Many problems with 2 unknowns can be solved using a system of 2 linear equations. To solve these types of problems you should:

1) Assign variables to each of the unknowns
2) Write 2 equations showing the relationships between the variables. Each equation should include both variables.
3) Solve the system of equations using any method (graphing, substitution, elimination)
4) Check your solution
5) Clearly communicate your final answer

Example 1: Find the value of two numbers if their sum is 13 and their difference is 5.

x = first #
y = second #

Eqn 1: $x + y = 13$
Eqn 2: $x - y = 5$

solve system with elimination

(1) $x + y = 13$
(2) $x - y = 5$ +

$2x = 18$
$x = 9$

$x + y = 13$
$9 + y = 13$
$y = 4$

$\therefore$ the two numbers are 9 and 4.

Example 2: The Sports Shop sells Nike running shoes for \$82 a pair and Air Jensen basketball shoes for \$95 a pair. One day, the Sports Shop sells 75 pairs of Nike and Air Jensen shoes totaling \$6241 in sales. How many pairs of each shoe were sold?

x = # of Nike sold
y = # of AJ sold.

(1) $x + y = 75$
(2) $82x + 95y = 6241$

solve system using substitution

(1) $x + y = 75$
$y = 75 - x$

(2) $82x + 95y = 6241$
$82x + 95(75 - x) = 6241$
$82x + 7125 - 95x = 6241$
$-13x = -884$
$x = 68$

$y = 75 - 68$
$y = 7$

$\therefore$ they sold 68 Nike's and 7 AJ's

Example 3: A blue spruce tree grows an average of 15 cm per year. An eastern hemlock grows an average of 10 cm per year. When they were planted, a blue spruce was 120 cm tall and an eastern hemlock was 180 cm tall. How many years after planting will the trees reach the same height? How tall will that be?

x = # of years

y = height

① $y = 120 + 15x$

② $y = 180 + 10x$

Solve system using substitution

① $y = 120 + 15x$ ② $y = 180 + 10x$

$120 + 15x = 180 + 10x$

$5x = 60$

$x = 12$

$y = 120 + 15(12)$

$y = 120 + 180$

$y = 300$

$\boxed{\therefore \text{ After 12 years both trees will be 300 cm tall.}}$

Example 4: Tia had \$10 000 to invest. She invested part of it in a term deposit paying 4% per annum and the remainder in bonds paying 5% per annum. If the total interest earned after one year was \$440, how much did she invest in each account?

x = amount in term deposit

y = amount in bonds

① $x + y = 10\,000$

② $0.04x + 0.05y = 440$

Solve using elimination:

$4 \times$ ① $4x + 4y = 40\,000$

$100 \times$ ② $4x + 5y = 44\,000$ −

$-y = -4000$

$y = 4000$

① $x + y = 10\,000$

$x + 4000 = 10\,000$

$x = 6000$

$\boxed{\therefore \text{ She invested \$6000 in a term deposit and \$4000 in bonds}}$

Example 5: A chemistry teacher needs to make 10L of 42% sulfuric acid solution. The acid solutions available are 30% sulfuric acid and 50% sulfuric acid, by volume. How many liters of each solution must be mixed to make the 42% solution?

x = amount of 30% acid
y = amount of 50% acid

① $x + y = 10$ (volume of solution)
② $0.3x + 0.5y = 0.42(10)$ (amount of pure acid)

solve using elimination:

$3 \times$① $3x + 3y = 30$
$10 \times$② $3x + 5y = 42$ −
$-2y = -12$
$y = 6$

① $x + y = 10$
$x + 6 = 10$
$x = 4$

$\therefore$ you need 4L of 30% acid and 6L of 50% acid.

Example 6: A riverboat took 2 hours to travel 24km, down a river with the current and 3 hours to make the return trip against the current. Find the speed of the boat in still water and the speed of the current.

Note:

Speed travelling with current = boat speed + current speed
Speed travelling against current = boat speed - current speed

Remember:

$distance = speed \times time$

x = speed of boat in still water
y = speed of current

① $2(x+y) = 24$ (with current)
② $3(x-y) = 24$ (against current)

solve using elimination

① $x + y = 12$
② $x - y = 8$ +
$2x = 20$
$x = 10$

① $x + y = 12$
$10 + y = 12$
$y = 2$

$\therefore$ the speed of the boat in still water is 10 km/h and the speed of the current is 2 km/h.