

### L3 – Equation of a Circle

MPM2D

Jensen

Unit 2

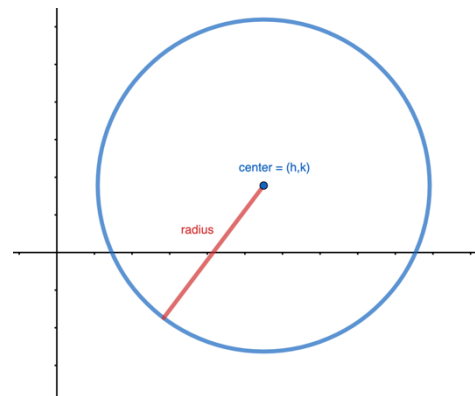
A **circle** is the set of all points that are the same distance from a fixed point, the center.

The **radius** is the distance from the center of the circle to any point on the circle.

#### Equation of any circle:

The equation of a circle is defined based on the location of its center  $(h, k)$  and length of its radius,  $r$ .

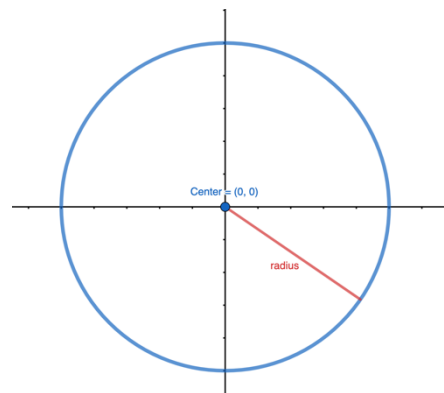
$$(x - h)^2 + (y - k)^2 = r^2$$



#### Equation of a circle with center at ORIGIN

If the radius of the circle is at the origin  $(0,0)$ , the equation simplifies.

$$x^2 + y^2 = r^2$$



**Example 1:** Write the equation of a circle with center  $(0,0)$  and a radius of

a) 3

$$x^2 + y^2 = 3^2$$

$$x^2 + y^2 = 9$$

b)  $\frac{1}{2}$

$$x^2 + y^2 = \left(\frac{1}{2}\right)^2$$

$$x^2 + y^2 = \frac{1}{4}$$

**Example 2:** What is the radius of a circle defined by the equation  $x^2 + y^2 = 36$

$$r^2 = 36$$

$$r = \sqrt{36}$$

$$r = 6 \text{ units}$$

**Example 3:** A circle has a center at the origin and passes through the point  $(5,3)$ . Determine the equation of the circle.

$$x^2 + y^2 = r^2$$

$$5^2 + 3^2 = r^2$$

$$r^2 = 34$$

$$\boxed{x^2 + y^2 = 34}$$

**Example 4:** Is the point  $(-5,9)$  inside, outside, or on the circle  $x^2 + y^2 = 100$

$$(-5)^2 + (9)^2 \stackrel{?}{=} 100$$

$$25 + 81 \stackrel{?}{=} 100$$

$$106 > 100$$

∴  $(-5,9)$  is **OUTSIDE** the circle.

**Tip:**

If point  $(x,y)$  is **ON** the circle  $\rightarrow x^2 + y^2 = r^2$

If point  $(x,y)$  is **OUTSIDE** the circle  $\rightarrow x^2 + y^2 > r^2$

If point  $(x,y)$  is **INSIDE** the circle  $\rightarrow x^2 + y^2 < r^2$

**Example 5:** Determine the equation of a circle with center at  $(3,4)$  and a radius of 8.

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-3)^2 + (y-4)^2 = 8^2$$

$$\boxed{(x-3)^2 + (y-4)^2 = 64}$$

**Example 6:** Determine the shortest distance from the point (10,7) to the edge of the circle  $x^2 + y^2 = 49$

Distance from origin to (10,7):

$$D = \sqrt{(10-0)^2 + (7-0)^2}$$

$$D = \sqrt{149}$$

Radius of circle :

$$r^2 = 49$$

$$r = \sqrt{49}$$

$$r = 7$$

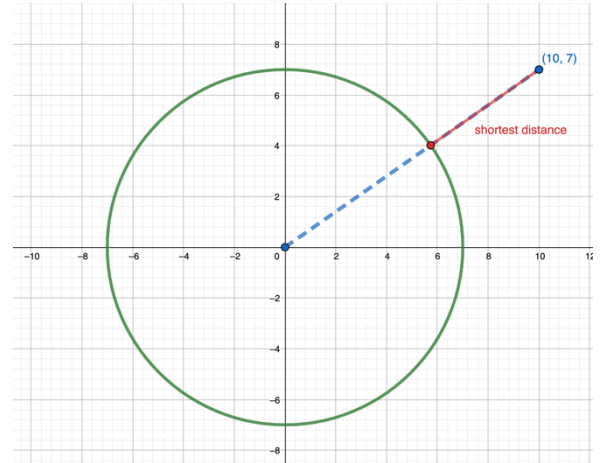
Distance from point to circle:

$$D = \sqrt{149} - 7$$

$$D \approx 5.21 \text{ units}$$

**Tip:**

The shortest distance is going to be in the direction of a line that goes through the center of the circle.



[geogebra link](#)