L3 – Solving Linear Systems by ELMINATION Unit 1
MPM2D
Jensen

Remember that **solving** a linear system means to find the values of the variables that satisfy ALL of the equations in the system. Graphically speaking, this means you will find the ordered pair (x, y) where the lines intersect.

There are 3 main methods for solving a linear system:

1) Graphing
2) Substitution
3) Elimination

Steps for Solving by ELIMINATION:

1) Get rid of decimals or fractions if necessary

2) Rewrite the equations with like terms in the same column $(x + y = \#)$

3) Multiply one or both equations by a number so that you have two equations in which the coefficients of one variable are the same or opposite

4) Add or subtract the equations to eliminate a variable and solve the resulting equation for the remaining variable

5) Substitute your solution for one of the variables in to either of the original equations to solve for the other variable

6) Check that the solutions satisfy BOTH of the original equations

Example 1: Solve each of the following linear systems using the method of ELIMINATION

a) ℓ_1: $3x + 2y = 19$
ℓ_2: $5x - 2y = 5$

$\ell_1 \rightarrow 3x + 2y = 19$
$\ell_2 \rightarrow 5x - 2y = 5$ +

$8x + 0y = 24$
$8x = 24$
$x = 3$

Sub $x=3$ into ℓ_1

$3x + 2y = 19$
$3(3) + 2y = 19$
$9 + 2y = 19$
$2y = 10$
$y = 5$

Check solution $x=3, y=5$

ℓ_1

LS	RS
$= 3x+2y$	$= 19$
$= 3(3)+2(5)$	
$= 19$ ✓	

ℓ_2

LS	RS
$= 5x-2y$	$= 5$
$= 5(3)-2(5)$	
$= 5$ ✓	

The solution is $x=3, y=5$
The POI is $(3,5)$

b) ℓ_1: $x + 4y = 6$
ℓ_2: $2x - 3y = 1$

$2 \times \ell_1 \rightarrow \quad 2x + 8y = 12$
$\ell_2 \rightarrow \quad 2x - 3y = 1 \quad -$

$0x + 11y = 11$
$11y = 11$
$y = 1$

sub $y=1$ into ℓ_1
$x + 4y = 6$
$x + 4(1) = 6$
$x = 2$

Check solution $x=2, y=1$

$\boxed{\ell_1}$

LS	RS
$= x + 4y$	$= 6$
$= 2 + 4(1)$	
$= 6$ ✓	

$\boxed{\ell_2}$

LS	RS
$= 2x - 3y$	$= 1$
$= 2(2) - 3(1)$	
$= 1$ ✓	

The solution is $x=2, y=1$
The POI is $(2,1)$

c) ℓ_1: $3x + 2y = 2$
ℓ_2: $4x + 5y = 12$

$4 \times \ell_1 \rightarrow \quad 12x + 8y = 8$
$3 \times \ell_2 \rightarrow \quad 12x + 15y = 36 \quad -$

$0x - 7y = -28$
$-7y = -28$
$y = 4$

sub $y=4$ into ℓ_1
$3x + 2y = 2$
$3x + 2(4) = 2$
$3x + 8 = 2$
$3x = -6$
$x = -2$

Check solution $x=-2, y=4$

$\boxed{\ell_1}$

LS	RS
$= 3x + 2y$	$= 2$
$= 3(-2) + 2(4)$	
$= 2$ ✓	

$\boxed{\ell_2}$

LS	RS
$= 4x + 5y$	$= 12$
$= 4(-2) + 5(4)$	
$= 12$ ✓	

The solution is $x=-2, y=4$
The POI is $(-2,4)$

d) $\ell_1: \ 0.6x - 0.3y = 2.4$
$\ell_2: -0.4y + 0.7x - 2.9 = 0 \rightarrow 0.7x - 0.4y = 2.9$

$10 \times \ell_1 \rightarrow 6x - 3y = 24 \xrightarrow{\times 4} 24x - 12y = 96$
$10 \times \ell_2 \rightarrow 7x - 4y = 29 \xrightarrow{\times 3} 21x - 12y = 87 \ -$

$3x + 0y = 9$
$3x = 9$
$x = 3$

sub $x = 3$ into ℓ_1
$6x - 3y = 24$
$6(3) - 3y = 24$
$18 - 3y = 24$
$-3y = 6$
$y = -2$

$\boxed{\text{The solution is } x = 3, y = -2 \quad \text{The POI is } (3, -2)}$

Check solution $x = 3, y = -2$

ℓ_1

LS
$= 0.6x - 0.3y$
$= 0.6(3) - 0.3(-2)$
$= 1.8 + 0.6$
$= 2.4$ ✓

RS
$= 2.4$

ℓ_2

LS
$= -0.4y + 0.7x - 2.9$
$= -0.4(-2) + 0.7(3) - 2.9$
$= 0.8 + 2.1 - 2.9$
$= 0$ ✓

RS
$= 0$

e) $\ell_1: \ \frac{x}{2} + \frac{y}{8} = 4$
$\ell_2: \ \frac{x}{3} - \frac{y}{2} = -2$

$8 \times \ell_1 \rightarrow 4x + y = 32 \xrightarrow{\times 3} 12x + 3y = 96$
$6 \times \ell_2 \rightarrow 2x - 3y = -12 \longrightarrow 2x - 3y = -12 \ +$

$14x + 0y = 84$
$14x = 84$
$x = 6$

sub $x = 6$ into ℓ_1
$4x + y = 32$
$4(6) + y = 32$
$24 + y = 32$
$y = 8$

$\boxed{\text{The solution is } x = 6, y = 8 \quad \text{the POI is } (6, 8)}$

Check solution

$\boxed{\ell_1}$

LS
$= \frac{x}{2} + \frac{y}{8}$
$= \frac{6}{2} + \frac{8}{8}$
$= 3 + 1$
$= 4$ ✓

RS
$= 4$

$\boxed{\ell_2}$

LS
$= \frac{x}{3} - \frac{y}{2}$
$= \frac{6}{3} - \frac{8}{2}$
$= 2 - 4$
$= -2$ ✓

RS
$= -2$

f) ℓ_1: $5x + 2y = 2$
ℓ_2: $10x + 4y = -4$

$$\begin{array}{rl} 2 \times \ell_1 \rightarrow & 10x + 4y = 4 \\ \ell_2 \rightarrow & \underline{10x + 4y = -4} \quad - \\ & 0x + 0y = 8 \\ & 0 = 8 \end{array}$$

There are NO SOLUTIONS to this equation.

There are NO SOLUTIONS to the linear system. The lines are parallel and distinct.

Helpful tip:

*When coefficients of a variable have opposite signs, **ADDING** will eliminate them*
*When coefficients of a variable have the same sign, **SUBTRACTING** will eliminate them*