 0$, **vertical reflection** (reflection over the x-axis)

Note: a vertical stretch or compression means that distance from the x-axis of each point of the parent function changes by a factor of a.

Note: for a vertical reflection, the point (x, y) becomes point $(x, -y)$

<u>Changes to the *x*-coordinates (horizontal changes)</u>

d: horizontal translation $g(x) = f(x - d)$

The graph of $g(x) = f(x - d)$ is a horizontal translation of the graph of $f(x)$ by d units.

If d > 0, the graph shifts **right**
If c < 0, the graph shifts **left**

k: horizontal stretch/compression

The graph of $g(x) = f(kx)$ is a horizontal stretch or compression

of the graph of $f(x)$ by a factor of $\frac{1}{k}$

If $k > 1$ or $k < -1$, **compressed horizontally** by a factor of $\frac{1}{k}$
If $-1 < k < 1$, **stretched horizontally** by a factor of $\frac{1}{k}$
If $k < 0$, **horizontal reflection** (reflection in the y-axis)

Note: a vertical stretch or compression means that distance from the y-axis of each point of the parent function changes by a factor of $1/k$.

Note: for a horizontal reflection, the point (x, y) becomes point $(-x, y)$

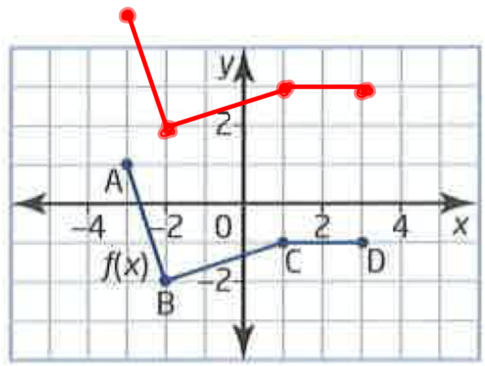
DO IT NOW!

a) Complete the table of values for the function $f(x)$ and $g(x)$. Then use the table of values to plot image points and graph the function $g(x)$

(x, y) $(x, y+4)$

$f(x):(x, f(x))$	$g(x):(x, f(x)+4)$
A(-3, 1)	A'(-3, 5)
B(-2, -2)	B'(-2, 2)
C(1, -1)	C'(1, 3)
D(3, -1)	D'(3, 3)

base function   image points



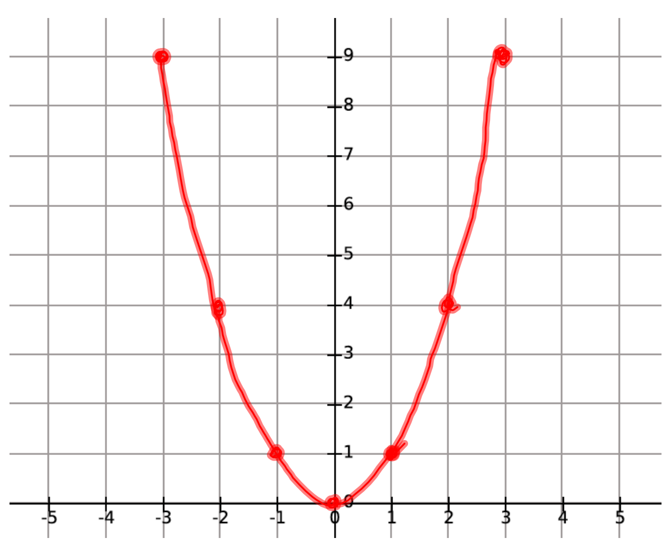
Quadratic Functions

Base Function:

Key Points:

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

Graph of Base Function



<u>**Order of Transformations**</u>

1. stretches, compressions, reflections

2. translations

$a \to k \to d \to c$

Example 1: If $f(x) = x^2$, describe the changes and write the transformed function:

a) $g(x) = 2f(x)$

vertical stretch b.a.f.o. 2

$g(x) = 2x^2$

b) $g(x) = f(2x)$

horizontal compression b.a.f.o. $\frac{1}{2}$

$g(x) = (2x)^2$

c) $g(x) = f(x)+4$

shift up 4 units

$g(x) = x^2+4$

d) $g(x)=f(x+3)$

shift left 3 units

$g(x) = (x+3)^2$

e) $g(x) = -f(x)$

vertical reflection

(flip over x-axis)

$g(x) = -x^2$

f) $g(x) = f(-x)$

horizontal reflection

(flip over y-axis)

$g(x) = (-x)^2$

Example 2: For each of the following functions, describe the transformations to $f(x) = x^2$ in order and write the transformed equation.

