W4 – Geometric Properties of Shapes Unit 2

MPM2D

Jensen

1) A triangle has vertices $C(1, 4)$, $D(-2, 2)$, and $E(3, 1)$. Determine if ΔCDE is a right triangle.

$slope_{CD} = \frac{2-4}{-2-1} = \frac{-2}{-3} = \frac{2}{3}$

$slope_{DE} = \frac{1-2}{3-(-2)} = \frac{-1}{5}$

$slope_{CE} = \frac{1-4}{3-1} = \frac{-3}{2}$

$slope_{CD} \times slope_{CE}$

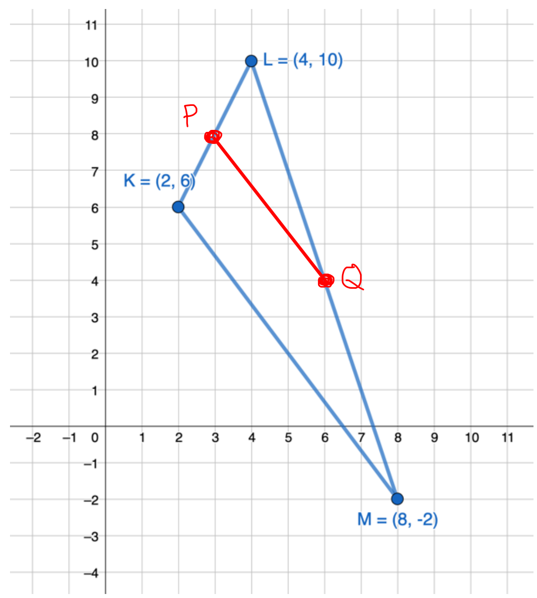
$= \left(\frac{2}{3}\right)\left(\frac{-3}{2}\right)$

$= -1$

∴ CD and CE are perpendicular.

There is a right angle at C.

2) The vertices of a triangle are $K(2,6)$, $L(4,10)$, and $M(8,-2)$. Let P be the midpoint of KL and Q be the midpoint of LM. Verify that…



a) PQ is parallel to KM

$P = mid_{KL} = \left(\frac{2+4}{2}, \frac{6+10}{2}\right) = (3, 8)$

$Q = mid_{LM} = \left(\frac{4+8}{2}, \frac{10+(-2)}{2}\right) = (6, 4)$

$slope_{PQ} = \frac{4-8}{6-3} = \frac{-4}{3}$

$slope_{KM} = \frac{-2-6}{8-2} = \frac{-8}{6} = \frac{-4}{3}$

∴ PQ and KM are parallel

b) PQ is half the length of KM

$length_{PQ} = \sqrt{(6-3)^2 + (4-8)^2} = \sqrt{25} = 5$

$length_{KM} = \sqrt{(8-2)^2 + (-2-6)^2} = \sqrt{100} = 10$

$length_{PQ} = \frac{1}{2} length_{KM}$

3) The equation of a circle with center $O(0,0)$ is $x^2 + y^2 = 10$. The points $C(3,1)$ and $D(1,-3)$ are the endpoints of chord CD. Verify that the center of the circles lies on the right bisector of chord CD.

