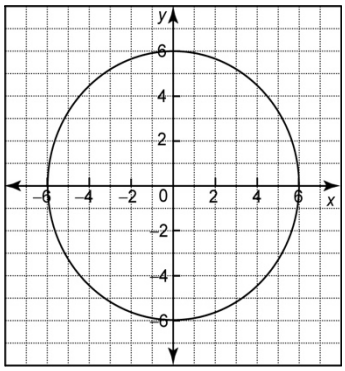
W3 – Equation of a Circle Unit 2
MPM2D
Jensen

1) Determine the equation of each circle.

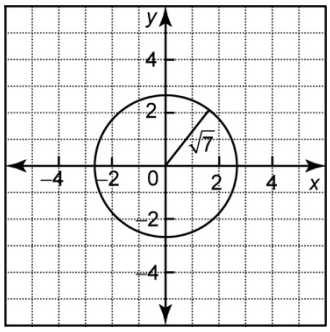
a)



$x^2 + y^2 = 6^2$

$x^2 + y^2 = 36$

b)



$x^2 + y^2 = (\sqrt{7})^2$

$x^2 + y^2 = 7$

2) State the radius of each of the following circles.

a) $x^2 + y^2 = 49$

$r^2 = 49$

$r = \sqrt{49}$

$r = 7$

b) $x^2 + y^2 = 16$

$r^2 = 16$

$r = \sqrt{16}$

$r = 4$

c) $x^2 + y^2 = 64$

$r^2 = 64$

$r = \sqrt{64}$

$r = 8$

d) $x^2 + y^2 = 1.44$

$r^2 = 1.44$

$r = \sqrt{1.44}$

$r = 1.2$

3) Find an equation for the circle centred at the origin that passes through each point.

a) (3, −4)

$x^2 + y^2 = r^2$

$(3)^2 + (-4)^2 = r^2$

$9 + 16 = r^2$

$r^2 = 25$

$x^2 + y^2 = 25$

b) (−5, 2)

$x^2 + y^2 = r^2$

$(-5)^2 + (2)^2 = r^2$

$25 + 4 = r^2$

$r^2 = 29$

$x^2 + y^2 = 29$

4) Determine whether each point is on, inside, or outside the circle defined by $x^2 + y^2 = 26$.

a) (1, 3)

$(1)^2+(3)^2 \stackrel{?}{=} 26$

$10 < 26$

(1,3) is inside the circle.

b) (−4, 6)

$(-4)^2+(6)^2 \stackrel{?}{=} 26$

$52 > 26$

(-4,6) is outside the circle

c) (1, 5)

$(1)^2+(5)^2 \stackrel{?}{=} 26$

$26 = 26$

(1,5) is on the circle

5) The point $A(4, b)$ lies on the circle defined by $x^2 + y^2 = 25$.

a) Find the possible value(s) of b.

$4^2+b^2=25$

$16+b^2=25$

$b^2=25-16$

$b^2=9$

$b=\pm\sqrt{9}$

$b=\pm 3$

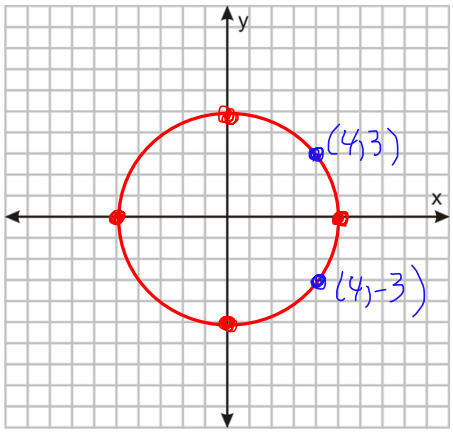
(4,3) and (4,-3) are on the circle.

b) Use a graph to show that the point(s) corresponding to the possible value(s) of b are on the circle.