a) $g(x) = -2f[-3(x+3)] - 1$ → plug in for x into x^2

vertical stretch by a factor of 2

vertical reflection

horizontal compression by a factor of 1/3

horizontal reflection

shift left 3 units and down 1 unit

$g(x) = -2[-3(x+3)]^2 - 1$

b)

$$y = \tfrac{1}{2} f[-3(x-2)] + 5$$

vertical compression by a factor of 1/2

horizontal compression by a factor of 1/3

horizontal reflection

shift right 2 units and up 5 units

$$g(x) = \frac{1}{2}[-3(x-2)]^2 + 5$$

Example 3: for each of the following functions…

i) make a table of values for the parent function

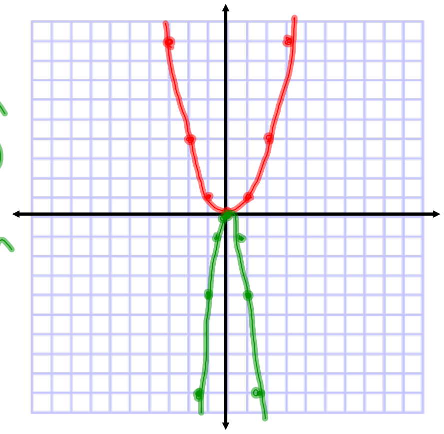
ii) graph the parent function $f(x) = x^2$

iii) describe the transformations

iv) make a table of values of image points

v) graph the transformed function and write it's equation

a) $g(x) = -f(2x)$



a = -1 ; vertical reflection (multiply y-values by -1)

k = 2 ; horizontal compression by a factor of 1/2 (divide x-values by 2)

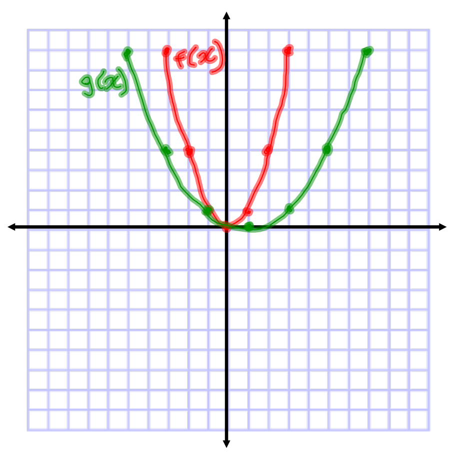
$f(x) = x^2$

x	*y*
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$g(x) = -(2x)^2$

$\frac{x}{2}$	$-y$
-1.5	-9
-1	-4
-0.5	-1
0	0
0.5	-1
1	-4
1.5	-9

b) $g(x)= f\,[-1/2(x-1)]$



horizontal stretch bafo 2 (2x)

horizontal reflection (-x)

shift right 1 unit. (x+1)

$f(x) = x^2$

x	*y*
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$g(x) = [-\frac{1}{2}(x-1)]^2$

-2x+1	y
7	9
5	4
3	1
1	0
-1	1
-3	4
-5	9

c) $g(x) = -2\,f[\,-3(x + 3)] - 1$

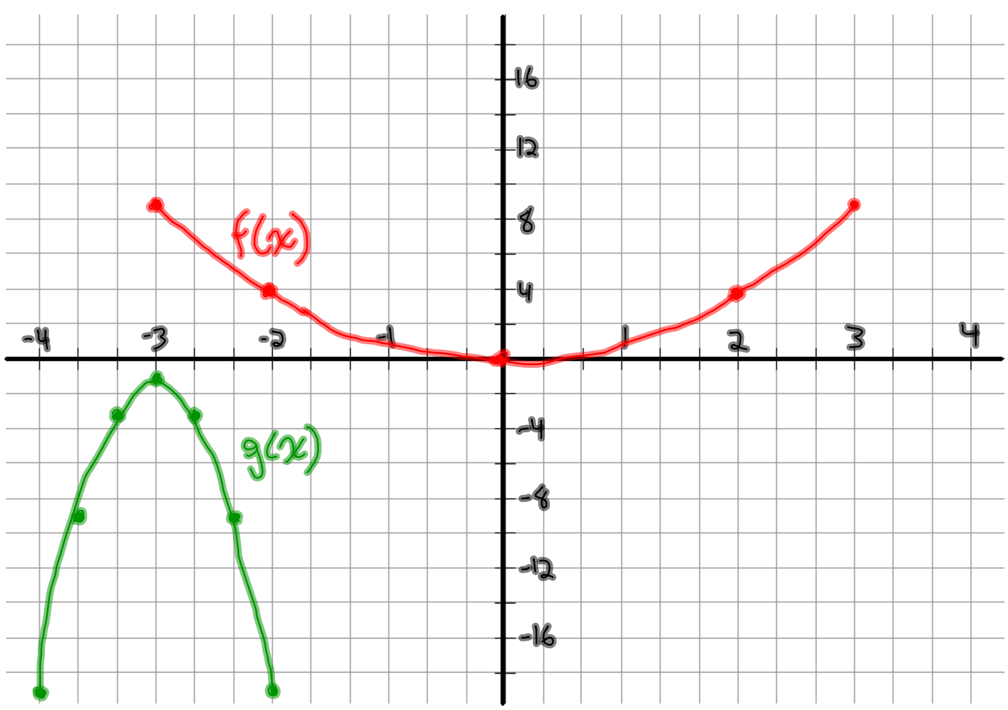
vertical stretch bafo 2. $(2y)$
vertical reflection $(-y)$
horizontal compression bafo $\frac{1}{3}$ $\left(\frac{x}{3}\right)$
horizontal reflection $(-x)$
shift left 3 units and down 1. $(x-3)$ $(y-1)$

$f(x) = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$g(x) = -2[-3(x+3)]^2 - 1$

$\frac{-x}{3} - 3$	$-2y-1$
-2	-19
-2.3	-9
-2.67	-3
-3	-1
-3.3	-3
-3.67	-9
-4	-19



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