W2 – Solving Linear Systems by SUBSTITUTION Unit 1

MPM2D

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1) Solve each linear system using the method of substitution. Check your answers.

a) $\ell_1: y = 3x - 4$
$\ell_2: x + y = 8$

① $y = 3x - 4$

$y = 3(3) - 4$

$y = 5$

② $x + y = 8$

$x + (3x - 4) = 8$

$4x - 4 = 8$

$4x = 12$

$x = 3$

Solution: $x = 3, y = 5$

b) $\ell_1: x = -4y + 5$
$\ell_2: x + 2y = 7$

① $x = -4y + 5$

$x = -4(-1) + 5$

$x = 9$

② $x + 2y = 7$

$(-4y + 5) + 2y = 7$

$-2y + 5 = 7$

$-2y = 2$

$y = -1$

Solution: $x = 9, y = -1$

c) $\ell_1: y = -2x + 3$
$\ell_2: 4x - 3y = 1$

① $y = -2x + 3$

$y = -2(1) + 3$

$y = 1$

② $4x - 3y = 1$

$4x - 3(-2x + 3) = 1$

$4x + 6x - 9 = 1$

$10x = 10$

$x = 1$

Solution: $x = 1, y = 1$

d) $\ell_1: 2x + 4y = 2$
$\ell_2: x = 1 - 2y$

① $2x + 4y = 2$

$2(1 - 2y) + 4y = 2$

$2 - 4y + 4y = 2$

$0y = 0$

② $x = 1 - 2y$

infinitely many solutions.
The lines are parallel and coincident.

e) $\ell_1: x + 2y = 3$
$\ell_2: 5x + 4y = 8$

① $x + 2y = 3$

$x = 3 - 2y$

$x = 3 - 2\left(\frac{7}{6}\right)$

$x = 3 - \frac{7}{3}$

$x = \frac{9}{3} - \frac{7}{3}$

$x = \frac{2}{3}$

② $5x + 4y = 8$

$5(3 - 2y) + 4y = 8$

$15 - 10y + 4y = 8$

$-6y = -7$

$y = \frac{7}{6}$

Solution: $x = \frac{2}{3}, y = \frac{7}{6}$

f) $\ell_1: 6x + 5y = 7$
$\ell_2: x - y = 3$

① $6x + 5y = 7$

$6(3 + y) + 5y = 7$

$18 + 6y + 5y = 7$

$11y = -11$

$y = -1$

② $x - y = 3$

$x = 3 + y$

$x = 3 + (-1)$

$x = 2$

Solution: $x = 2, y = -1$

g) $\ell_1: 2m + n = 2$
$\ell_2: 3m - 2n = 3$

① $2m+n=2$
$n = 2-2m$
$n = 2-2(1)$
$n = 0$

② $3m-2n=3$
$3m-2(2-2m)=3$
$3m-4+4m=3$
$7m=7$
$m=1$

Solution: $m=1$, $n=0$

h) $\ell_1: 3a + 2b = 4$
$\ell_2: 2a + b = 6$

① $3a+2b=4$
$3a+2(6-2a)=4$
$3a+12-4a=4$
$-a=-8$
$a=8$

② $2a+b=6$
$b=6-2a$
$b=6-2(8)$
$b=-10$

Solution: $a=8$, $b=-10$

i) $\ell_1: 2x + y = 4$
$\ell_2: 4x - y = 2$

① $2x+y=4$
$y=4-2x$
$y=4-2(1)$
$y=2$

②$4x-y=2$
$4x-(4-2x)=2$
$4x-4+2x=2$
$6x=6$
$x=1$

solution: $x=1$, $y=2$

2) Find the point of intersection of each pair of lines.

a) $\ell_1: 2x = y + 5$
$\ell_2: 3x + y = -9$

① $2x=y+5$
$2x-5=y$
$2\left(-\frac{4}{5}\right)-5=y$
$-\frac{8}{5}-\frac{25}{5}=y$
$-\frac{33}{5}=y$

