1) Solve using the method of elimination

a) $\ell_1: x + y = 2$
$\ell_2: 3x - y = 2$

$\ell_1 \rightarrow \quad x+y=2$
$\ell_2 \rightarrow \quad 3x-y=2$ $+$
$\qquad 4x \quad = 4$
$\qquad x = 1$

sub $x=1$ into ℓ_1
$x+y=2$
$1+y=2$
$y=1$

solution: $x=1, y=1$

b) $\ell_1: x - y = -1$
$\ell_2: 3x + y = -7$

$\ell_1 \rightarrow \quad x-y=-1$
$\ell_2 \rightarrow \quad 3x+y=-7$ $+$
$\qquad 4x \quad = -8$
$\qquad x = -2$

sub $x=-2$ into ℓ_1:
$x-y=-1$
$-2-y=-1$
$-1=y$

solution: $x=-2, y=-1$

c) $\ell_1: 3x + 3y = 7$
$\ell_2: x + y = 3$

$\ell_1 \rightarrow \quad 3x+3y=7$
$3 \times \ell_2 \rightarrow \quad 3x+3y=9$ $-$
$\qquad 0x+0y=-2$

No solutions. The lines are parallel and distinct.

d) $\ell_1: 5x + 2y = -11$
$\ell_2: 3x + 2y = -9$

$\ell_1 \rightarrow \quad 5x+2y=-11$
$\ell_2 \rightarrow \quad 3x+2y=-9$ $-$
$\qquad 2x \quad = -2$
$\qquad x = -1$

sub $x=-1$ into ℓ_2
$3x+2y=-9$
$3(-1)+2y=-9$
$-3+2y=-9$
$2y=-6$
$y=-3$

solution: $x=-1, y=-3$

2) Find the point of intersection of each pair of lines

a) $\ell_1: x + 2y = 2$
$\ell_2: 3x + 5y = 4$

$3 \times \ell_1 \rightarrow \quad 3x+6y=6$
$\ell_2 \rightarrow \quad 3x+5y=4$ $-$
$\qquad y=2$

sub $y=2$ into ℓ_1
$x+2y=2$
$x+2(2)=2$
$x+4=2$
$x=-2$

solution: $x=-2, y=2$

b) $\ell_1: 3x + 5y = 12$
$\ell_2: 2x - y = -5$

$\ell_1 \rightarrow \quad 3x+5y=12$
$5 \times \ell_2 \rightarrow \quad 10x-5y=-25$ $+$
$\qquad 13x = -13$
$\qquad x=-1$

sub $x=-1$ into ℓ_2
$2x-y=-5$
$2(-1)-y=-5$
$-2+5=y$
$y=3$

solution: $x=-1, y=3$

c)
$\ell_1: 3x + y = 13$
$\ell_2: 2x + 3y = 18$

$3 \times \ell_1 \to \quad 9x + 3y = 39$
$\ell_2 \to \quad \underline{2x + 3y = 18} \;-$
$\qquad\qquad 7x = 21$
$\qquad\qquad x = 3$

sub $x=3$ into ℓ_1
$3x + y = 13$
$3(3) + y = 13$
$9 + y = 13$
$y = 4$

solution: $x = 3, y = 4$

d)
$\ell_1: 6x + 5y = 12$
$\ell_2: 3x - 4y = 6$

$\ell_1 \to \quad 6x + 5y = 12$
$2 \times \ell_2 \to \quad \underline{6x - 8y = 12} \;-$
$\qquad\qquad 13y = 0$
$\qquad\qquad y = 0$

sub $y=0$ into ℓ_1
$6x + 5y = 12$
$6x + 5(0) = 12$
$6x = 12$
$x = 2$

solution: $x = 2, y = 0$

3) Solve by elimination

a)
$\ell_1: 3x - 2y = 5$
$\ell_2: 2x + 3y = 12$

$2 \times \ell_1 \to \quad 6x - 4y = 10$
$3 \times \ell_2 \to \quad 6x + 9y = 36 \;-$
$\qquad\qquad -13y = -26$
$\qquad\qquad y = 2$

sub $y=2$ into ℓ_2
$2x + 3y = 12$
$2x + 3(2) = 12$
$2x + 6 = 12$
$2x = 6$
$x = 3$

solution: $x = 3, y = 2$

b)
$\ell_1: 5m + 2n = 5$
$\ell_2: 2m + 3n = 13$

$2 \times \ell_1 \to \quad 10m + 4n = 10$
$5 \times \ell_2 \to \quad \underline{10m + 15n = 65} \;-$
$\qquad\qquad -11n = -55$
$\qquad\qquad n = 5$

sub $n=5$ into ℓ_1
$5m + 2n = 5$
$5m + 2(5) = 5$
$5m + 10 = 5$
$5m = -5$
$m = -1$

solution: $m = -1, n = 5$

c)
$\ell_1: 3a - 4b = 10$
$\ell_2: 5a - 12b = 6$

$3 \times \ell_1 \to \quad 9a - 12b = 30$
$\ell_2 \to \quad \underline{5a - 12b = 6} \;-$
$\qquad\qquad 4a = 24$
$\qquad\qquad a = 6$

sub $a=6$ into ℓ_1
$3a - 4b = 10$
$3(6) - 4b = 10$
$18 - 10 = 4b$
$8 = 4b$
$b = 2$

solution: $a = 6, b = 2$

d)
$\ell_1: 3h - 4k = 5$
$\ell_2: 5h + 3k = -11$

$3 \times \ell_1 \to \quad 9h - 12k = 15$
$4 \times \ell_2 \to \quad \underline{20h + 12k = -44} \;+$
$\qquad\qquad 29h = -29$
$\qquad\qquad h = -1$

sub $h=-1$ into ℓ_2
$5h + 3k = -11$
$5(-1) + 3k = -11$
$-5 + 3k = -11$
$3k = -6$
$k = -2$

solution: $h = -1, k = -2$

4) Mehrad works in a department store selling sports equipment. Baseball gloves cost $29 each and bats cost $14 each. One shift, he sells 28 items. His receipts total $647.

a) How many bats did Mehrab sell?

$x = $ # of gloves
$y = $ # of bats

① $x + y = 28$
② $29x + 14y = 647$

$14 \times ① \rightarrow 14x + 14y = 392$
② $\rightarrow 29x + 14y = 647$ —

$-15x = -255$
$x = 17$

sub $x = 17$ into ①
$x + y = 28$
$17 + y = 28$
$y = 11$

Mehrab sold 11 bats

b) How many gloves did he sell?

17 gloves

5) Maria rented the same car twice in one month. She paid $180 the first time for 3 days and she drove a total of 150 km. The next time, she also paid $180 and had the vehicle for only 2 days, but travelled 400 km.

a) What was the cost per day?

$x = $ cost per day
$y = $ cost per km

① $3x + 150y = 180$
② $2x + 400y = 180$

$2 \times ①$ $6x + 300y = 360$
$3 \times ②$ $6x + 1200y = 540$ —
$-900y = -180$
$y = 0.2$

sub $y = 0.2$ into ①
$3x + 150y = 180$
$3x + 150(0.2) = 180$
$3x + 30 = 180$
$3x = 150$
$x = 50$

$50 per day

b) What was the cost per km?

$0.20 per km