L1 – Solving Linear Systems by GRAPHING Unit 1
MPM2D
Jensen

Linear System: Two or more linear equations that are considered at the same time.

Point of Intersection: The point where 2 or more lines cross.

To **solve** a linear system means to find the values of the variables that satisfy ALL of the equations in the system. Graphically speaking, this means you will find the ordered pair (x, y) where the lines intersect.

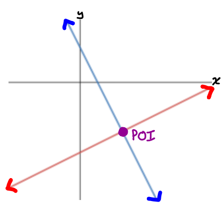
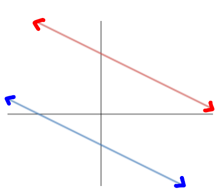
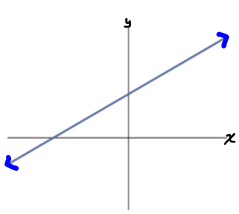
There are 3 main methods for solving a linear system:

1) Graphing
2) Substitution
3) Elimination

When solving by graphing, you can graph the lines by:

1) Using the slope and y-intercept (rearrange in to $y = mx + b$ form)
2) Use the x and y intercepts of each line
3) Create a table of values for each equation

A linear system could have 1, 0, or infinitely many solutions:

Graph	Slopes of Lines	Intercepts	Number of Solutions
Intersecting 	DIFFERENT	Usually different unless the lines intersect on an axis	1
Parallel & Distinct 	Same	Different	0
Parallel & Coincident 	Same	Same	Infinitely Many

Steps for Solving a Linear System by GRAPHING

1) Rearrange the equations in to slope y-intercept form ($y = mx + b$)

2) Graph equations and find the point of intersection

3) Verify that the point of intersection satisfies the equation of both lines

4) Clearly communicate your solution

Example 1: Find the point of intersection of the graphs of the following systems of equations.

a) ℓ_1: $y = x + 4$
ℓ_2: $y = -x + 2$

line 1

$y = \overset{m}{1}x + \overset{b}{4}$

slope $= m = \dfrac{1}{1}$ (rise / run)

y-int $= b = 4$

line 2

$y = \overset{m}{-1}x + \overset{b}{2}$

slope $= m = -1 = \dfrac{-1}{1}$ (rise / run)

check solution

ℓ_1

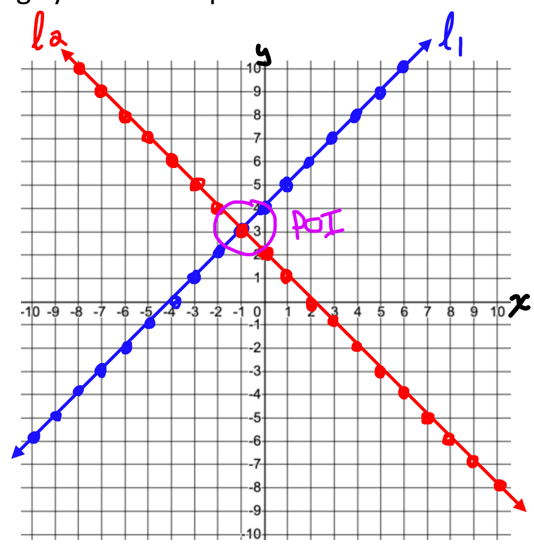
LS	RS
$= y$	$= x+4$
$= 3$	$= -1+4$
	$= 3$

✓

ℓ_2

LS	RS
$= y$	$= -x+2$
$= 3$	$= -(-1)+2$
	$= 3$

✓



The Point of Intersection is $(-1, 3)$

The solution is $x = -1$, $y = 3$

b) ℓ_1: $2x + y = 5$
ℓ_2: $x - 2y = 10$

line 1

$2x + y = 5$

$y = \overset{m}{-2}x + \overset{b}{5}$

slope $= m = -2 = \dfrac{-2}{1}$ (rise / run)

y-int $= b = 5$

line 2

$x - 2y = 10$

$x - 10 = 2y$

$\overset{m}{\tfrac{1}{2}}x - \overset{b}{5} = y$

slope $= m = \dfrac{1}{2}$ (rise / run)

y-int $= b = -5$

Check solution $x = 4$, $y = -3$

ℓ_1

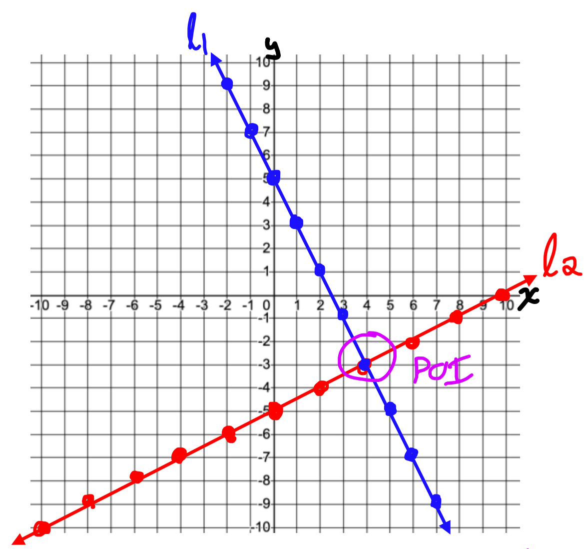
LS	RS
$= 2x+y$	$= 5$
$= 2(4)+(-3)$	
$= 5$	

