Transformations of $\sqrt{x}$

Transformations of Functions

Transformation:

A change made to a figure or a relation such that the figure or the graph of the relation is shifted or changed in shape.

Translations, stretches, and reflections are types of transformations.

The general function:

$f(x)$ parent function you are transforming

$$g(x) = af[k(x-d)] + c$$

a transformed function

takes f(x) and performs transformations to it

Changes to the y-coordinates (vertical changes)

c: vertical translation $g(x) = f(x) + c$

The graph of $g(x) = f(x) + c$ is a vertical translation of the graph of $f(x)$ by c units.

If $c > 0$, the graph shifts **up**
If $c < 0$, the graph shifts **down**

a: vertical stretch/compression $g(x) = af(x)$

The graph of $g(x) = af(x)$ is a vertical stretch or compression of the graph of $f(x)$ by a factor of a.

If $a > 1$ or $a < -1$, **vertical stretch** by a factor of a.
If $-1 < a < 1$, **vertical compression** by a factor of a.
If $a < 0$, **vertical reflection** (reflection over the x-axis)

Note: a vertical stretch or compression means that distance from the x-axis of each point of the parent function changes by a factor of a.

Note: for a vertical reflection, the point (x, y) becomes point $(x, -y)$

Changes to the x-coordinates (horizontal changes)

d: horizontal translation $g(x) = f(x - d)$

The graph of $g(x) = f(x - d)$ is a horizontal translation of the graph of $f(x)$ by d units.

If $d > 0$, the graph shifts **right**
If $d < 0$, the graph shifts **left**

k: horizontal stretch/compression

The graph of $g(x) = f(kx)$ is a horizontal stretch or compression of the graph of $f(x)$ by a factor of $\frac{1}{k}$

If $k > 1$ or $k < -1$, **compressed horizontally** by a factor of $\frac{1}{k}$
If $-1 < k < 1$, **stretched horizontally** by a factor of $\frac{1}{k}$
If $k < 0$, **horizontal reflection** (reflection in the y-axis)

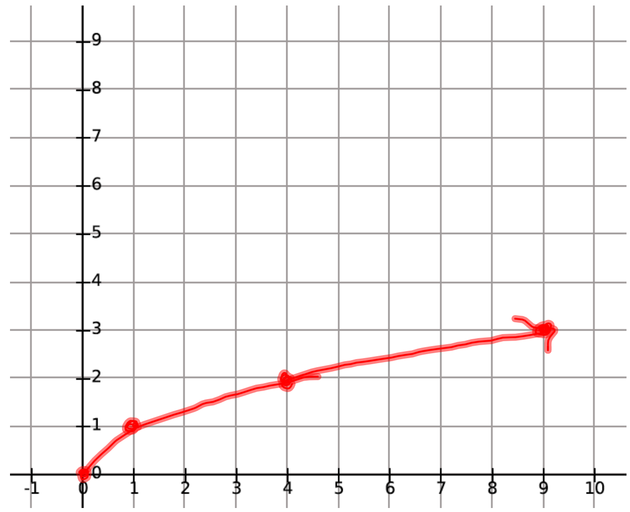
Note: a vertical stretch or compression means that distance from the y-axis of each point of the parent function changes by a factor of $1/k$.

Note: for a horizontal reflection, the point (x, y) becomes point $(-x, y)$

Radical (square root) Functions

Base Function: $f(x) = \sqrt{x}$

Graph of Base Function



Key Points:

x	y
0	0
1	1
4	2
9	3

Example 1: Using the parent function $f(x) = \sqrt{x}$, describe the transformations and write the equation of the transformed function $g(x)$.

$$g(x) = -2\, f\left[-\tfrac{1}{3}(x+6)\right] - 5$$

- *Vertical stretch bafo 2*
- *vertical reflection (reflection across the x-axis)*
- *Horizontal stretch bafo 3*
- *Horizontal reflection (reflection across the y-axis)*
- *Phase shift 6 units left*
- *Translate 5 units down*

$$g(x) = -2\sqrt{-\frac{1}{3}(x+6)} - 5$$

Example 2: for each of the following functions…

i) make a table of values for the parent function

ii) graph the parent function $f(x) = \sqrt{x}$

iii) describe the transformations

iv) make a table of values of image points

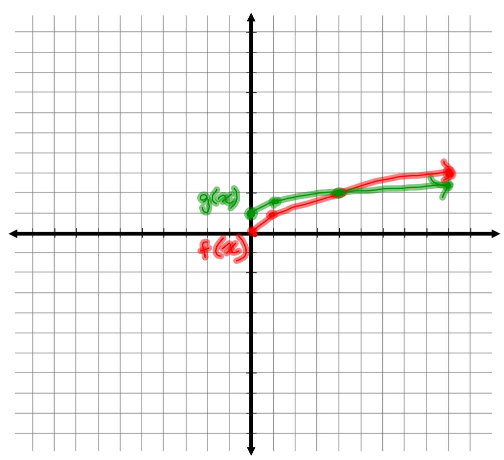
v) graph the transformed function and write it's equation

