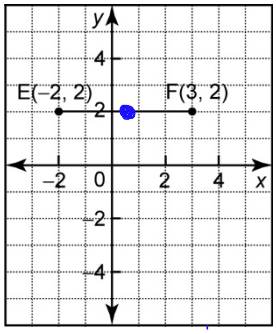
W1 – Midpoint and Length of a Line Segment Unit 2

MPM2D

Jensen

1) Determine the coordinates of the midpoint of each line segment.

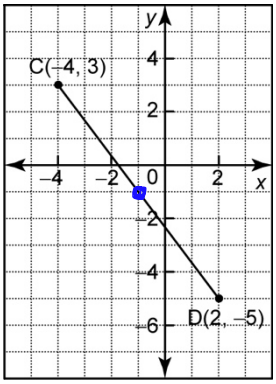
a)



$mid_{EF} = \left(\frac{-2+3}{2}, \frac{2+2}{2}\right)$

$= \left(\frac{1}{2}, 2\right)$

b)



$mid_{CD} = \left(\frac{-4+2}{2}, \frac{3+(-5)}{2}\right)$

$= (-1, -1)$

2) Determine the midpoint of the line segment defined by each pair of endpoints.

a) $J(5,7)$ and $K(3,9)$

$mid_{JK} = \left(\frac{5+3}{2}, \frac{7+9}{2}\right)$

$= (4, 8)$

b) $L(-1,0)$ and $M(1,-6)$

$mid_{LM} = \left(\frac{-1+1}{2}, \frac{0+(-6)}{2}\right)$

$= (0, -3)$

c) $A(5,9)$ and $B(-1,9)$

$mid_{AB} = \left(\frac{5+(-1)}{2}, \frac{9+9}{2}\right)$

$= (2, 9)$

d) $C(-7,8)$ and $D(-2,-9)$

$mid_{CD} = \left(\frac{-7+(-2)}{2}, \frac{8+(-9)}{2}\right)$

$= \left(\frac{-9}{2}, \frac{-1}{2}\right)$

e) $E(\frac{-1}{9}, \frac{-1}{2})$ and $F(\frac{14}{9}, \frac{4}{3})$

$$\text{mid}_{EF} = \left(\frac{\frac{-1}{9}+\frac{14}{9}}{2}, \frac{\frac{-1}{2}+\frac{4}{3}}{2}\right)$$
$$= \left(\frac{\frac{13}{9}}{2}, \frac{\frac{-3}{6}+\frac{8}{6}}{2}\right)$$
$$= \left(\frac{13}{18}, \frac{\frac{5}{6}}{2}\right)$$
$$= \left(\frac{13}{18}, \frac{5}{12}\right)$$

f) $A\left(\frac{5}{3}, 1\right)$ and $B(0,2)$

$$\text{mid}_{AB} = \left(\frac{\frac{5}{3}+0}{2}, \frac{1+2}{2}\right)$$
$$= \left(\frac{5}{6}, \frac{3}{2}\right)$$

g) $G\left(\frac{-3}{2}, \frac{-1}{3}\right)$ and $H\left(\frac{3}{4}, \frac{3}{5}\right)$

$$\text{mid}_{GH} = \left(\frac{\frac{-3}{2}+\frac{3}{4}}{2}, \frac{\frac{-1}{3}+\frac{3}{5}}{2}\right)$$
$$= \left(\frac{\frac{-6}{4}+\frac{3}{4}}{2}, \frac{\frac{-5}{15}+\frac{9}{15}}{2}\right)$$
$$= \left(\frac{\frac{-3}{4}}{2}, \frac{\frac{4}{15}}{2}\right)$$
$$= \left(\frac{-3}{8}, \frac{4}{30}\right)$$
$$= \left(\frac{-3}{8}, \frac{2}{15}\right)$$

h) $M(6.6, 8.52)$ and $N(-5.5, 4.07)$

$$\text{mid}_{MN} = \left(\frac{6.6+(-5.5)}{2}, \frac{8.52+4.07}{2}\right)$$
$$= \left(\frac{1.1}{2}, \frac{12.59}{2}\right)$$
$$= (0.55, 6.295)$$

3) The endpoints of the diameter of a circle are $A(-5, -3)$ and $B(3, 7)$. Find the coordinates of the center of this circle.

$$\text{center} = \text{mid}_{AB} = \left(\frac{-5+3}{2}, \frac{-3+7}{2}\right)$$
$$= \left(\frac{-2}{2}, \frac{4}{2}\right)$$
$$= (-1, 2)$$