✓

ℓ_2

LS	RS
$= x-2y$	$= 10$
$= 4-2(-3)$	
$= 10$	

✓



The point of intersection is $(4, -3)$

The solution is $x = 4$, $y = -3$

c) ℓ_1: $2x + 5y = -20$
ℓ_2: $5x - 3y = -15$

line 1

$2x+5y=-20$

$5y=-2x-20$

$y=\overset{m}{-\frac{2}{5}}x-4^{\,b}$

slope $= m = -\frac{2}{5}\begin{smallmatrix}\text{rise}\\ \text{run}\end{smallmatrix}$

y-int $= b = -4$

line 2

$5x-3y=-15$

$5x+15=3y$

$\overset{m}{\frac{5}{3}}x+\overset{b}{5}=y$

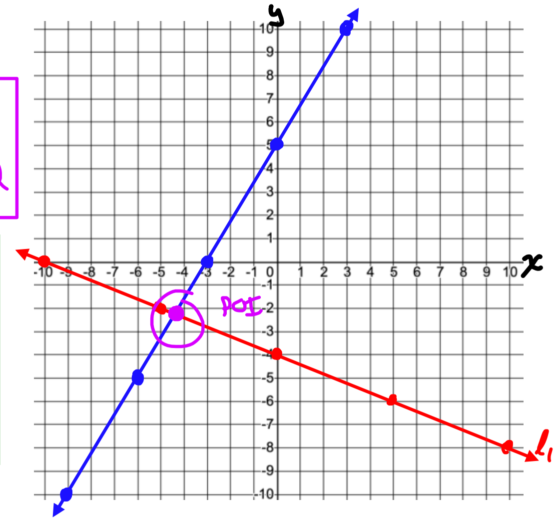
slope $= m = \frac{5}{3}\begin{smallmatrix}\text{rise}\\ \text{run}\end{smallmatrix}$

y-int $= b = 5$

POI : $(-4.3, -2.2)$

Solution : $x=-4.3,\ y=-2.2$

Note: *Our solution to this system is an estimate. The solution will not exactly verify the original equations but should be close!*



d) ℓ_1: $y = 2x + 3$
ℓ_2: $y = 2x - 4$

line 1

$y=\overset{m}{2}x+3^{\,b}$

slope $= m = 2 = \frac{2}{1}\begin{smallmatrix}\text{rise}\\ \text{run}\end{smallmatrix}$

y-int $= b = 3$

line 2

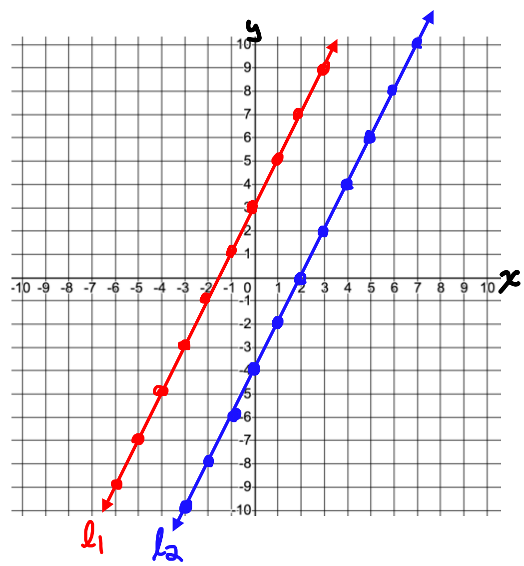
$y=\overset{m}{2}x-4^{\,b}$

slope $= m = 2 = \frac{2}{1}\begin{smallmatrix}\text{rise}\\ \text{run}\end{smallmatrix}$

y-int $= b = -4$

Notice the functions have the same slope but different y-intercepts. They will be parallel but distinct.

The lines are parallel and distinct.
$\therefore$ there are NO solutions.



e) ℓ_1: $x + y = 3$
ℓ_2: $2x + 2y = 6$

line 1

$x+y=3$

$y=\overset{m}{-}x+3^{\,b}$

slope $= m = -1 = \frac{-1}{1}\begin{smallmatrix}\text{rise}\\ \text{run}\end{smallmatrix}$

y-int $= b = 3$

line 2

$2x+2y=6$

$2y=-2x+6$

$y=\overset{m}{-}x+3^{\,b}$

slope $= m = -1 = \frac{-1}{1}\begin{smallmatrix}\text{rise}\\ \text{run}\end{smallmatrix}$

y-int $= b = 3$

Notice the lines have the same slope and same y-int. They will be parallel and coincident

The lines are parallel and coincident.
$\therefore$ there are INFINITE solutions.