$$\text{mid}_{CD} = \left(\frac{3+1}{2}, \frac{1+(-3)}{2}\right) = (2,-1)$$

$$\text{slope}_{CD} = \frac{-3-1}{1-3} = \frac{-4}{-2} = 2$$

slope of right bisector $= -\frac{1}{2}$

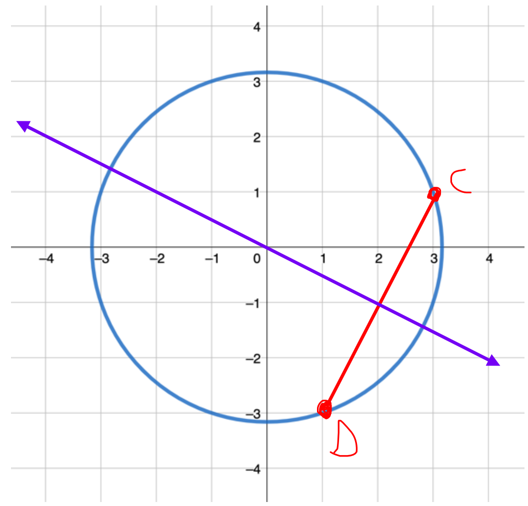
Eq^n: $y = mx + b$

$$-1 = \left(\tfrac{-1}{2}\right)(2) + b$$

$$-1 = -1 + b$$

$$b = 0$$

$$\boxed{y = -\tfrac{1}{2}x}$$



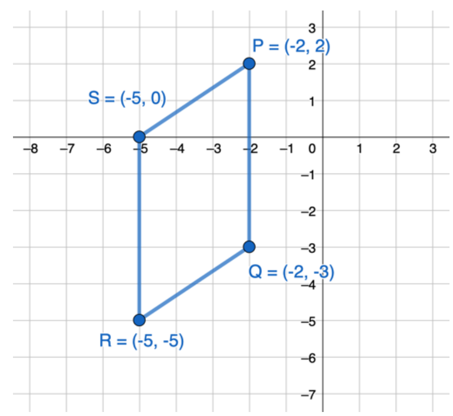
4) Verify that the quadrilateral with vertices $P(-2,2)$, $Q(-2,-3)$, $R(-5,-5)$, and $S(-5,0)$ is a parallelogram.

$$\text{slope}_{SP} = \frac{2-0}{-2-(-5)} = \frac{2}{3}$$

$$\text{slope}_{RS} = \frac{0-(-5)}{-5-(-5)} = \frac{5}{0} = \text{undefined}$$

$$\text{slope}_{RQ} = \frac{-3-(-5)}{-2-(-5)} = \frac{2}{3}$$

$$\text{slope}_{QP} = \frac{2-(-3)}{-2-(-2)} = \frac{5}{0} = \text{unefined}$$



$\boxed{\text{opposite sides are parallel; } \therefore \text{ PQRS is a parallelogram.}}$

5) A triangle has vertices of $K(-2,2)$, $L(1,5)$, and $M(3,-3)$. Verify that…

a) the triangle has a right angle.

$$\text{slope}_{KL} = \frac{5-2}{1-(-2)} = \frac{3}{3} = 1$$

$$\text{slope}_{LM} = \frac{-3-5}{3-1} = \frac{-8}{2} = -4$$

$$\text{slope}_{KM} = \frac{-3-2}{3-(-2)} = \frac{-5}{5} = -1$$

$\text{slope}_{KL} \times \text{slope}_{KM}$

$= (1)(-1)$

$= -1$

$\therefore$ KL and KM are perpendicular

$\therefore$ right angle at K.

b) the midpoint of the hypotenuse is the same distance from each vertex.

$$N = \text{mid}_{LM} = \left(\frac{1+3}{2}, \frac{5+(-3)}{2}\right) = (2,1)$$

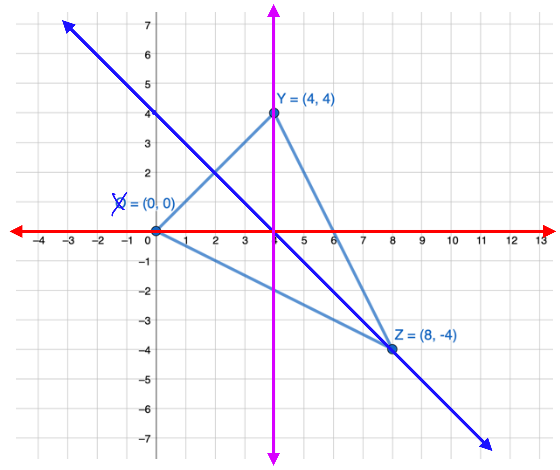
$$\text{length}_{KN} = \sqrt{[2-(-2)]^2 + (1-2)^2} = \sqrt{17}$$

$$\text{length}_{LN} = \sqrt{(2-1)^2 + (1-5)^2} = \sqrt{17}$$

$$\text{length}_{MN} = \sqrt{(2-3)^2 + [1-(-3)]^2} = \sqrt{17}$$

6) A triangle has vertices $X(0,0), Y(4,4)$, and $Z(8,-4)$

a) Write the equation for each of the three medians.



