Transformations of $f(x) = \frac{1}{x}$

<u>Transformations of Functions</u>

Transformation:

A change made to a figure or a relation such that the figure or the graph of the relation is shifted or changed in shape.

Translations, stretches, and reflections are types of transformations.

The general function:

$f(x)$ parent function you are transforming

$$g(x) = af[k(x - d)] + c$$

a transformed function

takes f(x) and performs transformations to it

Changes to the *y*-coordinates (vertical changes)

c: vertical translation $g(x) = f(x) + c$

The graph of $g(x) = f(x) + c$ is a vertical translation of the graph of $f(x)$ by c units.

If $c > 0$, the graph shifts **up**
If $c < 0$, the graph shifts **down**

a: vertical stretch/compression $g(x) = af(x)$

The graph of $g(x) = af(x)$ is a vertical stretch or compression of the graph of $f(x)$ by a factor of a.

If $a > 1$ or $a < -1$, **vertical stretch** by a factor of a.
If $-1 < a < 1$, **vertical compression** by a factor of a.
If $a < 0$, **vertical reflection** (reflection over the x-axis)

Note: a vertical stretch or compression means that distance from the x-axis of each point of the parent function changes by a factor of a.

Note: for a vertical reflection, the point (x, y) becomes point $(x, -y)$

Changes to the *x*-coordinates (horizontal changes)

d: horizontal translation $g(x) = f(x - d)$

The graph of $g(x) = f(x - d)$ is a horizontal translation of the graph of $f(x)$ by d units.

If $d > 0$, the graph shifts **right**
If $d < 0$, the graph shifts **left**

k: horizontal stretch/compression

The graph of $g(x) = f(kx)$ is a horizontal stretch or compression

of the graph of $f(x)$ by a factor of $\frac{1}{k}$

If $k > 1$ or $k < -1$, **compressed horizontally** by a factor of $\frac{1}{k}$
If $-1 < k < 1$, **stretched horizontally** by a factor of $\frac{1}{k}$
If $k < 0$, **horizontal reflection** (reflection in the y-axis)

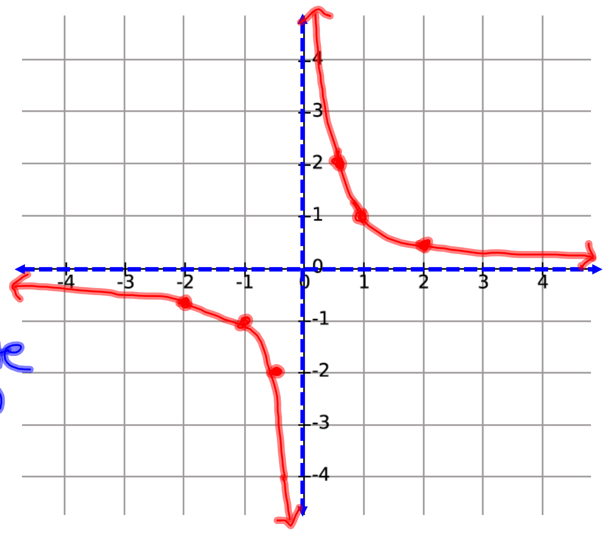
Note: a vertical stretch or compression means that distance from the y-axis of each point of the parent function changes by a factor of $1/k$.

Note: for a horizontal reflection, the point (x, y) becomes point $(-x, y)$

Rational Functions

Base Function: $f(x) = \frac{1}{x}$

Graph of Base Function



Key Points:

x	y
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
0	undefined
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$

vertical asymptote at $x=0$

Asymptotes

Asymptote: *a line that a curve approaches more and more closely but never touches.*

The function $f(x) = \frac{1}{x}$ has two asymptotes:

Vertical Asymptote: Division by zero is undefined. Therefore the expression in the denominator of the function can not be zero. Therefore $x \neq 0$. This is why the vertical line $x = 0$ is an asymptote for this function.

Horizontal Asymptote: For the range, there can never be a situation where the result of the division is zero. Therefore the line $y = 0$ is a horizontal asymptote. For all functions where the denominator is a higher degree than the numerator, there will by a horizontal asymptote at $y = 0$.