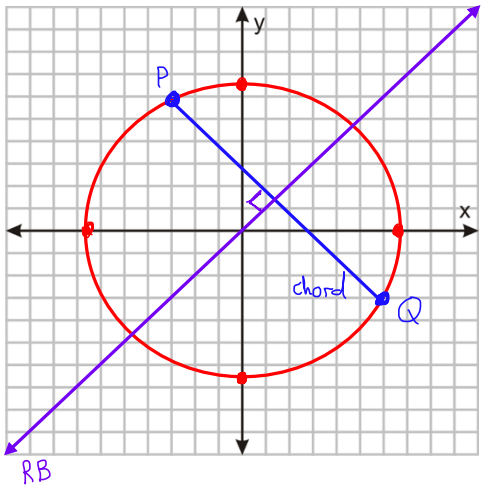
$r^2=25$

$r=\sqrt{25}$

$r=5$



6)a) Graph the circle defined by $x^2 + y^2 = 45$.



b) Verify algebraically that the line segment joining $P(-3, 6)$ and $Q(6, -3)$ is a chord of this circle. (In other words, verify that P and Q are points on the circle)

a) $r^2 = 45$

$r \simeq 6.7$

b) $P(-3,6)$

$x^2+y^2 \stackrel{?}{=} 45$

$(-3)^2+(6)^2 \stackrel{?}{=} 45$

$9+36 \stackrel{?}{=} 45$

$45 = 45$

Point P is on the circle.

$Q(6,-3)$

$x^2+y^2 \stackrel{?}{=} 45$

$(6)^2+(-3)^2 \stackrel{?}{=} 45$

$36+9 \stackrel{?}{=} 45$

$45 = 45$

Point Q is on the circle.

∴ PQ is a chord of the circle.

c) Find an equation in the form $y = mx + b$ for the right bisector of chord PQ.

$\text{slope}_{PQ} = \frac{-3-6}{6-(-3)} = \frac{-9}{9} = -1$

slope of right bisector $= 1$

$\text{mid}_{PQ} = \left(\frac{6+(-3)}{2}, \frac{-3+6}{2}\right) = \left(\frac{3}{2}, \frac{3}{2}\right)$

Eqⁿ: $y = mx + b$

$\frac{3}{2} = 1\left(\frac{3}{2}\right) + b$

$b = 0$

$\boxed{y = x}$

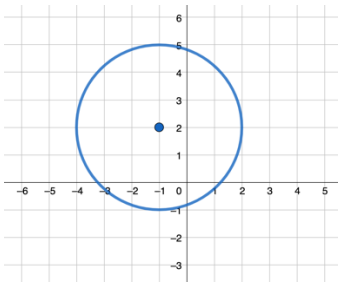
7) Determine an equation for each of the following circles.

a) centered at $(4,3)$ with a radius of 5

$(x-4)^2+(y-3)^2 = 5^2$

$\boxed{(x-4)^2+(y-3)^2 = 25}$

b)



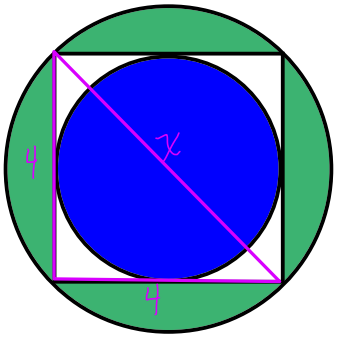
center: $(-1,2)$

radius $= 3$

$(x+1)^2+(y-2)^2 = 3^2$

$\boxed{(x+1)^2+(y-2)^2 = 9}$

8) An equation for the small circle in this diagram is $x^2 + y^2 = 4$. Determine the equation for the larger circle.