Median XY

$mid_{XY} = \left(\frac{0+4}{2}, \frac{0+4}{2}\right) = (2,2)$

slope of median $= \frac{-4-2}{8-2} = \frac{-6}{6} = -1$

Eqⁿ: $y = mx+b$
$2 = -1(2) + b$
$b = 4$

$y = -x+4$

Median YZ

$mid_{YZ} = \left(\frac{4+8}{2}, \frac{4+(-4)}{2}\right) = (6,0)$

slope of median $= \frac{0-0}{6-0} = \frac{0}{6} = 0$

Eqⁿ: $y = mx+b$
$0 = 0(6)+b$
$b = 0$

$y = 0$

Median XZ

$mid_{XZ} = \left(\frac{0+8}{2}, \frac{0+(-4)}{2}\right) = (4,-2)$

slope of median $= \frac{-2-4}{4-4} = \frac{-6}{0} =$ undefined

Eqⁿ: $x = 4$

b) The centroid of a triangle is the point of intersection of the medians of the triangle. Verify algebraically that the centroid of ΔXYZ is at $(4,0)$.

median of XY: $y = -x+4$
median of YZ: $y = 0$

solve using substitution:

$0 = -x+4$
$x = 4$

POI is $(4,0)$

7) The endpoints of the diameter of a circle are $M(-3,5)$ and $N(9,7)$. Determine…

a) the coordinates of the center of the circle.

center $= mid_{MN} = \left(\frac{-3+9}{2}, \frac{5+7}{2}\right) = (3,6)$

b) the length of the radius

$r =$ length from center to M $= \sqrt{(-3-3)^2 + (5-6)^2}$

$= \sqrt{37}$

≈ 6.08 units

8) Determine whether the triangle with vertices $A(-3,4)$, $B(-1,-2)$, and $C(3,2)$ is isosceles.

$$\text{length}_{AB} = \sqrt{[-1-(-3)]^2 + (-2-4)^2} = \sqrt{40} = 2\sqrt{10}$$

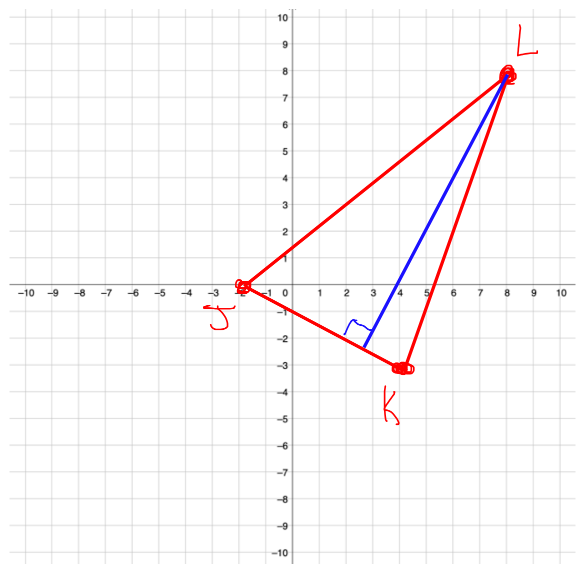
$$\text{length}_{BC} = \sqrt{[3-(-1)]^2 + [2-(-2)]^2} = \sqrt{32} = 4\sqrt{2}$$

$$\text{length}_{AC} = \sqrt{[3-(-3)]^2 + (2-4)^2} = \sqrt{40} = 2\sqrt{10}$$

Yes it is isosceles.

9) A triangle has vertices $J(-2,0)$, $K(4,-3)$, and $L(8,8)$.

a) Find an equation for the altitude from vertex L.



$$\text{slope}_{JK} = \frac{-3-0}{4-(-2)} = \frac{-3}{6} = \frac{-1}{2}$$

slope of altitude $= 2$

Eq^n: $y = mx + b$

$8 = 2(8) + b$

$8 = 16 + b$

$b = -8$

$$\boxed{y = 2x - 8}$$

b) Find the length of the altitude.