4) One endpoint of a diameter of a circle centered at the origin is $(-5, 2)$. Find the coordinates of the other endpoint of this diameter.

endpoint 1 : $(-5,2)$
endpoint 2 : (x,y)
midpoint : $(0,0)$

$$(0,0) = \left(\frac{-5+x}{2}, \frac{2+y}{2}\right)$$

$0 = \frac{-5+x}{2}$ $\qquad$ $0 = \frac{2+y}{2}$

$0 = -5+x$ $\qquad$ $0 = 2+y$

$x = 5$ $\qquad$ $y = -2$

The other endpoint is $(5, -2)$

5) For a line segment DE, one endpoint is $D(6,5)$, and the midpoint is $M(4,2)$. Find the coordinates of endpoint E.

$$(4,2) = \left(\frac{6+x}{2}, \frac{5+y}{2}\right)$$

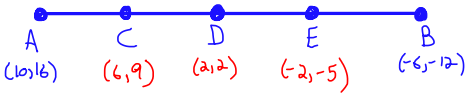
$4 = \frac{6+x}{2}$ $\qquad$ $2 = \frac{5+y}{2}$

$8 = 6+x$ $\qquad$ $4 = 5+y$

$x = 2$ $\qquad$ $y = -1$

The other endpoint is $E(2,-1)$

6) The endpoints of AB are $A(10,16)$ and $B(-6,-12)$. Find the coordinates of the points that divide the segment into four equal parts.



$D = \text{mid}_{AB} = \left(\frac{10+(-6)}{2}, \frac{16+(-12)}{2}\right) = (2,2)$

$C = \text{mid}_{AD} = \left(\frac{10+2}{2}, \frac{16+2}{2}\right) = (6,9)$

$E = \text{mid}_{DB} = \left(\frac{2+(-6)}{2}, \frac{2+(-12)}{2}\right) = (-2,-5)$

The points are $(6,9)$, $(2,2)$, and $(-2,-5)$.

7) The endpoints of PQ are $P(3,-4)$ and $Q(11,c)$. The midpoint of PQ is $M(d,3)$. Find the values of c and d.

$$(d,3) = \left(\frac{3+11}{2}, \frac{-4+c}{2}\right)$$

$d = \frac{3+11}{2}$ $\qquad$ $3 = \frac{-4+c}{2}$

$d = 7$ $\qquad$ $6 = -4+c$

$c = 10$

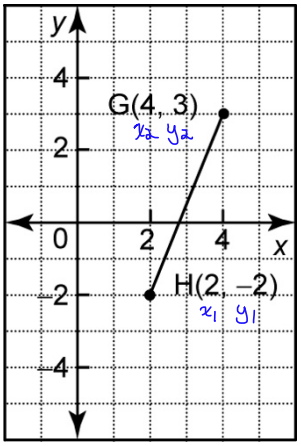
8) Find the exact length of the line segment.

$\text{length}_{HG} = \sqrt{(4-2)^2 + [3-(-2)]^2}$

$= \sqrt{(2)^2 + (5)^2}$

$= \sqrt{4+25}$

$= \sqrt{29}$



9) Calculate the exact length of the line segment defined by each pair of endpoints.

a) A(-6, -2) and B(4, 3)

$\text{length}_{AB} = \sqrt{[4-(-6)]^2 + [3-(-2)]^2}$

$= \sqrt{(10)^2 + (5)^2}$

$= \sqrt{125}$

$= \sqrt{25} \times \sqrt{5}$

$= 5\sqrt{5}$

b) C(-2, 0) and D(7, -3)

$\text{length}_{CD} = \sqrt{[7-(-2)]^2 + (-3-0)^2}$

$= \sqrt{(9)^2 + (-3)^2}$

$= \sqrt{90}$

$= \sqrt{9} \times \sqrt{10}$

$= 3\sqrt{10}$

c) E(-5, -6) and F(-1, -2)

$\text{length}_{EF} = \sqrt{[-1-(-5)]^2 + [-2-(-6)]^2}$

$= \sqrt{(4)^2 + (4)^2}$

$= \sqrt{32}$

$= \sqrt{16} \times \sqrt{2}$

$= 4\sqrt{2}$

d) G(0, 5) and H(8, -1)

