W1 – Solving Linear Systems by GRAPHING Unit 1

MPM2D

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1) Solve each system by graphing.

a) ℓ_1: $y = 2x + 1$
ℓ_2: $y = x - 2$

Solution: $x=-3, y=-5$

Line 1

$y = \overset{m}{2}x + \overset{b}{1}$

slope $= m = \frac{2}{1}$

y-int $= b = 1$

Line 2

$y = \overset{m}{1}x - \overset{b}{2}$

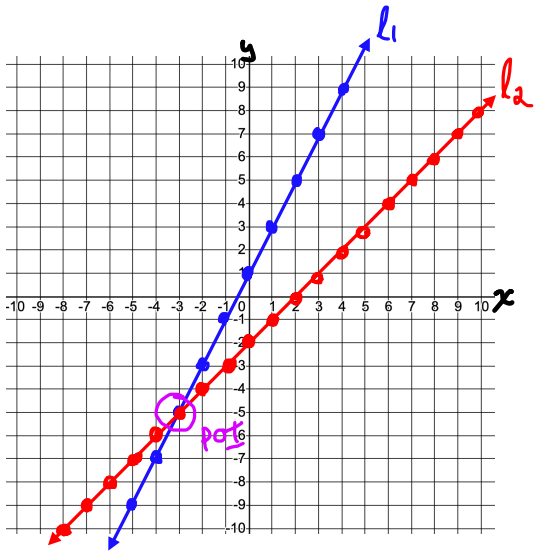
slope $= m = \frac{1}{1}$

y-int $= b = -2$

Check:

ℓ_1: LS $= y = -5$; RS $= 2x+1 = 2(-3)+1 = -5$ ✓

ℓ_2: LS $= y = -5$; RS $= x-2 = -3-2 = -5$ ✓



b) ℓ_1: $x + 2y = 2$
ℓ_2: $x + y = 3$

Solution: $x=4, y=-1$

Line 1

$x+2y=2$

$2y = -x+2$

$y = \overset{m}{-\frac{1}{2}}x + \overset{b}{1}$

slope $= m = \frac{-1}{2}$

y-int $= b = 1$

Line 2

$x+y=3$

$y = \overset{m}{-1}x + \overset{b}{3}$

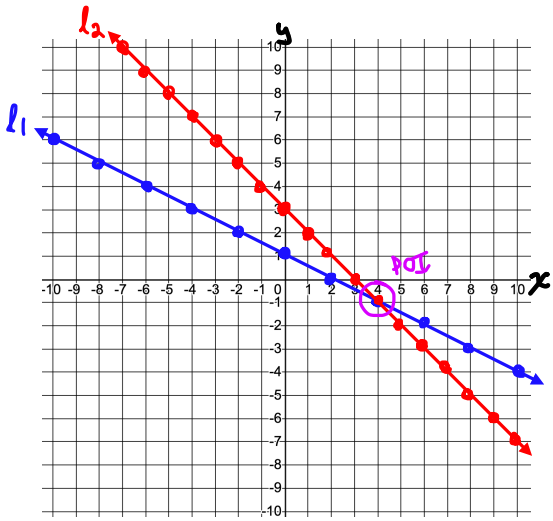
slope $= m = \frac{-1}{1}$

y-int $= b = 3$

Check:

ℓ_1: LS $= x+2y = 4+2(-1) = 2$; RS $= 2$ ✓

ℓ_2: LS $= x+y = 4+(-1) = 3$; RS $= 3$ ✓



c) ℓ_1: $y = 2x - 3$
ℓ_2: $2x - y = 5$

Line 1

$y = \overset{m}{2}x - \overset{b}{3}$

slope $= m = \frac{2}{1}$

y-int $= b = -3$

Line 2

$2x - y = 5$

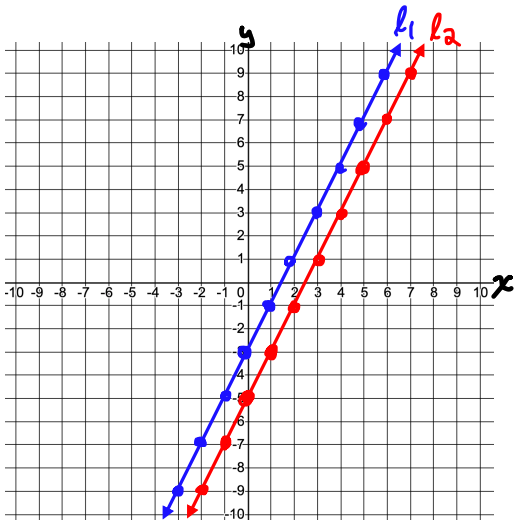
$2x - 5 = y$

$y = \overset{m}{2}x - \overset{b}{5}$

slope $= m = \frac{2}{1}$

y-int $= b = -5$

No solutions. The lines are parallel and distinct



d) ℓ_1: $3x = y + 4$
ℓ_2: $6x - 2y - 8 = 0$

Line 1

$3x = y + 4$

$3x - 4 = y$

$y = 3x - 4$ (m, b)

slope $= m = \frac{3}{1}$

y-int $= b = -4$

Line 2

$6x - 2y - 8 = 0$

$6x - 8 = 2y$

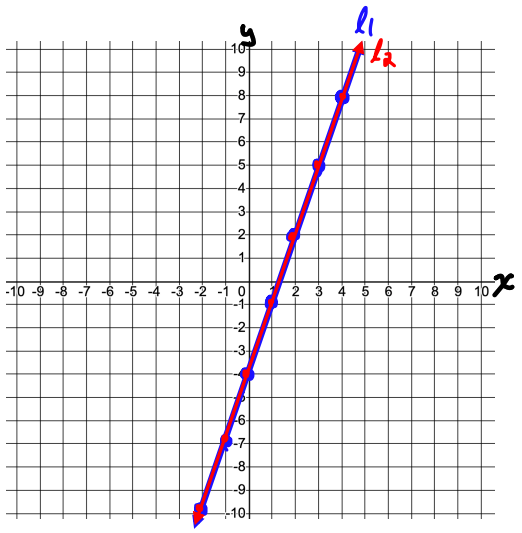
$3x - 4 = y$

$y = 3x - 4$ (m, b)

slope $= m = \frac{3}{1}$

y-int $= b = -4$

Infinitely many solutions. The lines are parallel and coincident.



e) ℓ_1: $3x + 2y = 3$
ℓ_2: $2x + 10y = -5$

Line 1

$3x + 2y = 3$

$2y = -3x + 3$

$y = -\frac{3}{2}x + \frac{3}{2}$ (m, b)

slope $= m = -\frac{3}{2}$

y-int $= b = \frac{3}{2}$

Line 2

$2x + 10y = -5$

$10y = -2x - 5$

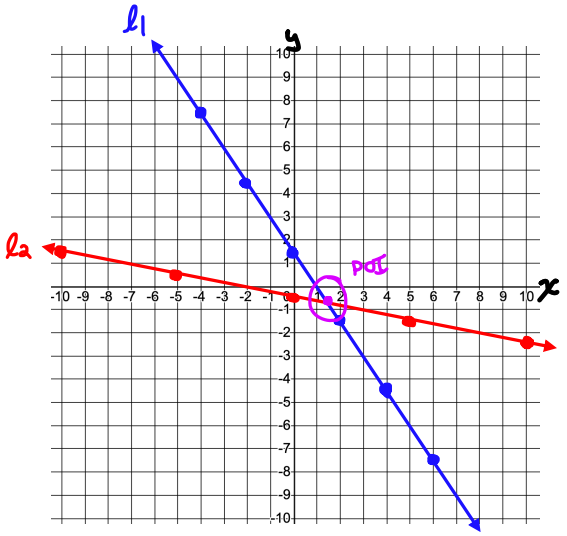
$y = -\frac{2}{10}x - \frac{5}{10}$

$y = -\frac{1}{5}x - \frac{1}{2}$ (m, b)