② $3x+y=-9$
$3x+(2x-5)=-9$
$5x-5=-9$
$5x=-4$
$x=-\frac{4}{5}$

solution: $x=-\frac{4}{5}$, $y=-\frac{33}{5}$

b) $\ell_1: 4x + 2y = 7$
$\ell_2: -x - 7y = 6$

① $4x+2y=7$
$4(-7y-6)+2y=7$
$-28y-24+2y=7$
$-26y=31$
$y=-\frac{31}{26}$

② $-x-7y=6$
$-7y-6=x$
$-7\left(\frac{-31}{26}\right)-6=x$
$\frac{217}{26}-\frac{156}{26}=x$
$x=\frac{61}{26}$

solution: $x=\frac{61}{26}$, $y=-\frac{31}{26}$

c) $\ell_1: p + 4q = 3$
$\ell_2: 5p = -2q + 3$

① $p + 4q = 3$
$p = 3 - 4q$
$p = 3 - 4\left(\frac{2}{3}\right)$
$p = \frac{9}{3} - \frac{8}{3}$
$p = \frac{1}{3}$

② $5p = -2q + 3$
$5(3 - 4q) = -2q + 3$
$15 - 20q = -2q + 3$
$12 = 18q$
$\frac{12}{18} = q$
$q = \frac{2}{3}$

solution: $p = \frac{1}{3}, \frac{2}{3}$

d) $\ell_1: a + b + 6 = 0$
$\ell_2: 2a - b - 3 = 0$

① $a + b + 6 = 0$
$a + (2a - 3) + 6 = 0$
$3a + 3 = 0$
$3a = -3$
$a = -1$

② $2a - b - 3 = 0$
$2a - 3 = b$
$2(-1) - 3 = b$
$b = -5$

solution: $a = -1, b = -5$

e) $\ell_1: x - 2y - 2 = 0$
$\ell_2: 3x + 4y - 16 = 0$

① $x - 2y - 2 = 0$
$x = 2y + 2$
$x = 2(1) + 2$
$x = 4$

② $3x + 4y - 16 = 0$
$3(2y + 2) + 4y - 16 = 0$
$6y + 6 + 4y - 16 = 0$
$10y - 10 = 0$
$10y = 10$
$y = 1$

solution: $x = 4, y = 1$

3) Samantha works twice as many hours per week as Adriana. Together they work a total of 39 hours in one week.

a) Write an equation to represent the information in the first sentence.

$S = 2A$

b) Write an equation to represent the information in the second sentence.

$S + A = 39$

c) Use the method of substitution to find the number of hours worked by each person.

$S = 2A$

$S = 2(13)$

$S = 26$

$S + A = 39$

$2A + A = 39$

$3A = 39$

$A = 13$

Solution: Samantha worked 26 hours
Adriana worked 13 hours

4) Ugo plays hockey and is awarded 2 points for each goal and 1 point for each assist. Last season he had a total of 86 points. He scored 17 fewer goals than assists.

a) Write a linear system to represent the information

① $2g + a = 86$

② $a - g = 17$

b) Solve the system

$2g + a = 86$

$2g + (17 + g) = 86$

$3g = 69$

$g = 23$

$a - g = 17$

$a = 17 + g$

$a = 17 + 23$

$a = 40$

c) What does the solution represent in the context of this question?

Ugo scored 23 goals and had 40 assists.

5) Joanne's family decides to rent a hall for her retirement party. Pin Hall charges $500 for the hall and $15 per meal. Bloom Place charges $350 for the hall and $18 per meal.

a) Write two equations to represent the information.

① $C = 500 + 15g$

② $C = 350 + 18g$

b) Solve the linear system to find the number of guests for which the charges are the same at both halls.

$C = 500 + 15g$

$350 + 18g = 500 + 15g$

$3g = 150$

$g = 50$

$C = 350 + 18g$

$C = 350 + 18(50)$

$C = 1250$

Solution: Both halls would charge $1250 if 50 guests attended.

Answers:

1) a) $x = 3, y = 5$ **b)** $x = 9, y = -1$ **c)** $x = 1, y = 1$ **d)** infinite solutions
e) $x = \frac{2}{3}, y = \frac{7}{6}$ **f)** $x = 2, y = -1$ **g)** $m = 1, n = 0$ **h)** $a = 8, b = -10$ **i)** $x = 1, y = 2$
2)a) $x = -\frac{4}{5}, y = -\frac{33}{5}$ **b)** $x = \frac{61}{26}, y = -\frac{31}{26}$ **c)** $p = \frac{1}{3}, q = \frac{2}{3}$ **d)** $a = -1, b = -5$ **e)** $x = 4, y = 1$
3)a) $S = 2a$ **b)** $S + A = 39$ **c)** Samantha worked 26 hours and Adriana worked 13 hours
4)a) $2g + a = 86; g = a - 17$ **b)** $g = 23, a = 40$ **c)** 23 goals; 40 assists
5)a) $C = 500 + 15n; C = 350 + 18n$ **b)** 50 guests