L4 – Geometric Properties of Shapes Unit 2
MPM2D
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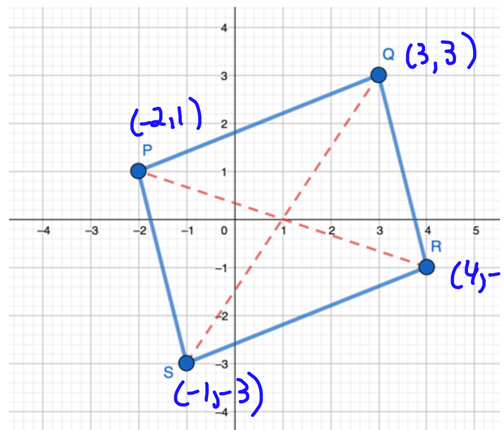
Formulas we will need:

Midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Length $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Equation of a circle: $x^2+y^2=r^2$

Example 1: Verify that the diagonals of the parallelogram with vertices $P(-2,1), Q(3,3), R(4,-1)$, and $S(-1,-3)$ bisect each other.



we can verify they bisect each other but showing they share the same midpoint.

$\text{midpoint}_{PR} = \left(\frac{-2+4}{2}, \frac{1+(-1)}{2}\right)$
$= (1,0)$

$\text{midpoint}_{SQ} = \left(\frac{-1+3}{2}, \frac{-3+3}{2}\right)$
$= (1,0)$

∴ the diagonals intersect at their midpoints (bisect each other)

Example 2: The vertices of a triangle are $A(-3,6)$, $B(1,-6)$ and $C(5,2)$. If M is the midpoint of AB and N is the midpoint of AC, verify that

a) MN is parallel to BC

$\text{mid}_{AB} = M = \left(\frac{-3+1}{2}, \frac{6+(-6)}{2}\right) = \overset{x_1\ \ y_1}{(-1,0)}$

$\text{mid}_{AC} = N = \left(\frac{-3+5}{2}, \frac{6+2}{2}\right) = \overset{x_2\ \ y_2}{(1,4)}$

$\text{slope}_{MN} = \frac{\overset{y_2}{4} - \overset{y_1}{0}}{\underset{x_2}{1} - \underset{x_1}{(-1)}} = \frac{4}{2} = 2$

$\text{slope}_{BC} = \frac{2-(-6)}{5-1} = \frac{8}{4} = 2$

∴ $MN \parallel BC$

b) MN is half the length of BC

$\text{length}_{MN} = \sqrt{[\overset{x_2}{1} - \overset{x_1}{(-1)}]^2 + (\overset{y_2}{4} - \overset{y_1}{0})^2}$
$= \sqrt{(2)^2+(4)^2}$
$= \sqrt{20}$

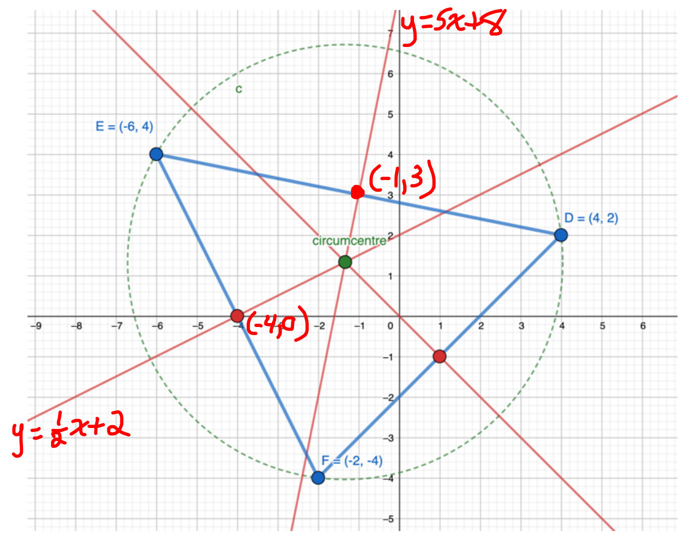
$\text{length}_{BC} = \sqrt{(5-1)^2 + [2-(-6)]^2}$
$= \sqrt{(4)^2+(8)^2}$
$= \sqrt{80}$

$\frac{\text{length}_{BC}}{\text{length}_{MN}} = \frac{\sqrt{80}}{\sqrt{20}} = 2$

∴ MN is half the length of BC

Example 3: ΔDEF has vertices $D(4,2)$, $E(-6,4)$, and $F(-2,-4)$. Determine the coordinates of the circumcentre of ΔDEF. The circumcentre is the point of intersection of the right bisectors of the sides of a triangle.

https://www.geogebra.org/calculator/brabwjsq



Right Bisector of EF

$$\text{Midpoint}_{EF} = \left(\frac{-6+(-2)}{2}, \frac{4+(-4)}{2}\right) = (-4,0)$$

$$\text{slope}_{EF} = \frac{-4-4}{-2-(-6)} = \frac{-8}{4} = -2$$

slope perpendicular to EF $= \frac{1}{2}$

y-int:

$y = mx+b$

$0 = (\frac{1}{2})(-4)+b$

$0 = -2+b$

$b = 2$

Eqn: $y = \frac{1}{2}x+2$

Right Bisector of ED

$$\text{midpoint}_{ED} = \left(\frac{-6+4}{2}, \frac{4+2}{2}\right) = (-1,3)$$

$$\text{slope}_{ED} = \frac{2-4}{4-(-6)} = \frac{-2}{10} = -\frac{1}{5}$$

slope perpendicular to ED $= 5$

y-int: $y = mx+b$

$3 = 5(-1)+b$

$3 = -5+b$

$b = 8$

Eqn: $y = 5x+8$

Intersection of Right Bisectors

① $y = \frac{1}{2}x+2$

$y = \frac{1}{2}(\frac{-4}{3})+2$

$y = \frac{-4}{6}+2$

$y = \frac{-2}{3}+\frac{6}{3}$

$y = \frac{4}{3}$

② $y = 5x+8$

$2(\frac{1}{2}x+2) = (5x+8)2$

$x+4 = 10x+16$

$4-16 = 10x-x$

$-12 = 9x$

$x = -\frac{12}{9}$

$x = -\frac{4}{3}$

So the circumcentre is at $(\frac{-4}{3}, \frac{4}{3})$

Example 4: The equation of a circle with centre $O(0,0)$ is $x^2 + y^2 = 25$. The points $A(-3,4)$ and $B(5,0)$ are the endpoints of chord AB. Verify that the centre of the circle lies on the right bisector of chord AB.

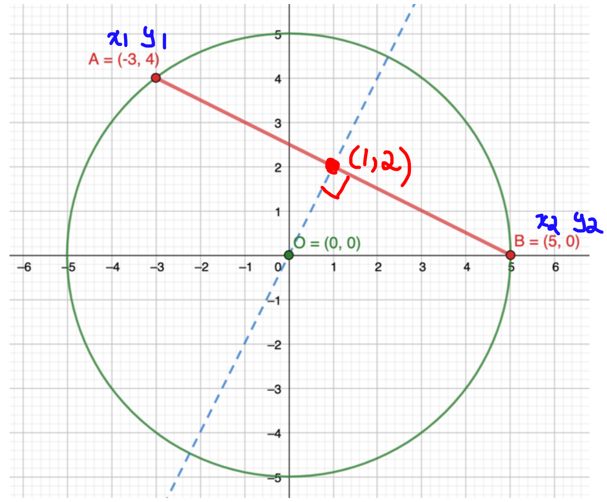
$\text{midpoint}_{AB} = \left(\frac{\overset{x_1}{-3}+\overset{x_2}{5}}{2}, \frac{\overset{y_1}{4}+\overset{y_2}{0}}{2}\right) = (1,2)$

$\text{slope}_{AB} = \frac{0-4}{5-(-3)} = \frac{-4}{8} = \frac{-1}{2}$

slope perpendicular to $AB = 2$

y-int: $y = mx+b$
$2 = 2(1)+b$
$2 = 2+b$
$b = 0$

Eqⁿ: $y = 2x$



Verify $(\overset{x}{0}, \overset{y}{0})$ is on the right bisector $y = 2x$

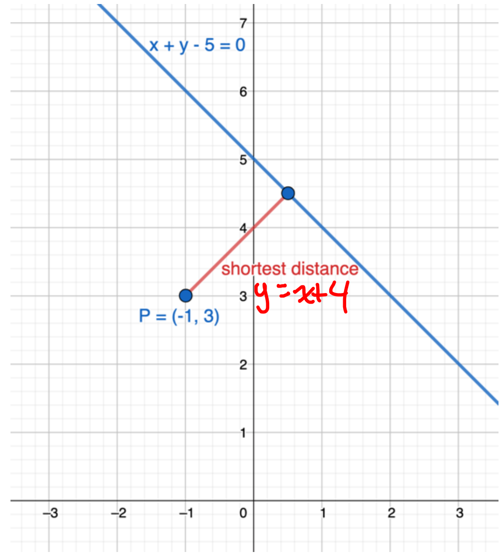
LS	RS
$= y$	$= 2x$
$= 0$	$= 2(0)$
	$= 0$

∴ the center of the circle lies on the right bisector of the chord.

Example 5: Find the distance from the point $P(-1,3)$ to the line $x + y - 5 = 0$, to the nearest tenth of a unit.

Steps to find shortest distance from a point to a line:

1) Write an equation for the line that is perpendicular to the given line and intersects the point given
2) Find the point of intersection of the perpendicular line with the given line
3) Find the distance between the POI and the given point.



1) slope of $x+y-5=0$:

$x+y-5=0$

$y=-x+5$

slope $=-1$

$\perp$ slope $= 1$

Eq^n of line along path of shortest distance:

$y=mx+b$

$3=1(-1)+b$

$3=-1+b$

$b=4$

$y=x+4$

2) POI of lines

① $x+y-5=0$

$x+(x+4)-5=0$

$2x-1=0$

$x=\frac{1}{2}$

② $y=x+4$

$y=\frac{1}{2}+4$

$y=\frac{1}{2}+\frac{8}{2}$

$y=\frac{9}{2}$

POI is $\left(\frac{1}{2},\frac{9}{2}\right)$

③ Distance from $(-1,3)$ to $\left(\frac{1}{2},\frac{9}{2}\right)$

$D=\sqrt{\left[\frac{1}{2}-(-1)\right]^2+\left(\frac{9}{2}-3\right)^2}$

$D=\sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{3}{2}\right)^2}$

$D=\sqrt{\frac{9}{2}}$

$\boxed{D \simeq 2.1 \text{ units}}$