L1 – Midpoint and Length of a Line Segment Unit 2

MPM2D

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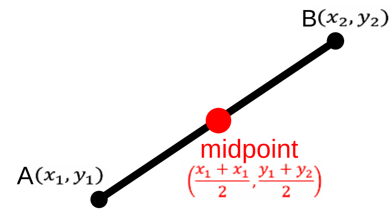
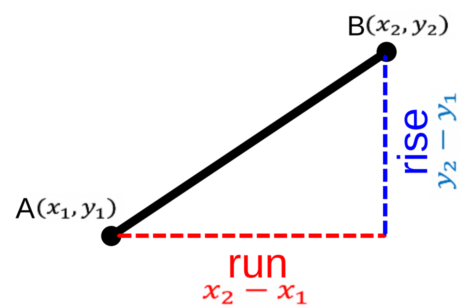
To find the **midpoint** of a line segment, you must find the middle (average) of both the x and y coordinates of the endpoints. If A has coordinates (x_1, y_1) and B has coordinates (x_2, y_2), then the coordinates of the midpoint of line segment AB are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$



To find the **length** of a line segment, we first construct a right-angle triangle using the rise and run of the line segment. The run is the difference in the x-coordinates of the endpoints, and the rise is the difference in the y-coordinates of the endpoints. You can then use Pythagorean Theorem, $a^2 + b^2 = c^2$ to calculate the length of the line segment.

$$(length\ of\ AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

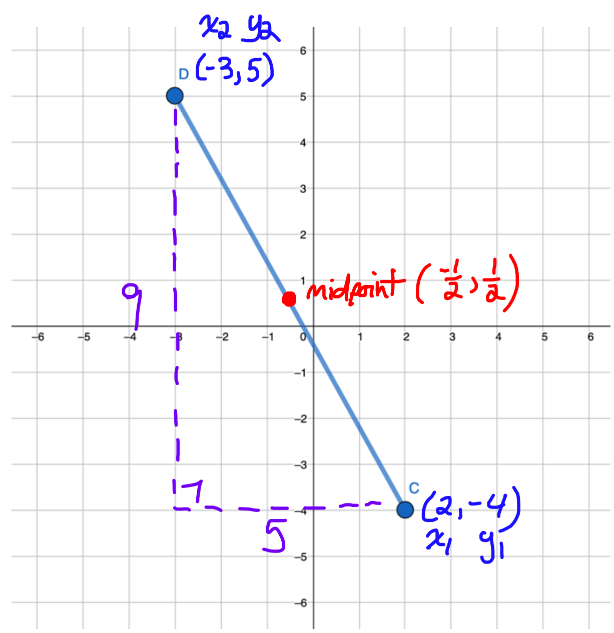
$$length\ of\ AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example 1: Calculate the length and midpoint of the line segment joining the endpoints $C(2, -4)$ and $D(-3, 5)$.

$$\text{midpoint}_{CD} = \left(\frac{2+(-3)}{2}, \frac{-4+5}{2}\right)$$

$$= \left(\frac{-1}{2}, \frac{1}{2}\right)$$



$$\text{length}_{CD} = \sqrt{(-3-2)^2 + [5-(-4)]^2}$$

$$= \sqrt{(-5)^2 + (9^2)}$$

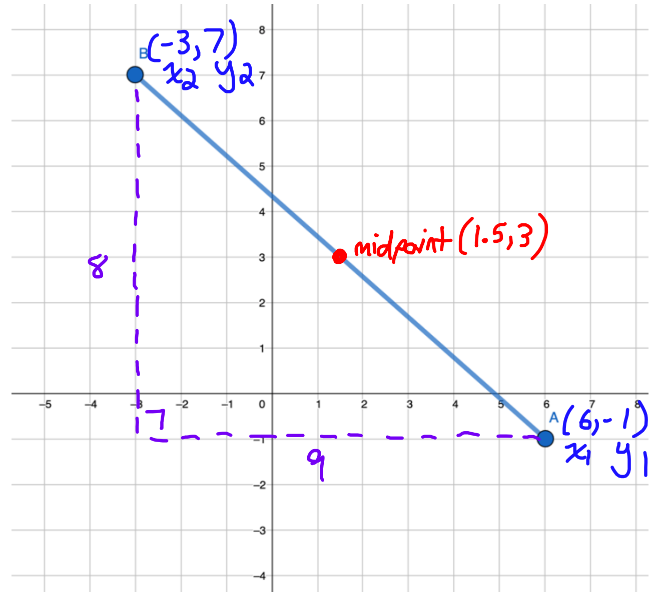
$$= \sqrt{106}$$

$$\simeq 10.3 \text{ units}$$

Example 2: Calculate the length and midpoint of the line segment joining the endpoints $A(6,-1)$ and $B(-3,7)$.