slope $= m = -\frac{1}{5}$

y-int $= b = -\frac{1}{2}$

Estimated solution:
$x = 1.5, y = -0.8$



f) ℓ_1: $2x + 6y - 12 = 0$
ℓ_2: $6x - 3y - 15 = 0$

solution: $x = 3, y = 1$

Line 1

$2x + 6y - 12 = 0$

$6y = -2x + 12$

$y = -\frac{2}{6}x + 2$

$y = -\frac{1}{3}x + 2$ (m, b)

slope $= m = -\frac{1}{3}$

y-int $= b = 2$

Line 2

$6x - 3y - 15 = 0$

$6x - 15 = 3y$

$2x - 5 = y$

$y = 2x - 5$ (m, b)

slope $= m = \frac{2}{1}$

y-int $= b = -5$

Check:

ℓ_1

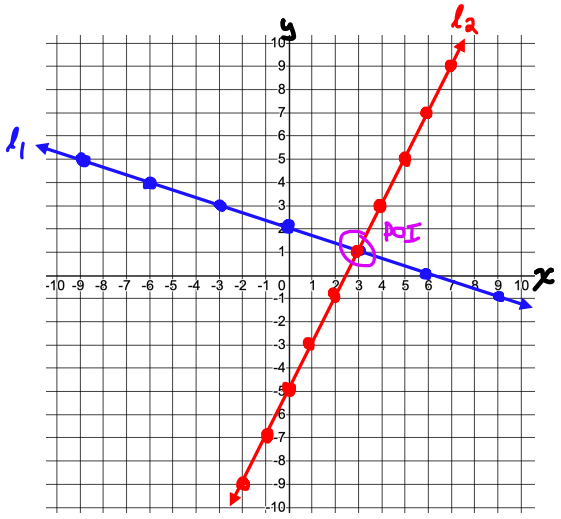
LS $= 2x + 6y - 12$
$= 2(3) + 6(1) - 12$
$= 0$

RS $= 0$

ℓ_2

LS $= 6x - 3y - 15$
$= 6(3) - 3(1) - 15$
$= 0$

RS $= 0$



2) Without graphing, determine whether each system has one solution, no solutions, or infinitely many solutions. Explain.

a) ℓ_1: $2x + y = 5$
ℓ_2: $2x + 10y = -5$

Line 1

$2x+y=5$

$y=-2x+5$

slope $= m = -2$

y-int $= b = 5$

Line 2

$2x+10y=-5$

$10y=-2x-5$

$y=-\frac{2}{10}x-\frac{5}{10}$

$y=-\frac{1}{5}x-\frac{1}{2}$

slope $= m = -\frac{1}{5}$

y-int $= b = -\frac{1}{2}$

The lines have different slopes; there is 1 solution to the system.

b) ℓ_1: $3x - y = 0$
ℓ_2: $6x - 2y = 3$

Line 1

$3x-y=0$

$3x=y$

$y=3x+0$

slope $= m = 3$

y-int $= b = 0$

Line 2

$6x-2y=3$

$6x-3=2y$

$3x-\frac{3}{2}=y$

$y=3x-\frac{3}{2}$

slope $= m = 3$

y-int $= b = -\frac{3}{2}$

Same slope but different y-intercepts; the lines are parallel and distinct and have NO solutions.

c) ℓ_1: $x + y = 2$
ℓ_2: $3x = 6 - 3y$

Line 1

$x+y=2$

$y=-x+2$

slope $= m = -1$

y-int $= b = 2$

Line 2

$3x=6-3y$

$3y=-3x+6$

$y=-1x+2$

slope $= m = -1$

y-int $= b = 2$

same slope and y-intercept. The lines are parallel and coincident. There are infinitely many solutions

Answers:

1)a) $(-3,-5)$ **b)** $(4,-1)$ **c)** no solutions; parallel and distinct **d)** infinite solutions; parallel and coincident

e) $(1.5,-0.8)$ this is an approximate answer **f)** $(3,1)$

2)a) one solution **b)** no solutions **c)** infinitely many solutions