radius small circle $= \sqrt{4} = 2$

diameter small circle $= 2(2) = 4$

diameter of large circle: $x^2 = 4^2 + 4^2$

$x^2 = 32$

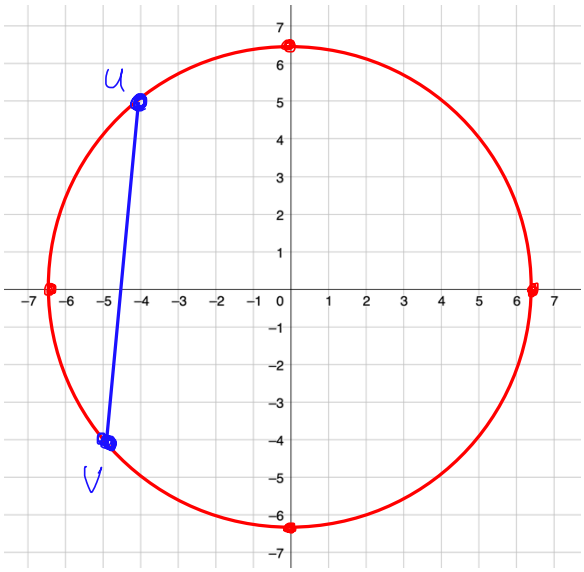
$x = \sqrt{32}$

$x = 4\sqrt{2}$

radius of large circle $= \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

Eqn: $x^2 + y^2 = (2\sqrt{2})^2 \longrightarrow \boxed{x^2 + y^2 = 8}$

9)a) Graph the circle defined by $x^2 + y^2 = 41$.



b) Verify algebraically that the line segment joining $U(-4,5)$ and $V(-5,-4)$ is a chord of this circle.

a) $r^2 = 41$

$r = \sqrt{41}$

$r \approx 6.4$

b) $U(-4,5)$

$(-4)^2 + (5)^2 \stackrel{?}{=} 41$

$41 = 41$

$V(-5,-4)$

$(-5)^2 + (-4)^2 \stackrel{?}{=} 41$

$41 = 41$

U and V are on the circle; $\therefore$ UV is a chord of the circle.

c) Determine an equation for the line that passes through the origin and is perpendicular to the chord UV.

$\text{slope}_{UV} = \frac{-4-5}{-5-(-4)} = \frac{-9}{-1} = 9$

$\therefore m = -\frac{1}{9}$

Eqn: $y = mx + b$

$0 = -\frac{1}{9}(0) + b$

$b = 0$

$\boxed{y = -\frac{1}{9}x}$

d) Verify that this line passes through the midpoint of the chord.

$\text{mid}_{UV} = \left(\frac{-4+(-5)}{2}, \frac{5+(-4)}{2}\right) = \left(-\frac{9}{2}, \frac{1}{2}\right)$

Check if $\left(-\frac{9}{2}, \frac{1}{2}\right)$ is on $y = -\frac{1}{9}x$:

LS

$= y$

$= \frac{1}{2}$

RS

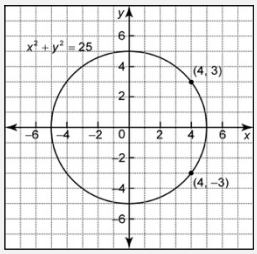
$= -\frac{1}{9}x$

$= -\frac{1}{9}\left(-\frac{9}{2}\right)$

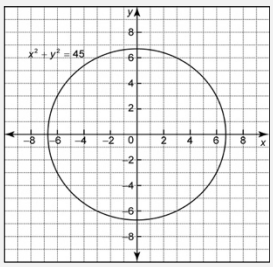
$= \frac{1}{2}$

Answers

1)a) $x^2 + y^2 = 36$ **b)** $x^2 + y^2 = 7$
2)a) 7 **b)** 4 **c)** 8 **d)** 1.2
3)a) $x^2 + y^2 = 25$ **b)** $x^2 + y^2 = 29$
4a) inside **b)** outside **c)** on
5)a) $(3, -3)$ **b)**



6)a)

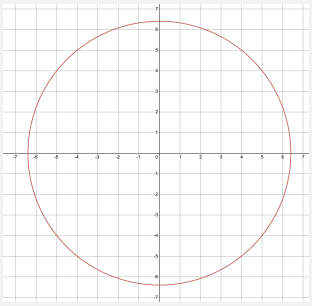


b) see solution for steps **c)** $y = x$

7)a) $(x - 4)^2 + (x - 3)^2 = 25$ **b)** $(x + 1)^2 + (y - 2)^2 = 9$

8) $x^2 + y^2 = 8$

9a)



b) see solutions **c)** $y = -\frac{1}{9}x$ **d)** The line passes through the midpoint $\left(-\frac{9}{2}, \frac{1}{2}\right)$