$$\text{midpoint}_{AB} = \left(\frac{6+(-3)}{2}, \frac{-1+7}{2}\right)$$

$$= \left(\frac{3}{2}, 3\right)$$



$$\text{length}_{AB} = \sqrt{(-3-6)^2 + [7-(-1)]^2}$$

$$= \sqrt{(-9)^2 + (8)^2}$$

$$= \sqrt{145}$$

$$\simeq 12.04 \text{ units}$$

Example 3: Calculate the length and midpoint of the line segment joining the endpoints $E\left(-\frac{5}{8}, \frac{1}{8}\right)$ and $F(4, \frac{3}{4})$.

$$\text{Midpoint}_{EF} = \left(\frac{-\frac{5}{8} + \frac{32}{8}}{2}, \frac{\frac{1}{8} + \frac{6}{8}}{2}\right)$$

$$= \left(\frac{\frac{27}{8}}{2}, \frac{\frac{7}{8}}{2}\right)$$

$$= \left(\frac{27}{16}, \frac{7}{16}\right)$$

$$\text{length}_{EF} = \sqrt{\left(-\frac{5}{8} - \frac{32}{8}\right)^2 + \left(\frac{1}{8} - \frac{6}{8}\right)^2}$$

$$= \sqrt{\left(-\frac{37}{8}\right)^2 + \left(-\frac{5}{8}\right)^2}$$

$$= \sqrt{\frac{697}{32}}$$

$$\simeq 4.67 \text{ units}$$

Example 4: If line segment AB has point $A(5,7)$ and a midpoint at $(4,8)$, what are the coordinates of point B?

x_1 y_1

$$\text{midpoint}_{AB} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$(4,8) = \left(\frac{5+x_2}{2}, \frac{7+y_2}{2}\right)$$

$$4 = \frac{5+x_2}{2} \qquad 8 = \frac{7+y_2}{2}$$

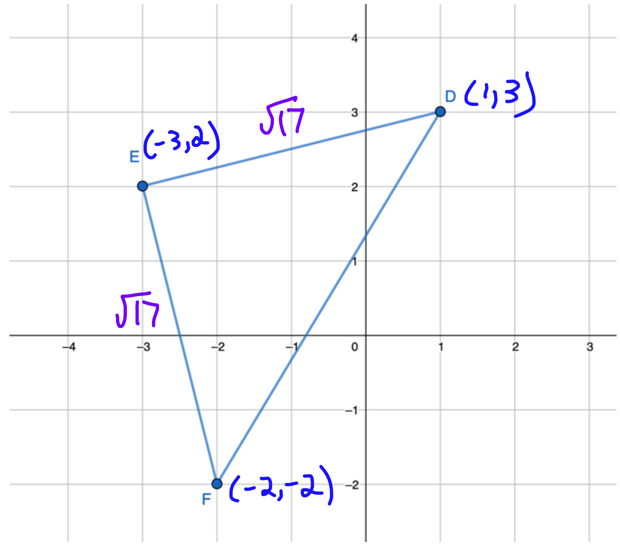
$$8 = 5 + x_2 \qquad 16 = 7 + y_2$$

$$x_2 = 3 \qquad y_2 = 9$$

$$\boxed{\therefore B(3,9)}$$

Example 5: Triangle DEF has vertices $D(1,3)$, $E(-3,2)$, and $F(-2,-2)$.

a) Classify the triangle by side length



$\text{length}_{EF} = \sqrt{[-2-(-3)]^2 + (-2-2)^2} = \sqrt{17}$

$\text{length}_{DE} = \sqrt{(-3-1)^2 + (2-3)^2} = \sqrt{17}$

$\text{length}_{DF} = \sqrt{(-2-1)^2 + (-2-3)^2} = \sqrt{34}$

$\therefore \triangle DEF$ is isosceles.

b) Determine the perimeter of the triangle rounded to the nearest tenth.

$\text{Perimeter} = \sqrt{17} + \sqrt{17} + \sqrt{34}$

$\simeq 14.1$ units

c) Is it a right-angle triangle? Give proof.

If it's a right triangle, $a^2 + b^2 = c^2$

Check:

$$\sqrt{17}^2 + \sqrt{17}^2 \stackrel{?}{=} \sqrt{34}^2$$

$$17 + 17 \stackrel{?}{=} 34$$

$$34 = 34$$

✓

$\therefore$ it is a right triangle.