Example 1: Describe the combination of transformations that must be applied to the base function $f(x) = \frac{1}{x}$ to obtain the transformed function. Then, write the corresponding equation.

a) $g(x) = 4f(x - 3) + 0.5$

vertical stretch by a factor of 4

shift right 3 units

shift up 0.5 units

$$g(x) = 4\left(\frac{1}{x-3}\right) + 0.5$$

$$\downarrow$$

$$g(x) = \frac{4}{x-3} + 0.5$$

b) $g(x) = f[-2(x + 0.5)] - 1$

horizontal compression by a factor of 1/2

horizontal reflection

shift left 0.5 units

shift down 1 unit

$$g(x) = \frac{1}{-2(x+0.5)} - 1$$

Example 2: for each of the following functions…

i) make a table of values for the parent function $f(x) = 1/x$

ii) describe the transformations

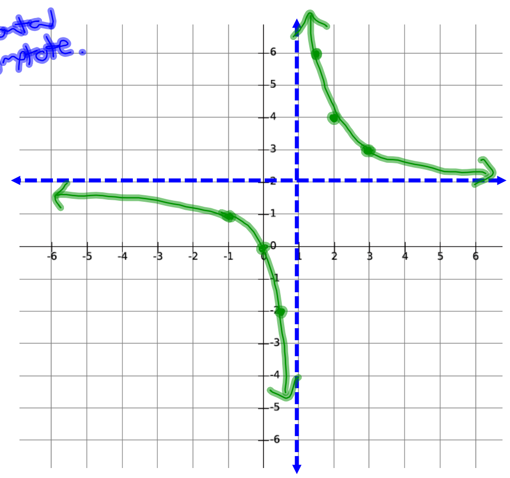
iii) make a table of values of image points

iv) graph the transformed function and write it's equation

a) $g(x)=2f(x - 1) + 2$

horizontal asymptote.

vertical asymptote



vertical stretch by a factor of 2 (2y)

shift right 1 unit (x+1)

shift up 2 units (y+2)

$f(x) = \frac{1}{x}$

x	*y*
-2	-½
-1	-1
-½	-2
0	undefined
½	2
1	1
2	½

x+1	2y+2
-1	1
0	0
0.5	-2
1	undefined
1.5	6
2	4
3	3

$g(x) = 2\left(\frac{1}{x-1}\right)+2$

↓

$g(x) = \frac{2}{x-1}+2$

b) $g(x) = -f\,[2(x + 0.5)]\ -1$ ← horizontal asymptote

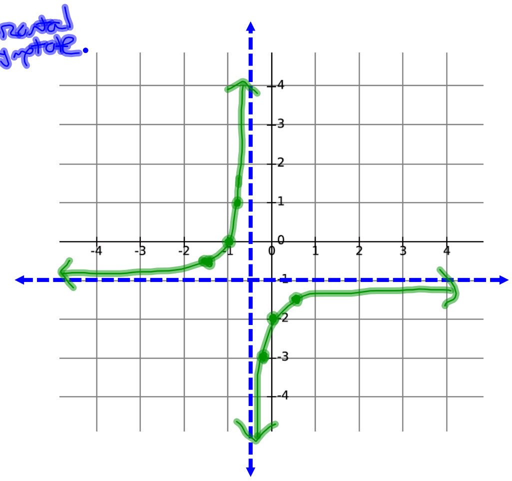
vertical asymptote

vertical reflection $(-y)$

horizontal compression by a factor of 1/2 $\left(\frac{x}{2}\right)$

shift left 0.5 units $(x-0.5)$

shift down 1 unit $(y-1)$



x	***y***
-2	-½
-1	-1
-½	-2
0	undefined
½	2
1	1
2	½

$\frac{x}{2}-0.5$	$-y-1$
-1.5	-0.5
-1	0
-0.75	1
-0.5	undefined
-0.25	-3
0	-2
0.5	-1.5

$$g(x) = -1\left(\frac{1}{2(x+0.5)}\right) - 1$$

↓

$$g(x) = \frac{-1}{2(x+0.5)} - 1$$

Complete Worksheet