$\text{length}_{GH} = \sqrt{(8-0)^2 + (-1-5)^2}$

$= \sqrt{(8)^2 + (-6)^2}$

$= \sqrt{100}$

$= 10$

e) (–5, 6) and (3, –2)

$\text{length} = \sqrt{[3-(-5)]^2 + (-2-6)^2}$

$= \sqrt{(8)^2 + (-8)^2}$

$= \sqrt{128}$

$= \sqrt{64} \times \sqrt{2}$

$= 8\sqrt{2}$

f) $\left(-\frac{3}{4}, -\frac{2}{5}\right)$ and $\left(\frac{1}{4}, \frac{3}{5}\right)$

$\text{length} = \sqrt{\left[\frac{1}{4} - \left(-\frac{3}{4}\right)\right]^2 + \left[\frac{3}{5} - \left(-\frac{2}{5}\right)\right]^2}$

$= \sqrt{(1)^2 + (1)^2}$

$= \sqrt{2}$

10) On a street map of his town, Jordan's house has coordinates (8,1). The town's two high schools are at (0,5) and (6,11). Which school is closer to Jordan's house?

distance to school @ (0,5)

$\text{length} = \sqrt{(0-8)^2+(5-1)^2}$

$= \sqrt{(-8)^2+(4)^2}$

$= \sqrt{80}$

≈ 8.94 units

distance to school @ (6,11)

$\text{length} = \sqrt{(6-8)^2+(11-1)^2}$

$= \sqrt{(-2)^2+(10)^2}$

$= \sqrt{104}$

≈ 10.2 units

∴ the school @ (0,5) is closer

11) The vertices of ΔABC are $A(2,5)$, $B(-6,-1)$ and $C(10,-1)$.

a) Determine the length of each side of this triangle.

$\text{length}_{AB} = \sqrt{(-6-2)^2+(-1-5)^2} = \sqrt{(-8)^2+(-6)^2} = \sqrt{100} = 10$

$\text{length}_{BC} = \sqrt{[10-(-6)]^2+[-1-(-1)]^2} = \sqrt{(16)^2+(0)^2} = \sqrt{256} = 16$

$\text{length}_{AC} = \sqrt{(10-2)^2+(-1-5)^2} = \sqrt{(8)^2+(-6)^2} = \sqrt{100} = 10$

b) What is the perimeter of the triangle?

Perimeter $= 10+16+10$

$= 36$ units

c) Classify the triangle.

2 equal side lengths; ∴ an isosceles triangle.

12) A circle has a diameter with endpoints $R(-4, 6)$ and $T(10, -8)$.

a) Find the length of this diameter exactly.

$$\text{length}_{RT} = \sqrt{[10-(-4)]^2 + (-8-6)^2}$$
$$= \sqrt{(14)^2 + (-14)^2}$$
$$= \sqrt{392}$$
$$= \sqrt{196} \times \sqrt{2}$$
$$= 14\sqrt{2} \text{ units}$$

b) Find the length of the radius of this circle. Round to the nearest tenth.

$$\text{Radius} = \frac{\text{diameter}}{2} = \frac{14\sqrt{2}}{2} = 7\sqrt{2} \approx 9.9 \text{ units}$$

Answers

1)a) $\left(\frac{1}{2}, 2\right)$ **b)** $(-1, -1)$

2)a) $(4,8)$ **b)** $(0, -3)$ **c)** $(2,9)$ **d)** $\left(-\frac{9}{2}, -\frac{1}{2}\right)$ **e)** $\left(\frac{13}{18}, \frac{5}{12}\right)$ **f)** $\left(\frac{5}{6}, \frac{3}{2}\right)$ **g)** $\left(\frac{-3}{8}, \frac{2}{15}\right)$ **h)** $(0.549, 6.295)$

3) $(-1, 2)$

4) $(5, -2)$

5) $(2, -1)$

6) $(6,9), (2,2), (-2, -5)$

7) $c = 10, d = 7$

8) $\sqrt{29}$

9)a) $\sqrt{125} = 5\sqrt{5}$ **b)** $\sqrt{90} = 3\sqrt{10}$ **c)** $\sqrt{32} = 4\sqrt{2}$ **d)** 10 **e)** $\sqrt{128} = 8\sqrt{2}$ **f)** $\sqrt{2}$

10) The school at $(0,5)$ is closer to Jordan's house.

11)a) $AB = AC = 10, BC = 16$ **b)** 36 units **c)** isosceles

12)a) $\sqrt{392} = 14\sqrt{2}$ **b)** $7\sqrt{2}$