a)

$$g(x) = \frac{1}{2}f(x) + 1$$

vertical compression by a factor of $\frac{1}{2}$ $\left(\frac{y}{2}\right)$

shift up 1 unit $(y+1)$



$f(x) = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3

$g(x) = \frac{1}{2}\sqrt{x} + 1$

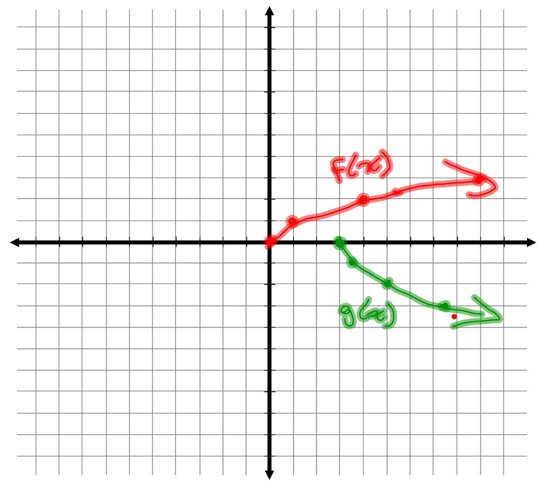
x	$\frac{y}{2}+1$
0	1
1	1.5
4	2
9	2.5

b) $g(x) = -f[2(x-3)]$

vertical reflection $(-y)$

horizontal compression by a factor of $\frac{1}{2}$ $\left(\frac{x}{2}\right)$

shift right 3 units $(x+3)$



$f(x) = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3

$g(x) = -\sqrt{2(x-3)}$

$\frac{x}{2}+3$	$-y$
3	0
3.5	-1
5	-2
7.5	-3

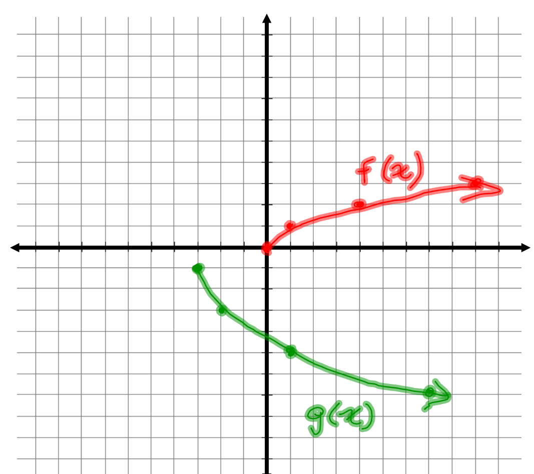
c) $g(x) = -2f(x+3) - 1$

vertical stretch by a factor of 2 $(2y)$

vertical reflection $(-y)$

shift left 3 units $(x-3)$

shift down 1 unit $(y-1)$



$f(x) = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3

$g(x) = -2\sqrt{x+3} - 1$

$x-3$	$-2y-1$
-3	-1
-2	-3
1	-5
6	-7

d) $g(x) = 3f\left[-\frac{1}{2}(x+2)\right] + 1$

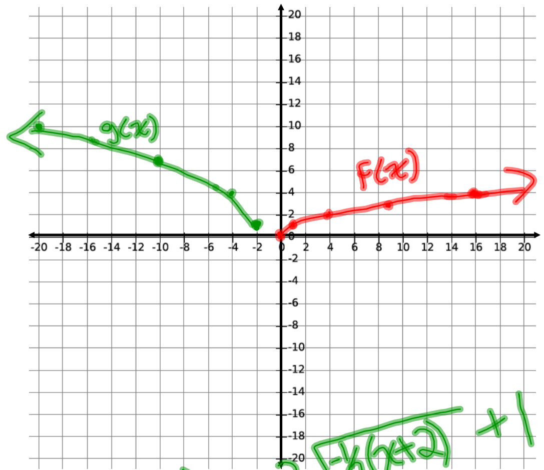
vertical stretch by a factor of 3 (3y)

horizontal stretch by a factor of 2 (2x)

horizontal reflection (-x)

shift left 2 units (x-2)

shift up 1 unit (y+1)



$g(x) = -3\sqrt{-\frac{1}{2}(x+2)} + 1$

$f(x) = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3

-2x-2	3y+1
-2	1
-4	4
-10	7
-20	10

Complete Worksheet