Eq^n of line containing JK:

$y = mx + b$

$0 = \left(\frac{-1}{2}\right)(-2) + b$

$0 = 1 + b$

$b = -1$

$y = \frac{-1}{2}x - 1$

① $y = \frac{-1}{2}x - 1$

$2x - 8 = \frac{-1}{2}x - 1$

$\frac{4}{2}x + \frac{1}{2}x = -1 + 8$

$\frac{5}{2}x = 7$

$5x = 14$

$x = \frac{14}{5}$

② $y = 2x - 8$

$y = 2\left(\frac{14}{5}\right) - 8$

$y = \frac{28}{5} - \frac{40}{5}$

$y = \frac{-12}{5}$

Length from $(8,8)$ to $\left(\frac{14}{5}, \frac{-12}{5}\right)$

$= \sqrt{\left(\frac{14}{5} - 8\right)^2 + \left(-\frac{12}{5} - 8\right)^2}$

$= \sqrt{\frac{676}{5}}$

≈ 11.63

c) Find the area of ΔJKL

$$\text{length } JK = \sqrt{[4-(-2)]^2 + (-3-0)^2}$$

$= \sqrt{45}$

$= 3\sqrt{5}$

$A = \frac{1}{2}bh$

$= \frac{1}{2}\sqrt{45}\sqrt{\frac{676}{5}}$

$= \frac{1}{2}(3)(\cancel{\sqrt{5}})\left(\frac{26}{\cancel{\sqrt{5}}}\right) = 39 \text{ units}^2$

10) ΔAOB has vertices $A(4,4)$, $O(0,0)$, and $B(8,0)$. Determine the coordinates of the circumcenter of ΔAOB.

Right Bisector of AO

$mid_{AO} = \left(\frac{4+0}{2}, \frac{4+0}{2}\right) = (2,2)$

$slope_{AO} = \frac{0-4}{0-4} = \frac{-4}{-4} = 1$

slope of RB $= -1$

Eq^n: $y = mx+b$

$2 = -1(2)+b$

$b = 4$

① $\boxed{y = -x+4}$

Right Bisector of OB

$mid_{OB} = \left(\frac{0+8}{2}, \frac{0+0}{2}\right) = (4,0)$

$slope_{OB} = \frac{0-0}{8-0} = 0$

slope of RB = undefined

② $\boxed{\text{Eq}^n\text{: } x = 4}$

Right Bisector of AB

$mid_{AB} = \left(\frac{4+8}{2}, \frac{4+0}{2}\right) = (6,2)$

$slope_{AB} = \frac{0-4}{8-4} = \frac{-4}{4} = -1$

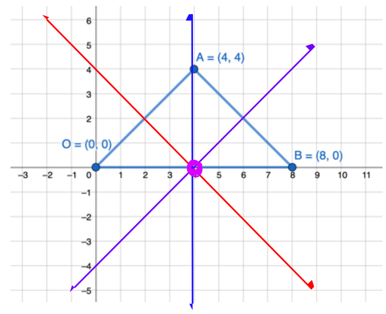
slope of RB $= 1$

Eq^n: $y = mx+b$

$2 = 1(6)+b$

$b = -4$

③ $\boxed{y = x-4}$



Solve for intersection of RB's

① $y = -x+4$

② $y = x-4$

$x-4 = -x+4$

$2x = 8$

$x = 4$

$y = 4-4$

$y = 0$

$(4,0)$

$\boxed{\therefore \text{ The circumcenter is } (4,0)}$

11) Find the exact distance from the point $D(4,-2)$ to the line segment joining the points $E(1,3)$ and $F(-4,-2)$.

Eq^n of EF

$slope_{EF} = \frac{3-(-2)}{1-(-4)} = \frac{5}{5} = 1$

$y = mx+b$

$3 = 1(1)+b$

$b = 2$

① $\boxed{y = x+2}$

Eq^n of line from D to EF

slope $= -1$

$y = mx+b$

$-2 = -1(4)+b$

$b = 2$

② $\boxed{y = -x+2}$

PoI of Lines

① $y = x+2$

② $y = -x+2$

$-x+2 = x+2$

$2-2 = x+x$

$0 = 2x$

$x = 0$

$y = -(0)+2$

$y = 2$

$\boxed{\text{PoI is } (0,2)}$

Distance from $D(4,-2)$ to $(0,2)$

$length = \sqrt{(0-4)^2 + [2-(-2)]^2}$

$= \sqrt{32}$

$= 4\sqrt{2}$

$\simeq 5.66$ units

$\boxed{\text{The shortest distance is } 5.66 \text{ units}}$