L2 – Solving Linear Systems by SUBSTITUTION Unit 1
MPM2D
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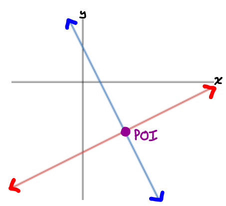
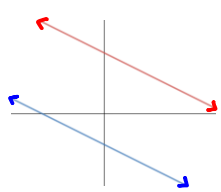
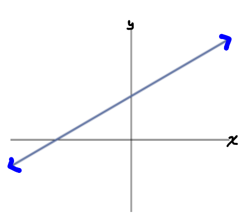
Remember that **solving** a linear system means to find the values of the variables that satisfy ALL of the equations in the system. Graphically speaking, this means you will find the ordered pair (x, y) where the lines intersect.

There are 3 main methods for solving a linear system:

1) Graphing
2) Substitution
3) Elimination

A linear system could have 1, 0, or infinitely many solutions:

Graph	Slopes of Lines	Intercepts	Number of Solutions	What happens algebraically
Intersecting 	DIFFERENT	Usually different unless the lines intersect on an axis	1	You will get a single solution for each variable that will satisfy both equations
Parallel & Distinct 	Same	Different	0	You will get an equation that is not true for any value of the variable Ex: $0x = 5$
Parallel & Coincident 	Same	Same	Infinitely Many	You will get an equation that is true for ALL values of the variable Ex: $0x = 0$

Steps for Solving by Substitution:

1) Rearrange either equation to isolate a variable (x or y)

2) Substitute what the isolated variable is equal to into the OTHER equation

3) Solve the new equation for the variable

4) Plug your answer back in to EITHER original equation to solve for the OTHER variable.

5) Check your answer in BOTH equations

Example 1: Solve the following systems using the method of substitution

a) ℓ_1: $x + 4y = 6$
ℓ_2: $2x - 3y = 1$

① Isolate x in l_1:
$x = -4y + 6$

② Sub in for x in l_2
$2x - 3y = 1$
$2(-4y+6) - 3y = 1$

③ Now solve for y
$-8y + 12 - 3y = 1$
$-11y = 1 - 12$
$-11y = -11$
$y = 1$

④ Sub $y=1$ in to l_1
$x = -4(1) + 6$
$x = 2$

⑤ Check solution $x=2$, $y=1$

l_1
LS $= x + 4y = 2 + 4(1) = 6$ ✓
RS $= 6$

l_2
LS $= 2x - 3y = 2(2) - 3(1) = 1$ ✓
RS $= 1$

The solution is $x=2$, $y=1$
The POI is $(2,1)$

b) ℓ_1: $5x - 3y - 2 = 0$
ℓ_2: $7x + y = 0$

① Isolate y in l_2
$7x + y = 0$
$y = -7x$

② sub in for y in l_1
$5x - 3y - 2 = 0$
$5x - 3(-7x) - 2 = 0$

③ Solve for x
$5x + 21x - 2 = 0$
$26x = 2$
$x = \frac{2}{26}$
$x = \frac{1}{13}$

④ sub $x = \frac{1}{13}$ into l_2
$y = -7x$
$y = -7\left(\frac{1}{13}\right)$
$y = \frac{-7}{13}$

⑤ Check solution $x = \frac{1}{13}$, $y = \frac{-7}{13}$

l_1
LS $= 5x - 3y - 2 = 5\left(\frac{1}{13}\right) - 3\left(\frac{-7}{13}\right) - 2 = \frac{5}{13} + \frac{21}{13} - 2 = \frac{26}{13} - 2 = 2 - 2 = 0$ ✓
RS $= 0$

l_2
LS $= 7x + y = 7\left(\frac{1}{13}\right) + \left(\frac{-7}{13}\right) = \frac{7}{13} - \frac{7}{13} = 0$ ✓
RS $= 0$

The solution is $x = \frac{1}{13}$, $y = \frac{-7}{13}$
The POI is $\left(\frac{1}{13}, \frac{-7}{13}\right)$

c) ℓ_1: $2x + 2y = 7$
ℓ_2: $x + y = 6$

① Isolate x in ℓ_2

$x + y = 6$
$x = 6 - y$

② sub in for x into ℓ_1

$2x + 2y = 7$
$2(6 - y) + 2y = 7$

③ solve for y

$12 - 2y + 2y = 7$
$-2y + 2y = 7 - 12$
$0y = -5$

There are NO solutions to this equation

The system has NO solutions.
The lines are parallel and distinct.

d) ℓ_1: $3x + 4y = 2$
ℓ_2: $9x + 12y = 6$

① Isolate for y in ℓ_1

$3x + 4y = 2$
$4y = -3x + 2$
$y = -\frac{3}{4}x + \frac{1}{2}$

② sub in for y in ℓ_2

$9x + 12y = 6$
$9x + 12\left(-\frac{3}{4}x + \frac{1}{2}\right) = 6$

③ solve for x

$9x - \frac{36}{4}x + \frac{12}{2} = 6$
$9x - 9x + 6 = 6$
$9x - 9x = 6 - 6$
$0x = 0$

There infinite solutions
to this equation

The system has infinitely many solutions.
The lines are parallel and coincident.